THE LOW FREQUENCY PROPERTIES OF A SEMICONDUCTING PLASMA SITUATED IN A CONSTANT ELECTRIC FIELD

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Submitted to JETP editor May 31, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 102–111 (January, 1966)

Some properties of electromagnetic wave propagation associated with the appearance of a negative differential conductivity are considered. Dispersion relations are obtained for transverse and longitudinal electromagnetic and electroacoustic waves in a semiconductor situated in constant electric and magnetic fields. It is shown that amplification or generation of waves as a result of change of sign of the conductivity is possible in a certain frequency range; the increments of amplitude increase of these waves are found.

As is well known, nonlinear effects begin to manifest themselves noticeably in semiconductors even for relatively small electric field strengths. These effects lead to a series of interesting phenomena: the appearance of a negative differential conductivity, the interaction of electromagnetic waves of different frequencies, etc. In this connection it is of definite interest to investigate the electromagnetic properties of a semiconducting plasma situated in a constant electric field. The present communication examines certain properties of electromagnetic wave propagation at low frequencies ($\omega \ll \nu$, where ν is the characteristic electron-collision frequency), associated with electron heating.

1. STATEMENT OF THE PROBLEM. INITIAL EQUATIONS

To solve the problem of electromagnetic wave propagation in a medium one must first of all determine the dielectric tensor of the medium, and then obtain dispersion relationships by using Maxwell's equations. Several authors[1, 2] have considered the low-frequency properties of a plasma in a constant electric field. These however did not take into account the possible appearance of a negative differential conductivity, which leads in many cases to instability of electromagnetic oscillations in a semiconductor. We confine ourselves to semiconductors in which inter-electron collisions play a substantial role. The kinetic equation for the electron distribution function $F(\mathbf{p}, \mathbf{r}, t)$ is written as

$$\frac{\partial F}{\partial t} + \mathbf{v}\nabla_{\mathbf{r}} F + e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right\}\nabla_{\mathbf{p}} F = \mathrm{St}\,\{F\} + \mathrm{St}_{ee}\{F\}. \qquad (1)^*$$

Here the following symbols have been introduced: $\mathbf{v}$ is the electron velocity, $\mathbf{E}$ the electric field, $\mathbf{H}$ the magnetic field, $\mathbf{p}$ the quasimomentum of the electrons, $\mathrm{St}\{F\}$ the collision integral of electrons with phonons and impurities, $\mathrm{St}_{ee}\{F\}$ the inter-electron collision integral. The dispersion law for electrons is assumed to be quadratic, $\epsilon = p^2/2m$ (ϵ is the electron energy).

We shall seek a solution of (1) in the form

$$F(\mathbf{p}, \mathbf{r}, t) = F_0(\varepsilon, \mathbf{r}, t) + (\mathbf{p}/p, \mathbf{F}_1(\varepsilon, \mathbf{r}, t)). \qquad (2)$$

As usual, we consider the second term on the right hand side of (2) to be much smaller than the first. If the electron concentration is large enough ($n \gg E^2 l/e^2$,[3, 4] where l is the mean free path of the electrons in their interaction with phonons, impurities, etc.), then only inter-electron collisions[1)] are significant in (1) to a first approximation, i.e.,

$$\mathrm{St}_{ee}\{F_0\} = 0,$$

whence we obtain, following [3, 5]

$$F_0(\varepsilon, \mathbf{r}, t) = \frac{n(\mathbf{r}, t)}{(2\pi m\theta(\mathbf{r}, t))^{3/2}} \exp\left[-\frac{\varepsilon}{\theta(\mathbf{r}, t)}\right]; \qquad (3)$$

*$[\mathbf{v}\mathbf{H}] \equiv \mathbf{v} \times \mathbf{H}$.

1)The condition imposed on the frequencies and wave vectors is given below.

$n(\mathbf{r}, t)$ and $\theta(\mathbf{r}, t)$ are found from the equations of continuity and energy balance (see [5]):

$$e\frac{\partial n}{\partial t} + \operatorname{div}\mathbf{j} = 0, \quad \frac{3}{2}\frac{\partial}{\partial t}n\theta + \operatorname{div}\mathbf{Q} - \mathbf{jE} = \mathcal{P}(n, \theta). \quad (4)$$

Here

$$\mathbf{j} = \frac{8\pi e m}{3}\int_0^\infty \varepsilon \mathbf{F}_1(\varepsilon, \mathbf{r}, t)\, d\varepsilon$$

is the electric current,

$$\mathbf{Q} = \frac{8\pi m}{3}\int_0^\infty \varepsilon^2 \mathbf{F}_1(\varepsilon, \mathbf{r}, t)\, d\varepsilon$$

is the heat flux, and

$$\mathcal{P}(n, \theta) = \int_0^\infty \varepsilon^{3/2}\,\mathrm{St}\,\{F_0(\varepsilon, \mathbf{r}, t)\}\, d\varepsilon$$

is the amount of heat transferred to the lattice by the electron gas.

We use the usual expressions for the collision integral[3]

$$\mathrm{St}\{F_0\} = -\frac{1}{v^3}\frac{\partial}{\partial \varepsilon}\left\{v^3 A(\varepsilon)\left[\frac{\partial F_0}{\partial \varepsilon} + \frac{F_0}{T}\right]\right\},$$

$$\mathrm{St}\left\{\frac{\mathbf{p}}{p}\mathbf{F}_1\right\} = -\frac{1}{\tau(\varepsilon)}\frac{\mathbf{p}}{p}\mathbf{F}_1 \quad (5)$$

where $A(\epsilon)$ is the diffusion coefficient in energy space, associated with the relaxation time for energies, T is the lattice temperature, $\tau(\epsilon)$ is the relaxation time for momenta. $\tau(\epsilon)$ and $A(\epsilon)$ may be represented in the following form:[6]

$$\tau(\varepsilon) = \tau(T)(\varepsilon/T)^q, \quad A(\varepsilon) = A(T)(\varepsilon/T)^r.$$

The numbers q and r characterize the type of scattering (phonons, impurities, etc.).

The kinetic equation for determining $\mathbf{F}_1$ is written as:[2)]

$$\frac{\partial \mathbf{F}_1}{\partial t} + \frac{e}{mc}[\mathbf{HF}_1] + \frac{1}{\tau(\varepsilon)}\mathbf{F}_1 = -\mathbf{v}\nabla_{\mathbf{r}}F_0 - e\mathbf{E}\frac{\partial F_0}{\partial \mathbf{p}}. \quad (6)$$

The distribution functions of the temperature, concentration, etc. may be obtained similarly.

We shall assume that the variable fields, which we shall designated by primed letters, are small in comparison with the constant fields, designated by zero indices:

$$\mathbf{E} = \mathbf{E}^0 + \mathbf{E}', \quad \mathbf{H} = \mathbf{H}^0 + \mathbf{H}',$$
$$|\mathbf{E}'| \ll |\mathbf{E}^0|, \quad |\mathbf{H}'| \ll \mathbf{H}^0.$$

2)The term in this equation associated directly with collisions between electrons does not play a significant part since the interaction between electrons changes neither the average electron energy directly, nor the current strength (see[3]).

We then obtain the following: Θ_0, the temperature in the zeroth approximation, is found from the equation (see also [6])

$$\mathbf{j}_0\mathbf{E}^0 = \sigma_{ik}{}^0 E_i{}^0 E_k{}^0 = \frac{2n_0}{\sqrt{\pi}\,\Theta_0^{3/2}}\left(1 - \frac{T}{\Theta_0}\right)\int_0^\infty A(\varepsilon)\,\varepsilon^{1/2} e^{-\varepsilon/\Theta_0}\, d\varepsilon, \quad (7)$$

where the components of the electric conductivity tensor are determined by the relation

$$\sigma_0\mathbf{E}^0 = \frac{8\pi\sqrt{2m}\,e^2}{3\Theta_0}\int_0^\infty d\varepsilon\,\frac{\varepsilon^{3/2}F_0{}^0(\varepsilon)\tau(\varepsilon)}{1 + \omega_H{}^2\tau^2(\varepsilon)}\left\{\mathbf{E}^0 - \frac{e\tau}{mc}[\mathbf{H}^0\mathbf{E}^0] + \frac{e^2\tau^2}{m^2c^2}\mathbf{H}^0(\mathbf{H}^0\mathbf{E}^0)\right\}, \quad \omega_H{}^2 = e^2H_0{}^2/m^2c^2. \quad (8)$$

Expanding (3) in a series in powers of n′ and Θ′, we obtain for F_0'

$$F_0' = F_0(\varepsilon)\left[\frac{n'(\mathbf{r}, t)}{n_0} + \frac{\Theta'(\mathbf{r}, t)}{\Theta_0}\left(\frac{\varepsilon}{\Theta_0} - \frac{3}{2}\right)\right]; \quad (9)$$

Θ′ and n′ satisfy the linearized system (4), and j′ and $\mathbf{Q}'$ are expressed in terms of $\mathbf{F}_1'$ with the help of the formulas given above. $\mathbf{F}_1'$ satisfies the equation obtained by linearizing Eq. (6).

If we assume that all primed quantities can be written in the form

$$\Psi'(\mathbf{r}, t) = \iint_{-\infty}^{\infty} \Psi'(\omega, \mathbf{k})\, e^{i(\mathbf{kr} - \omega t)}\, d\mathbf{k}\, d\omega,$$

then we may easily obtain for $\mathbf{F}_1'(\omega, \mathbf{k})$

$$\mathbf{F}_1' = \frac{\tau^*(\varepsilon)\sqrt{2\varepsilon/m}}{1 + \omega_H{}^2\tau^{*2}}F_0{}^0(\varepsilon)\left\{\mathbf{A} - \frac{e\tau^*}{mc}[\mathbf{H}^0\mathbf{A}] + \frac{e^2\tau^{*2}}{m^2c^2}\mathbf{H}_0(\mathbf{H}^0\mathbf{A})\right\}, \quad (10)$$

where $\tau^*(\epsilon) = \tau(\epsilon)/[1 - i\omega\tau(\epsilon)]$ is the effective relaxation time,

$$\mathbf{A} = \frac{e\mathbf{E}'}{\Theta_0} + \left\{\frac{e\mathbf{E}^0}{\Theta_0}\left[\frac{n'(\omega, \mathbf{k})}{n_0} + \frac{\Theta'(\omega, \mathbf{k})}{\Theta_0}\left(\frac{\varepsilon}{\Theta_0} - \frac{5}{2}\right)\right] + \frac{e}{\omega\Theta_0}[\mathbf{k}(\mathbf{v}_0(\varepsilon)\mathbf{E}') - \mathbf{E}'(\mathbf{k}\mathbf{v}_0(\varepsilon))]\right\} - i\mathbf{k}\left[\frac{n'(\omega, \mathbf{k})}{n_0} + \frac{\Theta'(\omega, \mathbf{k})}{\Theta_0}\left(\frac{\varepsilon}{\Theta_0} - \frac{3}{2}\right)\right],$$

$$\mathbf{v}_0(\varepsilon) = \frac{e\tau(\varepsilon)}{m(1 + \omega_H{}^2\tau^2)}\left\{\mathbf{E}^0 - \frac{e\tau}{mc}[\mathbf{H}^0\mathbf{E}^0] + \frac{e^2\tau^2}{m^2c^2}\mathbf{H}^0(\mathbf{H}^0\mathbf{E}^0)\right\}. \quad (11)$$

Formulas (2)–(11) are applicable if

$$v_T \gg v_0, \quad |\omega + i\nu| \gg k v_T,$$

where $v_T = \sqrt{2\Theta_0/m}$ is the average thermal velocity of electrons.[3)]

The first term in expression (11) describes the current created directly by the changing field. If we set $\mathbf{E}^0 = 0$ and $\mathbf{k} = 0$ we obtain an expression for the conductivity of $\sigma_{ik}(\omega)$, valid for arbitrary relations between ω and $1/\tau(\epsilon)$. (In what follows, the case for $\omega\tau(\epsilon) \ll 1$ is considered.) The second term is associated with the drift of electrons in a constant electric field, and finally the third corresponds to the diffusion and thermal diffusion current. Expressing the concentration $n'(\omega, \mathbf{k})$ and temperature $\Theta'(\omega, \mathbf{k})$ from Eqs. (8)–(9) in terms of the field $\mathbf{E}'(\omega, \mathbf{k})$, we determine the effective conductivity tensor $\sigma_{ik}(\omega, \mathbf{E}^0, \mathbf{k})$.

The expressions for $\sigma_{ik}(\omega, \mathbf{E}^0, \mathbf{k})$ are unwieldy in general form. We shall thus confine ourselves to considering individual cases.

2. PROPERTIES OF A PLASMA IN THE ABSENCE OF A CONSTANT MAGNETIC FIELD

If there is no magnetic field present, then σ^0 is a scalar and the equation of balance (7) has the form for $T \ll \Theta_0$

$$\sigma^0 E^{02} = \frac{2A(T)n_0}{T\sqrt{\pi}}\left(\frac{\Theta_0}{T}\right)^r \Gamma\left(r+\frac{3}{2}\right) \tag{12}$$

where $\sigma = e^2 n_0/m\nu(\Theta_0)$ and

$$\nu(\Theta_0) = \nu_0(T)\frac{3\sqrt{\pi}}{4\Gamma(5/2+q)}\left(\frac{T}{\Theta_0}\right)^q$$

is the effective electron collision frequency.

We shall consider the case of weak spatial dispersion when $eE^0/\Theta_0 \gg k$. This inequality can be re-written in the form

$$\delta\nu \gg kv_0, \quad \delta = v_0^2/v_T^2,$$

where $v_0 = eE^0/m\nu(\Theta)$ is the drift velocity. In addition to this we assume that

$$\omega \ll |r-q|\sigma^0 E^{02}/n_0\Theta_0 \quad (\omega \ll |r-q|\delta\nu).$$

In this approximation the conductivity tensor is equal to

$$\sigma_{ik} = \sigma^0\left\{\left(1-\beta\frac{\mathbf{kv}_0}{\omega}\right)\delta_{ik} + \beta\frac{k_i v_{0k}}{\omega} + \frac{1}{(r-q)(\omega-\mathbf{kv}_0)}\right.$$
$$\left.\times\left\{2\omega q\frac{E_i^0 E_i^0}{E^{02}} + (r-q)\frac{v_{0i}}{\omega}[k_k(\omega-\beta\mathbf{kv}_0) + \beta k^2 v_{0k}]\right\}\right\},$$
$$\beta = {}^3/_4\sqrt{\pi}\,\Gamma(5/2+2q)/\Gamma^2(5/2+q). \tag{13}$$

If $q = 0$, then formula (13) goes over into the well known expression obtained in [2].

From the dispersion equation

$$\left| k^2\delta_{ik} - k_i k_k - \frac{\omega^2}{c^2}\varepsilon_{ik}\right| = 0, \quad \varepsilon_{ik} = \varepsilon\delta_{ik} + \frac{4\pi i}{\omega}\sigma_{ik} \tag{14}$$

(ϵ is the dielectric constant of the medium) we obtain

$$k^2 - \frac{\omega^2}{c^2}\left[\varepsilon + \frac{4\pi i\sigma_0}{\omega^2}(\omega - \beta\mathbf{kv}_0)\right] = 0, \tag{15}$$

$$k^2\left[\varepsilon + \frac{4\pi i}{\omega - \mathbf{kv}_0}(\sigma_0\sin^2\alpha + \sigma_{\text{dif}}\cos^2\alpha)\right]$$
$$-\frac{\omega^2}{c^2}\left[\varepsilon + \frac{4\pi i\sigma_0(\omega-\beta\mathbf{kv}_0)}{\omega^2}\right]\left[\varepsilon + \frac{4\pi i\sigma_{\text{dif}}}{\omega - \mathbf{kv}_0}\right] = 0, \tag{16}$$

where $\sigma_{\text{dif}} = \sigma_0(r+q)/(r-q)$ is the differential conductivity, and α is the angle between $\mathbf{k}$ and $\mathbf{v}_0$.

Equation (16) describes oscillations polarized in the $(\mathbf{k}, \mathbf{E}^0)$ plane and Eq. (15) characterizes transverse oscillations in the plane perpendicular to $(\mathbf{k}, \mathbf{E}^0)$. For $\alpha = 0$ Eq. (16) breaks up into the dispersion relation for longitudinal oscillations

$$\varepsilon + \frac{4\pi i\sigma_{\text{dif}}}{\omega - \mathbf{kv}_0} = 0, \quad \omega' = \mathbf{kv}, \quad \omega'' = -\frac{4\pi\sigma_{\text{dif}}}{\varepsilon} \tag{17a}$$

($\omega = \omega' + i\omega''$) and the equation for transverse oscillations (see (15))

$$\omega' = \beta kv_0, \quad \omega'' = -k^2c^2/4\pi\sigma_0 \quad (k^2 \gg \omega^2\varepsilon/c^2). \tag{17b}$$

For

$$kv_0 \gg 4\pi\sigma_0|(r+q)/(r-q)|$$

the longitudinal oscillations are weakly damped if

$$(r+q)/(r-q) > 0.$$

As is well known, negative differential conductivity arises for $r + q > 0$, and $r - q < 0$.[6]

The appearance of a negative differential conductivity has the following physical significance. The effective electron collision frequency increases as the electric field becomes stronger, which results in the decrease of the electron drift velocity and consequently the current decreases, i.e., $dj_0/dE^0 < 0$. (It is assumed that the concentration is independent of the field.) We then have growing oscillations. This means that it is possible to have amplification or generation of longitudinal waves propagating a constant field in a semiconductor situated in a constant electric field. If $k \sim 2\pi n/a$, where a is the length of the sample, n is an integer, then a spectrum of frequencies is generated in the system. The upper limiting frequency ω is here determined by the condition $\omega_{gr} \lesssim |r-q|\delta\nu$; the lower frequency is unre-

[3)]The limits of applicability of the expressions obtained are discussed in detail in[1-3,5].

stricted. The transverse waves are damped.

For $\alpha = \pi/2$ we obtain from formula (16)

$$k^2 = \frac{\omega^2}{c^2}\varepsilon + \frac{4\pi i\omega}{c^2}\sigma_0\frac{r+q}{r-q}, \qquad (18)$$

$$\omega\varepsilon + 4\pi i\sigma_0 = 0. \qquad (19)$$

Here longitudinal oscillations are strongly damped; transverse waves can propagate, weakly damped (or growing), on condition that

$$\frac{\omega\varepsilon}{4\pi\sigma_0} \gg \left|\frac{r+q}{r-q}\right|.$$

If

$$\left|\frac{r+q}{r-q}\right| \gg \frac{\omega}{4\pi\sigma_0},$$

then the field is strongly damped as is clear from (18). In this case the surface impedance of the semiconductor is of interest.

The impedance ζ for a field polarized in the $(\mathbf{k}, \mathbf{E}^0)$ plane, equals

$$\zeta = \left(\frac{\omega}{8\pi|\sigma_{\rm dif}|}\right)^{1/2}(1+i) = \frac{\omega}{ck}, \qquad (20)$$

for normal incidence of waves on the surface ($\mathbf{E}^0$ parallel to the boundary), on condition that $\sigma_{\rm dif} < 0$. That is to say, the amplitude of the field in the sample increases when $\operatorname{Re}\zeta > 0$.

The surface impedance for a field polarized perpendicular to $(\mathbf{k}, \mathbf{E}^0)$ is determined by the usual expression (normal skin effect). For wave propagation at an angle $(\alpha \neq \pi/2)$ to the electric field the dispersion equation has the form

$$\omega' = \beta k v_0, \qquad \omega'' = -\frac{k^2c^2(\sigma_0\sin^2\alpha + \sigma_{\rm dif}\cos^2\alpha)}{4\pi\sigma_0\sigma_{\rm dif}}, \qquad (21)$$

if the inequality

$$\varepsilon|\omega - \mathbf{k}\mathbf{v}_0| \ll 4\pi|\sigma_0\sin^2\alpha + \sigma_{\rm dif}\cos^2\alpha|$$

is fulfilled. In this case longitudinal and transverse waves intermingle and when

$$\sigma_{\rm dif} < 0, \qquad \sigma_0\sin^2\alpha > |\sigma_{\rm dif}|\cos^2\alpha$$

the amplitude of such a wave increases.

We note that negative differential conductivity can be used for sound amplification in piezo-semiconductors. It is well known that amplification of sound oscillations is possible in piezo-semiconductors if the carrier drift velocity exceeds the velocity of sound.[7] In this case the electronic conductivity changes sign. Negative differential conductivity also leads to a similar effect. For amplification it is of course necessary that in both cases the increment of oscillation growth caused by the electronic conductivity should exceed the damping decrement brought about by other dissipation processes.

Using Poisson's equations for a piezo-semiconductor in a constant electric field and the equation of elasticity, and assuming that the piezo-semiconductor has isotropic elastic properties, we obtain the dispersion equations for electro-acoustic oscillations:[7,8]

$$\left(\frac{\omega'}{k}\right)_1 = c_t + \frac{1}{2c_t}(F_1 - F_2)\operatorname{Re} v_\gamma^2,$$

$$\left(\frac{\omega''}{k}\right)_1 = \frac{1}{2c_t}(F_1 - F_2)\operatorname{Im} v_\gamma^2;$$

$$\left(\frac{\omega'}{k}\right)_2 = c_l + \frac{1}{2c_l}F_2\operatorname{Re} v_\gamma^2, \quad \left(\frac{\omega''}{k}\right)_2 = \frac{1}{2c_l}F_2\operatorname{Im} v_\gamma^2; \qquad (22)$$

here c_l and c_t are the longitudinal and transverse velocities of sound respectively:

$$F_1(\alpha, \varphi) = \sin^2\alpha(\cos^2\alpha + \sin^2\alpha\cos^2\varphi\sin^2\varphi),$$

$$F_2(\alpha, \varphi) = 9\sin^4\alpha\cos^2\alpha\sin^2\varphi\cos^2\varphi,$$

where α and φ are the polar and azimuthal angles of the $\mathbf{k}$ vector, with the polar axis directed along $\mathbf{E}^0$;

$$v_\gamma^2 = \frac{16\pi\gamma^2k^2}{\rho\varepsilon_{ik}k_ik_k} = \frac{16\pi\gamma^2(\omega_0' - \mathbf{k}\mathbf{v}_0)[(\omega_0' - \mathbf{k}\mathbf{v}_0)\varepsilon - 4\pi i(\sigma_0\sin^2\alpha + \sigma_{\rm dif}\cos^2\alpha)]}{\rho[\varepsilon^2(\omega_0' - \mathbf{k}\mathbf{v}_0)^2 + 16\pi^2(\sigma_0\sin^2\alpha + \sigma_{\rm dif}\cos^2\alpha)^2]}, \qquad (23)$$

where $\omega_0' = kc_t$, kc_l respectively for transverse and longitudinal waves, γ is a constant characteristic of the elastic properties of the sample, ρ is the mass density of the crystal. If we set $q = 0$ in (23) then expression (22) goes over into Hutson's well known formula for sound amplification.[7]

Increase of acoustic oscillations is possible for $\omega > kv_0$ if

$$\sigma_{\rm dif} < 0, \qquad |\sigma_{\rm dif}|\cos^2\alpha > \sigma_0\sin^2\alpha.$$

If diffusion and thermal diffusion effects are significant ($eE^0/\Theta_0 \ll k$—strong spatial dispersion), then the differential conductivity does not characterize energy dissipation processes. In this case both longitudinal ($\mathbf{k} \parallel \mathbf{E}^0$) and transverse ($\mathbf{k} \perp \mathbf{E}^0$) oscillations are strongly damped.

The damping of longitudinal oscillations ω'' is proportional to $\sigma_0 k^2 r_d^2$, where $r_d^2 = \Theta_0/4\pi e^2 n_0$ is the Debye screening radius. (A similar result is obtained in [9].) The damping of transverse oscillations is determined by the static conductivity:

$$\omega'' \sim c^2k^2/4\pi\sigma_0.$$

3. A SEMICONDUCTOR IN A STRONG MAGNETIC FIELD

Low-frequency weakly damped oscillations, namely helical waves ($\omega \ll \tau^{-1} \ll \omega_H$) with a dispersion law[5]

$$\omega' = \frac{k^2c^2}{\omega_0^2}\omega_H \cos\alpha, \quad \omega'' = -\frac{k^2c^2}{\omega_0^2}\nu; \quad \omega_0^2 = \frac{4\pi e^2 n_0}{m} \tag{24}$$

can exist in semiconductors or in a plasma in a strong magnetic field. Here α is the angle between the direction of the magnetic field and the wave vector.

Under certain conditions a constant electric field may lead to the amplification of helical waves. One of the conditions is that if the carrier drift velocity exceeds the phase velocity of the spiral wave, then as a result of the Cerenkov effect the amplitude of such a wave is amplified. Moreover, in the low-frequency region of the spectrum a state of affairs is possible where the differential conductivity plays the part of the real conductivity. Increase of field amplitude on account of the negative differential conductivity then takes place instead of damping.

Wave amplification for $v_0 > \omega/k$ is considered in the paper of Bok and Nozieres.[10]

We shall consider the possibility of helical wave amplification in the case when the semiconductor has a negative differential conductivity. We shall not take into account the effect of carrier drift on the conductivity, i.e., we assume that the frequency satisfies the condition $\delta\nu \gg \omega \gg kv_0$. Let

$$\mathbf{E}^0 \perp \mathbf{H}^0, \quad \mathbf{E}^0 = (0, E_y^0, 0), \quad \mathbf{H}^0 = (0, 0, H_z^0);$$

then

$$\sigma_{xx} = \sigma_{xx}^0 = \sigma_{zz}^0 \frac{\nu^2}{\omega_H^2}\,\frac{\Gamma(5/2 - q)}{\Gamma(5/2 + q)},$$

$$\sigma_{xy} = \sigma_{xy}^0 = -\sigma_{yx} = \sigma_{zz}^0 \frac{\nu}{\omega_H}\,\frac{\Gamma(5/2)}{\Gamma(5/2 + q)},$$

$$\sigma_{yy} = \sigma_{xx}^0 \frac{r - q}{r + q},$$

$$\sigma_{zz} = \sigma_{zz}^0 = \frac{4e^2 n_0 \Gamma(5/2 + q)}{3\sqrt{\pi}\, m v_0}\left(\frac{\Theta_0}{T}\right)^q. \tag{25}$$

As is clear from (25) the part of the true component of the conductivity σ_{yy} at a frequency $\omega \ll \delta\nu$ is played not by the static conductivity $\sigma_{yy} = \sigma_{xx}^0$, but by the differential conductivity.

$$\sigma_{yy} = \sigma_{xx}^0 \frac{r - q}{r + q}.$$

This circumstance can lead to amplification of helical waves if $r + q > 0$ and $r - q < 0$, since damping depends on the diagonal components of the conductivity tensor.

Substituting the value of σ_{ik} from (25) into (14), we obtain the dispersion equation for helical waves in the form

$$k^4\cos^2\alpha - \left(\frac{4\pi\omega}{c^2}\sigma_{xy}^0\right)^2 - \frac{4\pi i \sigma_{xx}^0 \omega k^2}{c^2}\left[\frac{r-q}{r+q} + \cos^2\alpha + \frac{\Gamma^2(5/2)}{\Gamma(5/2+q)\Gamma(5/2-q)}\sin^2\alpha\right] = 0, \tag{26}$$

whence it follows that

$$\omega' = \frac{k^2c^2\cos\alpha}{4\pi\sigma_{xy}^0},$$

$$\omega'' = -\frac{k^2c^2}{8\pi\sigma_{xy}^{02}}\sigma_{xx}^0\left[\frac{r-q}{r+q} + \cos^2\alpha + \frac{\Gamma^2(5/2)}{\Gamma(5/2+q)\Gamma(5/2-q)}\sin^2\alpha\right]. \tag{27}$$

If $r + q > 0$, $r - q < 0$, and

$$\left|\frac{r-q}{r+q}\right| > \cos^2\alpha + \frac{\Gamma^2(5/2)}{\Gamma(5/2+q)\Gamma(5/2-q)}\sin^2\alpha,$$

then the helical wave is amplified. This condition may be fulfilled if, for example, energy transfer takes place by scattering from optical phonons ($r = -1/2$), and momentum transfer by scattering from charged impurities ($q = 3/2$).[6] The increment of amplitude growth for such a wave with frequency ω' is equal to

$$\xi = \frac{\nu}{2|\omega_H\cos\alpha|}\left|\frac{r-q}{r+q} + \cos^2\alpha + \frac{\Gamma^2(5/2)}{\Gamma(5/2+q)\Gamma(5/2-q)}\sin^2\alpha\right| \ll 1. \tag{28}$$

In conclusion we shall examine the case of electromagnetic wave propagation transverse to the magnetic field ($\alpha = \pi/2$). Here, as is well known, the dispersion equation breaks up into two equations for the ordinary and extraordinary waves:

$$k^2 - \frac{\omega^2}{c^2}\varepsilon_{zz} = 0, \quad k^2\varepsilon_{yy} + \frac{\omega^2}{c^2}(\varepsilon_{xy}\varepsilon_{yx} - \varepsilon_{xx}\varepsilon_{yy}) = 0. \tag{29}$$

In this case weakly damped waves do not arise in the plasma. The surface impedance for normal wave incidence is of interest. If the vector $\mathbf{k}$ is directed along the constant electric field $\mathbf{E}^0$, then we obtain

$$\zeta^{(0)} = \left(\frac{\omega}{8\pi\sigma_{zz}^0}\right)^{1/2}(1 - i),$$

$$\zeta^{(H)} = \left(\frac{\omega}{8\pi\sigma_{\text{eff}}}\right)^{1/2}(1 + i) \quad \left(\zeta = \frac{\omega}{ck}\right),$$

$$\sigma_{\text{eff}} = \frac{\sigma_{xy}^{02}}{\sigma_{xx}^{0}} \left| \frac{r+q}{r-q} \right| \quad (r+q>0, \quad r-q<0). \quad (30)$$

The amplitude of the extraordinary wave increases within the interior of the medium, and the energy is released in the medium.

If the vector **k** vector is directed transverse to the field $\mathbf{E}^0$ the surface impedance for the extraordinary wave is determined from the equation

$$k^2 = \frac{4\pi i\omega}{c^2} \frac{\sigma_{xy}^{02}}{\sigma_{xx}^{0}},$$

and the wave amplitude does not increase.

The high frequency case when $\omega \gg kv_0$, ν is of definite interest. Under these assumptions the conductivity tensor and refractive index are determined by the usual formulas (see for example [5]), in which, however, the effective electron temperature Θ, which depends on the electric and magnetic fields as well as on the form of the boundary conditions, must be taken as the temperature. It is easy to see that in the case where $\omega \gg \nu$ the damping of electromagnetic waves κ is determined by the formula $\kappa = \kappa_0(T/\Theta_0)^q$, where κ_0 is the damping in the absence of an electric field.

The line width $\Delta\omega$ and electromagnetic wave damping for magnetoplasma and cyclotron resonances are written in the form

$$\Delta\omega = \Delta\omega_0 \left(\frac{T}{\Theta_0}\right)^q, \quad \varkappa = \varkappa_0 \left(\frac{\Theta_0}{T}\right)^{q/2}.$$

Here Θ_0 is a function of r, q, $\tau_0(T)$, and A(T), so that by measuring the dependence of line width for resonance or damping of electromagnetic waves one can obtain some idea of the nature of electron-phonon interaction.

Note added in proof (October 29, 1965). In the present paper the magnetic field of the current itself is not taken into account. It is possible to show that allowance for this field imposes the following limitations on the semiconductor parameters:

$$\frac{v_0^2}{4c^2} \ll \frac{r_d^2}{d^2}, \quad \frac{v_0\omega_0^2 d}{2c^2} \ll \nu$$

(d is the dimension of the sample in the direction perpendicular to the constant current).

The first condition corresponds to neglect of the pinch effect, the second to the neglect of the magnetic field of the constant current itself.

The authors are grateful to A. A. Rukhadze for having drawn their attention to this fact.

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Translated by B. Uscinski
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