# *INDUCED RAMAN SCATTERING OF LONGITUDINAL WAVES IN A MAGNETOACTIVE PLASMA*

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Decay interaction involving three longitudinal natural oscillations is studied by means of the nonlinear equation for the evolution of field fluctuations in a homogeneous magnetoactive plasma. In concrete particular cases of longitudinal spectra, expressions are obtained for the kernels of equations that describe such a decay in an almost isothermal plasma.

WE investigate in this paper induced Raman scattering of longitudinal waves in a homogeneous unbounded plasma situated in a constant homogeneous magnetic field. It is known that waves in such a plasma can in the general case not be separated into longitudinal and transverse. However, for waves whose refractive index is large ($n^2 \gg 1$), the ratio of the longitudinal component of the electric field to the transverse component becomes very large [1,2] and such waves can be regarded as longitudinal. These are precisely the waves considered here.

Having a large refractive index (small phase velocity), longitudinal waves interact effectively with the plasma particles, and this leads to many specific effects, viz. an increase in the damping of the waves and an increase in the fluctuation level. At the same time, an analysis of the decay interaction of the longitudinal waves is much simpler than in the general case of arbitrary polarization, and makes it possible to obtain easily understood concrete equations describing the interaction of waves belonging to different branches of the longitudinal-wave spectrum.

Decay interaction of longitudinal waves in a magnetoactive homogeneous plasma is analogous to the interaction of phonons in a uniaxial crystal, which is investigated in detail in solid-state physics (see [3]). The equations obtained below are identical in form to the equations of phonon interaction in a solid[1)]. We note that general expression describing the nonlinear interaction of electromagnetic waves in a magnetoactive plasma are obtained by a method used for an isotropic plasma in [7] (see also [8]). In this paper we obtain from these general equations more particular equations corresponding only to decay processes (induced Raman scattering of waves).

Unlike the numerous investigations by others [6,9-20], we consider in detail all possible concrete processes of induced Raman scattering of longitudinal waves with fully defined spectra, excited and weakly damped in a quasi-isothermal magnetoactive plasma.

The need for investigating nonlinear interactions in a magnetoactive plasma is demonstrated by the experiments devoted to nonlinear effects in such a plasma (see, for example, [21]).

## 1. EQUATION OF DECAY INTERACTION OF WAVES

An interaction of the decay type for waves in a homogeneous unbounded magnetoactive plasma is described by the equation (see [7,8])

$$\frac{1}{\omega}\frac{d}{dt}(E_jE_i)_{\omega,\mathbf{k}}\left\{\frac{\partial\omega\varepsilon_{ij}{}^H(\omega,\mathbf{k})}{\partial\omega}+\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\right\}$$
$$=\operatorname{Im}\int d\omega'\,d\mathbf{k}'\,(E_sE_c)_{\omega',\mathbf{k}'}S_{ijs}(\omega,\mathbf{k};\omega',\mathbf{k}')$$
$$\times\{\tilde{A}_{ia}{}^*(\omega,\mathbf{k})S^*_{abc}(\omega,\mathbf{k};\omega',\mathbf{k}')(E_jE_b)_{\omega-\omega',\mathbf{k}-\mathbf{k}'}$$
$$+2\tilde{A}_{jb}(\omega-\omega',\mathbf{k}-\mathbf{k}')$$
$$\times S_{bca}(\omega-\omega',\mathbf{k}-\mathbf{k}';\omega,\mathbf{k})(E_aE_i)_{\omega,\mathbf{k}}\}. \tag{1.1}$$

We have used here the following notation: $(E_jE_i)_{\omega,\mathbf{k}}$ is the correlator of the Fourier components of the electric field of the wave with frequency $\omega$ and wave vector $\mathbf{k}$; $\epsilon_{ij}(\omega,\mathbf{k})$ is the dielectric tensor of a plasma situated in a constant homogeneous magnetic field $B_0$ (see [22]); $\epsilon^H_{ij}(\omega,\mathbf{k})$ is the Hermitian part of this tensor;

$$A_{ij}(\omega,\mathbf{k})=\left[\varepsilon_{ij}(\omega,\mathbf{k})-\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\right]^{-1},$$

$\tilde{A}_{ij}$ is the singular part of the tensor $A_{ij}(\omega,\mathbf{k})$,

1)A detailed comparison of such equations in general form was made in[4,6], so that we do not discuss it here.

proportional to $\delta(\det A_{ij}^{-1})$ (see definition (1.4) below);

$$S_{ijs}(\omega, \mathbf{k};\ \omega', \mathbf{k}') = \varepsilon_{ijs}(\omega, \mathbf{k};\ \omega', \mathbf{k}') + \varepsilon_{isj}(\omega, \mathbf{k};\ \omega - \omega',\ \mathbf{k} - \mathbf{k}'),$$

where

$$\varepsilon_{ijs}(\omega, \mathbf{k};\ \omega', \mathbf{k}') = i \sum \frac{4\pi e^3}{\omega} \int d\mathbf{p} v_i \int_{-\infty}^{0} d\tau_0 \int_{-\infty}^{0} d\tau_1$$

$$\times \exp[-i\omega\tau_0 + i\mathbf{k}\delta\mathbf{R}(\tau_0, \mathbf{v})]\ \alpha_{nj}(\omega - \omega', \mathbf{k} - \mathbf{k}';\ \tau_0)$$

$$\frac{\partial}{\partial p_n(\tau_0, \mathbf{v})} \exp[-i\omega'\tau_1 + i(\mathbf{k}', \delta\mathbf{R}(\tau_0 + \tau_1, \mathbf{v})$$

$$- \delta\mathbf{R}(\tau_0, \mathbf{v}))]\ \alpha_{ms}(\omega', \mathbf{k}';\ \tau_0 + \tau_1)\ \frac{\partial f_0(\mathbf{p}(\tau_0 + \tau_1, \mathbf{v}))}{\partial p_m(\tau_0 + \tau_1, \mathbf{v})}.$$

In the last expression the summation is over the species of plasma particles, e is the charge of the particle of the given species, $f_0(\mathbf{p})$ the equilibrium distribution function of such particles, and $\mathbf{p}(\tau, \mathbf{v}) = m\mathbf{v}(\tau, \mathbf{v})$ where m is the particle mass and *

$$\mathbf{v}(\tau, \mathbf{v}) = \mathbf{h}(\mathbf{v}\mathbf{h}) + [\mathbf{v}\mathbf{h}] \sin \Omega\tau - [\mathbf{h}[\mathbf{h}\mathbf{v}]] \cos \Omega\tau,$$

$$\mathbf{v}(0, \mathbf{v}) \equiv \mathbf{v}, \qquad \mathbf{h} = \mathbf{B}_0/B_0, \qquad \Omega = eB_0/mc,$$

$$\delta\mathbf{R}(\tau, \mathbf{v}) = \int_0 \mathbf{v}(\tau', \mathbf{v})\, d\tau',$$

$$\alpha_{ij}(\omega, \mathbf{k};\ \tau) = \frac{1}{\omega}[k_i v_j(\tau, \mathbf{v}) + \delta_{ij}(\omega - \mathbf{k}\mathbf{v}(\tau, \mathbf{v}))].$$

If three longitudinal waves participate in the interaction, a case which we shall consider below, the tensors in (1.1) have the simple form

$$(E_j E_i)_{\omega, \mathbf{k}} = (E_l^2)_{\omega, \mathbf{k}} k_i k_j / k^2, \tag{1.2}$$

where $(E_l^2)_{\omega,\mathbf{k}}$ is the spectral density of the square of the electric field;

$$A_{ij}(\omega, \mathbf{k}) = \left[\frac{k_a \varepsilon_{ab}(\omega, \mathbf{k}) k_b}{k^2}\right]^{-1} \frac{k_i k_j}{k^2} \equiv \frac{1}{\varepsilon(\omega, \mathbf{k})} \frac{k_i k_j}{k^2} \tag{1.3}$$

$$\tilde{A}_{ij}(\omega, \mathbf{k}) = -i\pi \left|\frac{\partial \varepsilon}{\partial \omega}\right|^{-1}_{\omega = \omega(\mathbf{k})} \frac{k_i k_j}{k^2}$$

$$\times \operatorname{sign} \varepsilon''(\omega, \mathbf{k})\, \delta(\omega - \omega(\mathbf{k})), \tag{1.4}$$

where $\omega(\mathbf{k})$ is the spectrum of the given longitudinal wave, and $\epsilon''(\omega, \mathbf{k}) = \operatorname{Im} \epsilon(\omega, \mathbf{k})$.

It follows from relations (1.1)—(1.3) that to describe the investigated processes it is sufficient to know only the longitudinal contraction of the tensors $S_{ijs}$:

$$S \equiv S_{ijs}(\omega, \mathbf{k};\ \omega', \mathbf{k}')\ \frac{k_i k_j'' k_s'}{k k' k''}$$

$$= i \sum \frac{e}{m} \frac{\omega_L^2}{k k' k''} \int_{-\infty}^{+\infty} \frac{dv_z}{\sqrt{2\pi}\, v_T} \exp(-v_z^2/2v_T^2)$$

$$\times \int_0^{\infty} \frac{v_\perp\, dv_\perp}{v_T^2} \exp(-v_T^2/2v_T^2)$$

$$\times \sum_{n,\, l=-\infty}^{+\infty} J_n\left(\frac{k_\perp' v_\perp}{\Omega}\right) J_l\left(\frac{k_\perp'' v_\perp}{\Omega}\right) J_{l+n}\left(\frac{k_\perp v_\perp}{\Omega}\right)$$

$$\times \exp[il\theta'' + in\theta' - i(l+n)\theta]$$

$$\times \Bigg\{ -\frac{k_i \Gamma_{ij}(\omega_0) k_j''}{\omega_0} \left[\frac{k_z k_z'}{\omega_0 \omega_1} + \frac{k_z k_z'}{\omega_1^2} - \frac{k_j'' \Gamma_{js}(\omega_1) k_s'}{\omega_1}\right]$$

$$+ \frac{k_z k_z'}{\omega_0 \omega_1} k_i \Gamma_{ij}'(\omega_0) k_j'' - \frac{k_i \Gamma_{is}(\omega_0) k_s'}{\omega_0}$$

$$\times \left[\frac{k_z k_z''}{\omega_0 \omega_2} + \frac{k_z k_z''}{\omega_2^2} - \frac{k_s' \Gamma_{sj}(\omega_2) k_j''}{\omega_2}\right] + \frac{k_z k_z''}{\omega_0 \omega_2} k_i \Gamma_{is}'(\omega_0) k_s'$$

$$+ \frac{i}{\Omega} \frac{(\mathbf{k}'[\mathbf{k}\mathbf{h}])}{\omega_0} \left[\frac{k_i \Gamma_{is}(\omega_0) k_s'}{\omega_2} - \frac{k_i \Gamma_{ij}(\omega_0) k_j''}{\omega_1}\right]$$

$$+ \frac{1}{2\Omega(\omega_0 + \Omega)} \left[\frac{(k_i a_{si} k_s')}{\omega_1 + \Omega} k_i \Gamma_{ij}(\omega_0 + \Omega) k_j''\right.$$

$$\left. + \frac{(k_i a_{ij} k_j'')}{\omega_2 + \Omega} k_i \Gamma_{is}(\omega_0 + \Omega) k_s'\right] - \frac{1}{2\Omega(\omega_0 - \Omega)}$$

$$\times \left[\frac{(k_i a_{si} k_s')}{\omega_1 - \Omega} k_i \Gamma_{ij}(\omega_0 - \Omega) k_j''\right.$$

$$\left. + \frac{(k_i a_{ji} k_j'')}{\omega_2 - \Omega} k_i \Gamma_{is}(\omega_0 - \Omega) k_s'\right]\Bigg\}. \tag{1.5}$$

Here and below $\omega'' = \omega - \omega'$, $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$, $\omega_L = (4\pi N e^2/m)^{1/2}$ is the Langmuir frequency of the particles of a definite species with charge e and mass m, and N is the number of particles per unit volume. In deriving this formula, we assumed a Maxwellian distribution for the particles in the ground state of the plasma, the thermal velocity of the plasma particles being $v_T = (\kappa T/m)^{1/2}$, where T is the temperature of the given species of particle and $\kappa$ is Boltzmann's constant. The symbols $k_z$, $k_z'$, $k_z''$, and $v_z$ and $k_\perp$, $k_\perp'$, $k_\perp''$, and $v_\perp$ are connected with the choice of the coordinate frame and determine the projections of the corresponding vectors on the z axis, which is directed along the external magnetic field $\mathbf{B}_0 = \mathbf{h}B_0$ and on the plane perpendicular to it. The angles $\theta$, $\theta'$, and $\theta''$ are those between the components $\mathbf{k}_\perp$, $\mathbf{k}_\perp'$, and $\mathbf{k}_\perp''$ (of the vectors $\mathbf{k}$, $\mathbf{k}'$, and $\mathbf{k}''$), which lie in this plane, and the x axis, and $J_n(x)$ is a Bessel function of order n. The tensors $\Gamma_{ij}(\omega)$, which enter the

*$[\mathbf{v}\mathbf{h}] \equiv \mathbf{v} \times \mathbf{h}$.

right side of (1.5), are defined by the formulas

$$\Gamma_{ij}(\omega)=\frac{h_ih_j}{\omega}+\frac{1}{2}\frac{a_{ij}}{\omega+\Omega}+\frac{1}{2}\frac{a_{ij}}{\omega-\Omega},$$

$$\Gamma_{ij}'(\omega)\equiv\frac{\partial\Gamma_{ij}(\omega)}{\partial\omega}, \tag{1.6}$$

$$a_{ij}=\delta_{ij}-h_ih_j+ie_{isj}h_s, \tag{1.7}$$

where $e_{isj}$ is a completely antisymmetrical tensor of third rank. Finally,

$$\omega_0\equiv\omega-k_zv_z-(l+n)\Omega+i0,$$
$$\omega_1\equiv\omega'-k_z'v_z-n\Omega+i0,$$
$$\omega_2\equiv\omega''-k_z''v_z-l\Omega+i0,$$

where +i0 must be regarded an infinitesimally small positive imaginary part of the frequencies $\omega$, $\omega'$ and $\omega''$. Relations (1.1)—(1.7) describe completely the decay interactions of the longitudinal waves in a magnetoactive plasma.

Let us present expressions for (1.5) in three particular cases. If we neglect the external magnetic field ($\Omega = 0$), then the right side of (1.5), with suitable substitution of the arguments of the tensor, goes over (accurate to a factor which results from the difference in the notation) into formula (1.2) of the paper of Gorbunov and Silin [23], in which the Coulomb interaction of the longitudinal waves in an isotropic plasma was investigated. In the approximation of a cold ($v_T = 0$) magnetoactive plasma ($\Omega \neq 0$) Eq. (1.5) takes the form

$$S_{ijs}(\omega,\mathbf{k};\omega',\mathbf{k}')\frac{k_ik_j''k_s'}{kk'k''}=i\sum\frac{e}{m}\frac{\omega_L^2}{kk'k''}$$
$$\times\Big\{-\frac{k_i\Gamma_{ij}(\omega)k_j''}{\omega}\Big[\frac{k_zk_z'}{\omega\omega'}+\frac{k_zk_z'}{\omega'^2}-\frac{k_j''\Gamma_{js}(\omega')k_s'}{\omega'}\Big]$$
$$-\frac{k_i\Gamma_{is}(\omega)k_s'}{\omega}\Big[\frac{k_zk_z''}{\omega\omega''}+\frac{k_zk_z''}{\omega''^2}-\frac{k_s'\Gamma_{sj}(\omega'')k_j''}{\omega''}\Big]$$
$$+\frac{k_zk_z'}{\omega\omega'}\Gamma_{ij}'(\omega)k_ik_j''+\frac{k_zk_z''}{\omega\omega''}\Gamma_{is}'(\omega)k_ik_s'$$
$$+\frac{i}{\Omega}\frac{(\mathbf{k}'[\mathbf{kh}])}{\omega}\Big[\frac{k_i\Gamma_{is}(\omega)k_s'}{\omega''}-\frac{k_i\Gamma_{ij}(\omega)k_j''}{\omega'}\Big]$$
$$+\frac{1}{2\Omega(\omega+\Omega)}\Big[\frac{(k_ia_{is}k_s')}{\omega'+\Omega}k_i\Gamma_{ij}(\omega+\Omega)k_j''$$
$$+\frac{(k_ia_{ij}k_j'')}{\omega''+\Omega}k_i\Gamma_{is}(\omega+\Omega)k_s'\Big]-\frac{1}{2\Omega(\omega-\Omega)}$$
$$\times\Big[\frac{(k_ia_{si}k_s')}{\omega'-\Omega}k_i\Gamma_{ij}(\omega-\Omega)k_j''$$
$$+\frac{(k_ia_{ji}k_j'')}{\omega''-\Omega}k_i\Gamma_{is}(\omega-\Omega)k_s'\Big]\Big\}. \tag{1.8}$$

This expression can be obtained directly from the tensor $S_{ijs}(\omega, \mathbf{k}; \omega', \mathbf{k}')$ (see (1.4) in [8]), calculated in the approximation of a cold plasma, by contraction with $k_ik_j''k_s')/(kk'k'')$. Neglecting thermal motion ($v_T = 0$) and the external magnetic field ($\omega = 0$), we obtain from (1.5)

$$S_{ijs}(\omega,\mathbf{k};\omega',\mathbf{k}')\frac{k_ik_j''k_s'}{kk'k''}=-\frac{i}{kk'k''}$$
$$\times\sum\frac{e}{m}\frac{\omega_L^2}{\omega\omega'\omega''}\Big[\frac{k^2}{\omega}\mathbf{k}'\mathbf{k}''+\frac{k'^2}{\omega'}\mathbf{k}\mathbf{k}''+\frac{k''^2}{\omega''}\mathbf{k}\mathbf{k}'\Big], \tag{1.9}$$

which coincides with the "longitudinal" contraction of the tensor $S_{ijs}(\omega, \mathbf{k}; \omega', \mathbf{k}')$ calculated for an isotropic cold plasma in [7] (formula (2.8)). We present another rather useful form of (1.1) for the particular case of coalescence of longitudinal waves. Going over from the correlators $(E_jE_i)_{\omega,\mathbf{k}}$ to the longitudinal-wave energy densities $W_l(\mathbf{k})$ in the volume $d\mathbf{k}$ of the space of the wave vectors $\mathbf{k}$, in accordance with the definition

$$W_l(\mathbf{k})=(2\pi)^3\int_0^\infty d\omega\frac{(E_jE_i)_{\omega,\mathbf{k}}}{4\pi}\frac{\partial}{\partial\omega}[\omega\varepsilon_{ij}^H(\omega,\mathbf{k})], \tag{1.10}$$

we obtain from (1.1) and (1.2)

$$\frac{dW_l(\mathbf{k})}{dt}=\frac{\omega(\mathbf{k})}{2\pi}\int d\mathbf{k}'d\mathbf{k}''\int_{-\infty}^{+\infty}d\omega'\int_{-\infty}^{+\infty}d\omega''\,\delta(\omega-\omega'-\omega'')$$
$$\times\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')Q(\omega,\mathbf{k};\omega',\mathbf{k}';\omega'',\mathbf{k}'';\mathbf{B}_0)$$
$$\times\{W_l(\mathbf{k}')W_l(\mathbf{k}'')\omega(\mathbf{k})\,\mathrm{sign}\,\varepsilon''(\omega,\mathbf{k})$$
$$-2W_l(\mathbf{k})W_l(\mathbf{k}')\omega''\,\mathrm{sign}\,\varepsilon''(\omega'',\mathbf{k}'')\}$$
$$\times\{\delta(\omega'-\omega'(\mathbf{k}'))\delta(\omega''-\omega''(\mathbf{k}''))$$
$$+\delta(\omega'+\omega'(-\mathbf{k}'))\delta(\omega''+\omega''(-\mathbf{k}''))$$
$$+\delta(\omega'-\omega'(\mathbf{k}'))\delta(\omega''+\omega''(-\mathbf{k}''))$$
$$+\delta(\omega'+\omega'(-\mathbf{k}'))\delta(\omega''-\omega''(\mathbf{k}''))\}, \tag{1.11}$$

where

$$Q(\omega,\mathbf{k};\omega',\mathbf{k}';\omega'',\mathbf{k}'';\mathbf{B}_0)\equiv Q$$
$$=\Big|S_{ijs}(\omega,\mathbf{k};\omega',\mathbf{k}')\frac{k_ik_j''k_s'}{kk'k''}\Big|^2$$
$$\times\Big(\omega\frac{\partial\varepsilon}{\partial\omega}\Big)^{-1}_{\omega=\omega(\mathbf{k})}\Big(\omega'\frac{\partial\varepsilon}{\partial\omega'}\Big)^{-1}\Big(\omega''\frac{\partial\varepsilon}{\partial\omega''}\Big)^{-1} \tag{1.12}$$

and $\omega(\mathbf{k})$, $\omega'(\mathbf{k})$, and $\omega''(\mathbf{k}'')$ are the dispersion laws of the interacting longitudinal waves in the corresponding transparency regions, defined by the equation

$$\varepsilon\equiv\varepsilon(\omega,\mathbf{k})=k_i\varepsilon_{ij}(\omega,\mathbf{k})k_j/k^2=0. \tag{1.13}$$

We have used here the symmetry property of the contraction (1.15), a property following directly from its construction

$$S_{ijs}(\omega,\mathbf{k};\omega',\mathbf{k}')\frac{k_ik_j''k_s'}{kk'k''}=S^*_{jsi}(\omega'',\mathbf{k}'';\omega,\mathbf{k})\frac{k_j''k_s'k_i}{kk'k''} \tag{1.14}$$

in the case of pure decay processes, when all the integrals with respect to $v_z$ in the right side of (1.5) are taken in the sense of principal value (see, for example, the article by Silin [5]).

From now on, when considering concrete interactions of longitudinal waves, we shall specifically write out the kernel Q for $\omega' = \omega'(\mathbf{k}')$ and $\omega'' = \omega''(\mathbf{k}'')$, that is, with account of the first term in the second curly bracket of the right side of (1.11).

## 2. INTERACTION OF LONGITUDINAL WAVES IN ISOTHERMAL PLASMA

Some of the longitudinal waves in a magnetoactive plasma exist only if the temperatures of the electrons and ions differ greatly. In this paper we consider the interaction of longitudinal waves, for the existence of which no such non-isothermal behavior is required. The spectra of the longitudinal waves, both in an isothermal and in a non-isothermal magnetoactive plasma have been obtained in several papers [2,24]. We use here the dispersion laws and the transparency conditions in the form derived by A. A. Rukhadze [24]. We confine ourselves throughout to a plasma consisting of electrons and a single species of ions.

In an almost-isothermal plasma ($T_e \sim T_i$) there are possible long-wave longitudinal oscillations, obtainable in the cold-plasma approximation, as well as cyclotron (ion and electron) and high-frequency Langmuir oscillations ($\omega \approx \omega_{Le} > \Omega_e$).

The subscripts e and i pertain throughout to quantities characterizing electrons and ions, respectively.

Long-wave oscillations are possible under the conditions

$$(k_\perp v_{Te,\,i} / \Omega_{e,\,i})^2 \ll 1, \quad |(\omega - n\Omega_{e,\,i}) / k_z v_{Te,\,i}| \gg 1, \quad n = 0, \pm 1, \pm 2, \ldots, \tag{2.1}$$

which allow us to neglect the thermal motion of the particles and to use the oscillation spectra of the cold plasma. These spectra are solutions of the dispersion equation

$$\varepsilon(\omega, \mathbf{k}) = 1 + \sum \left( \frac{\omega_L^2}{\Omega^2 - \omega^2} \frac{k_\perp^2}{k^2} - \frac{\omega_L^2}{\omega^2} \frac{k_z^2}{k^2} \right) = 0 \tag{2.2}$$

and are of the following form:

a) $\omega \ll \Omega_i$:

$$\omega^2 = k_z^2 \omega_{Le}^2 \Omega_i^2 / (k^2 \Omega_i^2 + k_\perp^2 \omega_{Li}^2); \tag{2.3}$$

b) $\omega \ll \Omega_e$ (arbitrary $\omega/\Omega_i$):

$$\omega^2 = \frac{1}{2} \left( 1 + \frac{k_\perp^2}{k^2} \frac{\omega_{Le}^2}{\Omega_e^2} \right)^{-1} \left[ \Omega_i^2 + \frac{k_\perp^2}{k^2} \omega_{Li}^2 + \frac{k_z^2}{k^2} \omega_{Le}^2 \pm a \right], \tag{2.4}$$

$$a \equiv \left[ \left( \Omega_i^2 + \frac{k_\perp^2}{k^2} \omega_{Li}^2 + \frac{k_z^2}{k^2} \omega_{Le}^2 \right)^2 - 4\Omega_i^2 \frac{k_z^2}{k^2} \omega_{Le}^2 \left( 1 + \frac{k_\perp^2}{k^2} \frac{\omega_{Le}^2}{\Omega_e^2} \right) \right]^{1/2}. \tag{2.5}$$

If the angle $\varphi$ between the wave vector and the direction of the magnetic field is not too close to $\pi/2$, ($k_\perp/k_z < (|e|/e_i)(M/m)^{1/2}$), then the spectra take the form

$$\omega^2 = k^2 \frac{\Omega_i^2 \Omega_e^2}{k^2 \Omega_e^2 + k_\perp^2 \omega_{Le}^2}, \tag{2.6}$$

$$\omega^2 = k_z^2 \frac{\omega_{Le}^2 \Omega_e^2}{k^2 \Omega_e^2 + k_\perp^2 \omega_{Le}^2}. \tag{2.7}$$

On the other hand, if $\varphi = \pi/2$, then

$$\omega^2 = \frac{\Omega_i^2 + \omega_{Li}^2}{\Omega_e^2 + \omega_{Le}^2} \Omega_e^2; \tag{2.8}$$

c) $\omega - \Omega_e \sim \Omega_e$ (more accurately, $\omega \gg (M/m)^{1/2} \Omega_i$):

$$\omega^2 = \tfrac{1}{2} (\omega_{Le}^2 + \Omega_e^2) \mp \tfrac{1}{2} \sqrt{(\omega_{Le}^2 + \Omega_e^2)^2 - 4\omega_{Le}^2 \Omega_e^2 k_z^2 / k^2}. \tag{2.9}$$

When $(M/m)^{1/2} \Omega_i \ll \omega \ll \Omega_e$, this spectrum coincides with (2.6), and when $\omega \gg \Omega_e$ under the condition $\omega_{Le} \gg \Omega_e$ it goes over into the spectrum of the Langmuir oscillations in a cold isotropic plasma:

$$\omega^2 = \omega_{Le}^2. \tag{2.10}$$

Weakly interacting ion cyclotron oscillations are possible under the following conditions:

$$\omega \approx n\Omega_i, \quad \left( \frac{k_\perp v_{Ti}}{\Omega_i} \right)^2 \gtrsim 1, \quad k_z v_{Te} \ll \Omega_e, \quad \left| \frac{\omega - n\Omega_i}{k_z v_{Ti}} \right| \gg 1, \quad \omega \gg k_z v_{Te}. \tag{2.11}$$

The spectrum of such oscillations is given by the formula

$$\omega = n\Omega_i \left\{ 1 + A_n \left( \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2} \right) \times \left[ 1 + \frac{k^2 v_{Ti}^2}{\omega_{Li}^2} + \frac{|e| T_i}{e_i T_e} (1 - A_0 (k_\perp^2 v_{Te}^2 / \Omega_e^2)) \right]^{-1} \right\}, \tag{2.12}$$

where $A_n(x) = e^{-x} I_n(x)$, and $I_n(x)$ is a modified Bessel function of order n.

The conditions for the existence of weakly damped electron cyclotron oscillations are expressed by the inequality

$$\omega \approx n\Omega_e, \tag{2.13}$$

$$|\omega - n\Omega_e| \gg |k_z| v_{Te}, \tag{2.14}$$

and the spectrum of such oscillations is given by the formula

$$\omega = n\Omega_e\{1 + A_n(k_\perp^2 v_{Te}^2/\Omega_e^2) \times [1 + (k_\perp^2 v_{Te}^2/\Omega_e^2)]^{-1}\}. \quad (2.15)$$

Finally, the Langmuir oscillations with frequency $\omega \approx \omega_{Le} \gg \Omega_e$ can exist, of course, not only in the long-wave limit (formula (2.10)), but also in the short-wave region, where the spatial dispersion should also be taken into account, and the spectrum takes the form

$$\omega^2 = \omega_{Le}^2 + 3k^2 v_{Te}^2. \quad (2.16)$$

We shall start our analysis of the decay interactions between waves in an almost-isothermal plasma with a case in which all three interacting waves have identical spectra. The presence of interaction between waves with identical spectra is a specific feature of decay interaction of waves in a magnetoactive plasma, for in an isotropic plasma such processes are made impossible by the non-decay character of the spectra of the Langmuir and ion-sound waves.

1. Interaction of low-frequency long-wave oscillations with the spectrum (2.3) is described by Eq. (1.11), in which the contraction

$$S \equiv S_{ijs}(\omega, k;\ \omega', k') k_i k_j'' k_s' / kk'k'' \quad (2.17)$$

can be obtained from (1.8), by assuming $\omega \ll \Omega_i$, $\omega' \ll \Omega_i$, and $\omega'' \ll \Omega_i$:

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega\omega'\omega''}\frac{k_z k_z' k_z''}{kk'k''}\left[\frac{k_z}{\omega} + \frac{k_z'}{\omega'} + \frac{k_z''}{\omega''}\right]. \quad (2.18)$$

We have retained here only the electronic part of the contraction, which exceeds the ionic part by approximately $(eM/e_i m)^2$ times (M—mass of the ion, m—mass of the electron, $e_i$—charge of the ion).

Substituting in (2.18) the spectra (2.3) for all three interacting waves, and using the expression for $\epsilon(\omega, \mathbf{k})$ in the cold-plasma approximation as well as formula (1.12), we obtain the following expression for the kernel Q of the integro-differential equation (1.11):

$$Q = (32\pi N_e mc^2)^{-1}\left(\frac{ck}{\omega_{Le}}\right)^2\left\{\left(1 + \frac{k_\perp^2\omega_{Li}^2}{k^2\Omega_i^2}\right)^{1/2} + \frac{k'}{k}\left[1 + \left(\frac{k_\perp'\omega_{Li}}{k'\Omega_i}\right)^2\right]^{1/2} + \frac{k''}{k}[1 + (k_\perp''\omega_{Li}/k''\Omega_i)^2]^{1/2}\right\}^2. \quad (2.19)$$

2. Expression (2.18) remains valid also in the regions $\omega - \Omega \sim \Omega_i$ or $\Omega_i \ll \omega \ll \Omega_e$, when the frequencies of all three long-wave interacting oscillations lie in these regions. Substituting in (1.12) the expression (2.18) and the spectra (2.6) for all three waves, we obtain an expression for the kernel Q in the case of interaction of three long-wave oscillations belonging to the first branch of the spectrum when $\omega, \omega', \omega'' \sim \Omega_i$:

$$Q = (32\pi N_e mc^2)^{-1}\left(\frac{ck_z}{\Omega_i}\right)^2\left\{\frac{k_z}{k}\left[1 + \left(\frac{k_\perp\omega_{Le}}{k\Omega_e}\right)^2\right]^{1/2} + \frac{k_z'}{k}\left[1 + \left(\frac{k_\perp'\omega_{Le}}{k'\Omega_e}\right)^2\right]^{1/2} + \frac{k_z''}{k}\left[1 + \left(\frac{k_\perp''\omega_{Le}}{k''\Omega_e}\right)^2\right]^{1/2}\right\}^2. \quad (2.20)$$

3. In perfect analogy, we can obtain for the second branch of oscillations with $\omega - \Omega_i \sim \Omega_i$, described by the spectrum (2.7) and going over into the region $\Omega_i \ll \omega \ll \Omega_e$, the following formula for the kernel Q:

$$Q = (32\pi N_e mc^2)^{-1}\left(\frac{ck}{\omega_{Le}}\right)^2\left\{\left[1 + \left(\frac{k_\perp\omega_{Le}}{k\Omega_e}\right)^2\right]^{1/2} + \frac{k'}{k}\left[1 + \left(\frac{k_\perp'\omega_{Le}}{k'\Omega_e}\right)^2\right]^{1/2} + (k''/k)[1 + (k_\perp''\omega_{Le}/k''\Omega_e)^2]^{1/2}\right\}^2. \quad (2.21)$$

4. A decay interaction is possible also between waves with spectra (2.9) having frequencies $\omega, \omega', \omega'' \sim \Omega_e$. It is described by (1.11), in which it is necessary to substitute the expression for S in the form (1.8) as well as the spectra (2.9).

5. When considering the interaction of ion-cyclotron waves described by the spectrum (2.12) and having frequencies $\omega \approx (l + n)\Omega_i$ and $\omega' \approx n\Omega_i$, $\omega'' \approx l\Omega_i$, it is necessary to use the following expression for S, obtained with allowance for the inequalities $\omega, \omega', \omega'' \ll \Omega_e$, $\omega \gg k_z v_{Te}$, $\omega' \gg k_z' v_{Te}$, and $\omega'' \gg k_z'' v_{Te}$:

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega\omega'\omega''}\frac{k_z k_z' k_z''}{kk'k''}\left(\frac{k_z}{\omega} + \frac{k_z'}{\omega'} + \frac{k_z''}{\omega''}\right) \times \int_0^\infty \frac{v_\perp dv_\perp}{v_{Te}^2}\exp(-v_\perp^2/2v_{Te}^2) J_0\left(\frac{k_\perp v_\perp}{\Omega_e}\right) \times J_0\left(\frac{k_\perp' v_\perp}{\Omega_e}\right) J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right). \quad (2.22)$$

The kernel of (1.11) then takes the form

$$Q = \frac{1}{4\pi N_e mc^2}\left(\frac{Me^2}{me_i^2}\right)^3\left(\frac{k_z k_z' k_z'' v_{Ti}^3}{nl(n+l)\Omega_i^3}\right)^2\left(\frac{k_z}{n+l} + \frac{k_z'}{n} + \frac{k_z''}{l}\right)^2 \times \frac{c^2}{\Omega_i^2} A_{l+n}\left(\frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}\right) A_n\left(\frac{k_\perp'^2 v_{Ti}^2}{\Omega_i^2}\right) A_l\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right) \times \left\{1 + \left(\frac{kv_{Ti}}{\omega_{Li}}\right)^2 + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2} \times \left\{1 + \left(\frac{k' v_{Ti}}{\omega_{Li}}\right)^2 + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2} \times \left\{1 + \left(\frac{k'' v_{Ti}}{\omega_{Li}}\right)^2 + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}$$

$$\times\left\{\int_0^\infty \frac{v_\perp dv_\perp}{v_{Te}^2}\exp\left(-\frac{v_\perp^2}{2v_{Te}^2}\right)\right.$$
$$\left.\times J_0\left(\frac{k_\perp v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right)\right\}^2. \quad (2.23)$$

We note that we have retained only the electronic terms in (2.22). In the case of the spectra (2.12)' this is always justified, since the ionic part of the contraction is always smaller than the electronic one:

$$S_i \sim M^{-2}[\omega-(l+n)\Omega_i]^{-4} \ll M^{-2}(k_z v_{Ti})^{-4}$$
$$\sim M^{-2}(k_\perp v_{Ti})^{-4} \lesssim M^{-2}\Omega_i^{-4} \ll m^{-2}\Omega_i^{-4} \sim S_e. \quad (2.24)$$

6. Three electron-cyclotron harmonics with frequencies $\omega \approx (l+n)\Omega_e$, $\omega' \approx n\Omega_e$ and $\omega'' \approx l\Omega_e$ interact with one another.

To describe such a process it is necessary to use the contraction S obtained from (1.5) by separating the terms containing in the denominator small differences $\omega-(l+n)\Omega_e$, $\omega'-n\Omega_e$, and $\omega''-l\Omega_e$. In Sec. 3 we shall estimate the kernel Q of the integro-differential equation (1.11) that describes this process and indicate the characteristic time of the decay or coalescence.

Having considered all possible decay interactions between the waves with identical spectra, let us explain now when waves with different spectra can interact, and let us determine the forms of (1.11) in these cases.

7. We start with the interaction between Langmuir waves with spectrum (2.16) and waves of other types. A Langmuir wave with a wave vector k, is transformed by interaction with one of the oscillations described above (of frequency $\omega''$ and wave vector $\mathbf{k}''$) into another Langmuir wave. Let $\omega'' \ll \Omega_i$ and let its spectrum be given by (2.2). Then, for

$$\omega = \omega_{Le}(1+3k^2v_{Te}^2/2\omega_{Le}^2),$$
$$\omega' = \omega_{Le}(1+3k'^2v_{Te}^2/2\omega_{Le}^2)$$

we obtain

$$S \approx S_e = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega^2\omega''^2}\frac{(\mathbf{kk}')k_z''^2}{kk'k''}. \quad (2.25)$$

Taking (2.25) into account, the kernel Q, according to the definition (1.12), becomes equal to

$$Q=(32\pi N_e mc^2)^{-1}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\left(\frac{ck''}{\omega_{Le}}\right)^2\left[1+\left(\frac{k_\perp''\omega_{Li}}{k''\Omega_i}\right)^2\right]. \quad (2.26)$$

8. If a wave with frequency $\omega''$ and wave vector $\mathbf{k}''$ is defined by spectra (2.6) and (2.7), and two other waves are of the Langmuir type and are described by the spectrum (2.16), then

$$S=S_e+S_i, \quad S_e=-i\frac{e}{m}\frac{\omega_{Le}^2}{kk'k''}\frac{(\mathbf{kk}')k_z''^2}{\omega^2\omega''^2},$$
$$S_i=+i\frac{e_i}{M}\frac{\omega_{Li}^2}{\omega^2}\frac{\mathbf{kk}'}{kk'k''}\frac{k_j''\Gamma_{js}(\omega'')k_s''}{\omega''}. \quad (2.27)$$

Assuming $\omega''-\Omega_i \sim \omega''$ we obtain $S_i \ll S_e$, so that

$$Q=(32\pi N_e mc^2)^{-1}(\mathbf{kk}'/kk')^2(ck_z''/\Omega_i)^2$$
$$\times[1+(k_\perp''\omega_{Le}/k''\Omega_e)^2] \quad (2.28)$$

for the spectrum $\omega''(\mathbf{k}'')$ given by formula (2.6), and

$$Q=(32\pi N_e mc^2)^{-1}(\mathbf{kk}'/kk')^2(ck''/\omega_{Le})^2$$
$$\times[1+(k_\perp''\omega_{Le}/k''\Omega_e)^2] \quad (2.29)$$

for the spectrum (2.7).

9. The last interaction between a long-wave oscillation with spectrum (2.9) (frequency $\omega''$ and wave vector $\mathbf{k}''$) and Langmuir waves (2.16) is characterized by the contraction

$$S=i\frac{e}{m}\frac{\omega_{Le}^2}{\omega^2}\frac{\mathbf{kk}'}{kk'k''}\left\{\frac{k_z''^2}{\omega''^2}+\frac{[\mathbf{k}''\mathbf{h}]^2}{\omega''^2-\Omega_e^2}\right\}-i\frac{e_i}{M}\frac{\omega_{Li}^2}{\omega''^2}\frac{(\mathbf{kk}')k''}{kk'\omega\omega'}, \quad (2.30)$$

in which the electronic term exceeds the ionic term.

Since it is essential for the existence of Langmuir waves that the condition $\omega_{Le} \gg \Omega_e$ be satisfied, the spectra $\omega''(\mathbf{k}'')$ given by formula (2.9) simplify in this case to

$$\omega''(\mathbf{k}'')=k_z''\Omega_e/k'' \quad \text{or} \quad \omega''(\mathbf{k}'')=\omega_{Le}. \quad (2.31)$$

Taking the same condition into account, we get

$$\omega''(\partial\varepsilon/\partial\omega'')=2\omega_{Le}^2k''^2/\Omega_e^2k_\perp''^2 \quad \text{or} \quad \omega''(\partial\varepsilon/\partial\omega'')=2.$$

We see from (2.31) that the oscillations of the second branch cannot participate in the decay process with Langmuir waves $\omega$ and $\omega'$, since they themselves are of the Langmuir type. For oscillations of the first branch, the kernel $Q = 0$ if we take into account only the higher-order electronic term (2.30). Account of the second, ionic term yields

$$Q=\frac{1}{32\pi}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\left(\frac{ek_\perp''}{\Omega_e}\right)^2\left\{\frac{1}{\sqrt{N_e mc^2}}\left[\frac{\Omega_e^2}{\omega_{Le}^2}\frac{k_z''^2}{k''^2}\right.\right.$$
$$\left.\left.+\frac{k_\perp''^2k''^2\omega_{Le}^2}{\Omega_e^2k_z''^4-k''^4\omega_{Le}^2}\right]-\frac{1}{\sqrt{N_iMc^2}}\left(\frac{\omega_{Li}}{\omega_{Le}}\right)^3\frac{k''^2}{k_z''^2}\right\}^2. \quad (2.32)$$

10. Let us consider now the interaction between shorter longitudinal waves with Langmuir oscillations (2.16). Let a wave with frequency $\omega'' \approx n\Omega_i$ and wave vector $\mathbf{k}''$ have a spectrum (2.12). Then

(1.5) takes the form

$$S = S_e + S_i = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega^2}\frac{(\mathbf{kk}')k_z''^2}{kk'k''}A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega^2}\right)$$
$$\times\int_{-\infty}^{+\infty}\frac{dv_z}{\sqrt{2\pi}\,v_{Te}}\frac{\exp(-v_z^2/2v_{Te}^2)}{(\omega''-k_z''v_z+i0)^2} - i\frac{e_i}{M}\frac{\omega_{Li}^2}{\omega^2}\frac{(\mathbf{kk}')k_z''^2}{kk'k''}$$
$$\times A_n\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right)(\omega''-n\Omega_i)^{-2}. \qquad (2.33)$$

Using $\omega'' \gg k_z'' v_T$ and assuming that

$$A_0(k_\perp''^2 v_{Te}^2/\Omega_e^2) \sim A_n(k_\perp''^2 v_{Ti}^2/\Omega_i^2)$$

we find, in accord with (2.24), that $S_e \gg S_i$, so that we can write for the kernel Q the relation

$$Q = \frac{1}{16\pi N_e mc^2}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\left(\frac{k_z''}{n\Omega_i}\right)^4 (cv_{Ti})^2\frac{M}{m}A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)$$
$$\times A_n\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right)\left\{1+\left(\frac{k''v_{Ti}}{\omega_{Li}}\right)^2\right.$$
$$\left.+\frac{|e|}{e_i}\frac{T_i}{T_e}\left[1-A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}. \qquad (2.34)$$

11. Finally, for an electron-cyclotron wave frequency $\omega'' \approx n\Omega_e$ and spectrum (2.15) interacting with a Langmuir wave, the contraction (1.5) is equal to

$$S = -i\frac{e_i}{M}\frac{\omega_{Li}^2}{\omega\omega'\omega''^2}\frac{\mathbf{kk}'}{kk'}k'' - i\frac{e}{m}\frac{\omega_{Le}^2}{\omega^2}\frac{(\mathbf{kk}')k_z''^2}{kk'k''}$$
$$\times(\omega''-n\Omega_e)^{-2}A_n(k_\perp''^2 v_{Te}^2/\Omega_e^2), \qquad (2.35)$$

from which we see that when $A_n(k_\perp''^2 v_{Te}^2/\Omega_e^2) \sim 1$ the electronic term is always larger than the ionic term. Taking into account only the second, electronic term in the right side of (2.35), we obtain the kernel of (1.11) in the form

$$Q = \frac{1}{16\pi N_e mc^2}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\left(\frac{\sqrt{cv_{Te}}\,k_z''}{n\Omega_e}\right)^4$$
$$\times A_n^{-3}\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\left[1+\left(\frac{k''v_{Te}}{\omega_{Le}}\right)^2\right]^2. \qquad (2.36)$$

The cases considered (see items 7–11) exhaust the possible variants of the interaction between Langmuir waves and longitudinal waves in an almost-isothermal magnetoactive plasma.

12. We proceed to study interactions of long-wave oscillations, having frequencies $\omega' - \Omega_e \sim \Omega_e$ of the order of the electronic cyclotron oscillation and spectra (2.9), with long wave oscillations whose frequencies $\omega''$ are considerably lower than the electronic gyroscopic frequency $\Omega_e$, and whose spectra are given by (2.3), (2.6), and (2.7). The interaction produces in this case a wave with frequency $\omega - \Omega_e \sim \Omega_e$ and spectrum (2.9). From the general relation (1.8) for a cold magnetoactive plasma we obtain for all three processes the same contraction

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{kk'k''}\frac{k_z''^2}{\omega''^2}\frac{k_i\Gamma_{is}(\omega)k_s'}{\omega}, \qquad (2.37)$$

but the kernel Q of the equation depends on the type of spectrum, namely

$$Q = \frac{1}{32\pi N_e mc^2}\left(\frac{ck''\omega_{Le}}{kk'\omega}\right)^2|k_i\Gamma_{is}(\omega)k_s'|^2\left[1+\left(\frac{k_\perp''\omega_{Li}}{k''\Omega_i}\right)^2\right]$$
$$\times\left[\left(\frac{k_\perp}{k}\right)^2\frac{\omega^2\omega_{Le}^2}{(\omega^2-\Omega_e^2)^2}+\left(\frac{k_z}{k}\right)^2\frac{\omega_{Le}^2}{\omega^2}\right]^{-1}$$
$$\times\left[\left(\frac{k_\perp'}{k'}\right)^2\frac{\omega'^2\omega_{Le}^2}{(\omega'^2-\Omega_e^2)^2}+\left(\frac{k_z'}{k'}\right)^2\frac{\omega_{Le}^2}{\omega'^2}\right]^{-1}, \qquad (2.38)$$

if $\omega'' \ll \Omega_i$ and $\omega''(\mathbf{k}'')$ is determined from (2.3), and

$$Q = \frac{1}{32\pi N_e mc^2}\left(\frac{ck_z''}{kk'}\right)^2\frac{\omega_{Le}^4}{\omega^2}|k_i\Gamma_{is}(\omega)k_s'|^2$$
$$\times\left[\left(\frac{k_\perp}{k}\right)^2\frac{\omega^2\omega_{Le}^2}{(\omega^2-\Omega_e^2)^2}+\left(\frac{k_z}{k}\right)^2\frac{\omega_{Le}^2}{\omega^2}\right]^{-1}$$
$$\times\left[\left(\frac{k_\perp'}{k'}\right)^2\frac{\omega'^2\omega_{Le}^2}{(\omega'^2-\Omega_e^2)^2}+\left(\frac{k_z'}{k'}\right)^2\frac{\omega_{Le}^2}{\omega'^2}\right]^{-1}\frac{1}{\omega''^2}, \qquad (2.39)$$

if $\omega'' - \Omega_i \sim \Omega_i$ and $\omega''(\mathbf{k}'')$ is determined from (2.6) or (2.7). The frequencies $\omega$ and $\omega'$ in the right sides of (2.38) and (2.39) must be taken throughout to mean the corresponding functions of the wave vectors $\mathbf{k}$ and $\mathbf{k}'$, in accordance with (2.9). In the right side of (2.39), the last factor, which depends on $\omega''$, is determined by formulas (2.6) and (2.7). In addition,

$$\omega^{-2}|k_i\Gamma_{is}(\omega)k_s'|^2 = \left[\frac{k_zk_z'}{\omega^2}+\frac{[\mathbf{kh}][\mathbf{k}'\mathbf{h}]}{\omega^2-\Omega_e^2}\right]^2$$
$$+\left(\frac{\Omega_e}{\omega}\right)^2\frac{(\mathbf{k}'[\mathbf{kh}])^2}{(\omega^2-\Omega_e^2)^2}. \qquad (2.40)$$

The long-wave oscillations (2.9) can also interact with a short wave ion-cyclotron and electron-cyclotron oscillations. The resultant wave produced will again have a spectrum (2.9). If the frequency of one of the interacting waves is $\omega'' \approx n\Omega_i$ and its dispersion law is determined by (2.12), then the contraction corresponding to the process takes the form

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{kk'k''}\frac{k_z''^2}{\omega}k_i\Gamma_{is}(\omega)k_s'$$
$$\times\int_{-\infty}^{+\infty}\frac{dv_z}{\sqrt{2\pi}v_{Te}}\frac{\exp(-v_z^2/2v_{Te}^2)}{(\omega''-k_z''v_z+i0)^2}, \qquad (2.41)$$

and the kernel Q is obtained from (2.12) and (2.41), with account of the fact that $\omega'' \gg k_z'' v_{Te}$:

$$Q=\frac{1}{16\pi N_e mc^2}\frac{M}{m}\frac{e^2}{e_i^2}\left(\frac{\omega_{Le}k_z''\sqrt{cv_{Ti}}}{\omega}\right)^4 A_n\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right)$$
$$\times\left|\frac{k_i\Gamma_{is}(\omega)k_s'}{kk'n\Omega_i}\right|^2\left[\left(\frac{k_\perp}{k}\right)^2\frac{\omega^2\omega_{Le}^2}{(\omega^2-\Omega_e^2)^2}+\frac{k_z^2}{k^2}\frac{\omega_{Le}^2}{\omega^2}\right]^{-1}$$
$$\times\left[\left(\frac{k_\perp'}{k'}\right)^2\frac{\omega'^2\omega_{Le}^2}{(\omega'^2-\Omega_e^2)^2}+\left(\frac{k_z'}{k'}\right)^2\frac{\omega_{Le}^2}{\omega'^2}\right]^{-1}$$
$$\times\left\{1+\left(\frac{k''v_{Ti}}{\omega_{Li}}\right)^2+\frac{|e|}{e_i}\frac{T_i}{T_e}\left[1-A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}. \tag{2.42}$$

We have neglected the small ionic term in the contraction (2.41). The notation is the same as in (2.38). The interaction with the electron-cyclotron wave of frequency $\omega''\approx$ e [and spectrum (2.15)] is determined by the general formula (1.5), in the right side of which it is necessary to retain terms with small denominators $\omega''-\Omega_e$. In Sec. 3 we present an estimate of the resultant kernel Q [formula (3.7)].

13. In the vicinity of the gyrofrequency $\Omega_e$, a short wave oscillation with spectrum (2.15) can also be produced by interaction of the same oscillation with long-wave oscillations having spectra (2.3), (2.6), and (2.7). Let the frequencies of the electron-cyclotron waves be $\omega\approx n\Omega_e$ and $\omega'\approx n\Omega_e$ and let the spectrum $\omega''(k'')$ be given by (2.3). Then

$$S=-i\frac{e}{m}\frac{\omega_{Le}^2}{\omega''^2}\frac{k_z''^2}{kk'k''}\frac{k_i\Gamma_{is}(\omega-n\Omega_e)k_s'}{\omega-n\Omega_e}$$
$$\times\exp\left[-\frac{v_{Te}^2}{2\Omega^2}(k_\perp^2+k_\perp'^2)\right]I_n\left(k_\perp k_\perp'\frac{v_{Te}^2}{\Omega^2}\right), \tag{2.43}$$

$$Q=\frac{1}{8\pi N_e mc^2}\left(\frac{ck''}{\omega_{Le}}\right)^2\frac{v_{Te}^4}{n^2\Omega_e^2}A_n^{-1}\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)A_n^{-1}\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)$$
$$\times\left[1+\left(\frac{k'v_{Te}}{\omega_{Le}}\right)^2\right]^2$$
$$\times\left[1+\left(\frac{k_\perp''\omega_{Li}}{k''\Omega_i}\right)^2\right]^2|k_i\Gamma_{is}(\omega-n\Omega_e)k_s'|^2$$
$$\times\exp\left[-\frac{v_{Te}^2}{\Omega_e^2}(k_\perp^2+k_\perp'^2)\right]I_n^2\left(k_\perp k_\perp'\frac{v_{Te}^2}{\Omega_e^2}\right). \tag{2.44}$$

If $\omega''-\Omega_i\sim\Omega_i$ (spectrum (2.6) and (2.7)), then S is given by the same formula (2.43), and

$$Q=\frac{1}{8\pi N_e mc^2}\left(\frac{ck_z''}{\omega''}\right)^2\frac{v_{Te}^4}{(n\Omega_e)^2}A_n^{-1}\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)A_n^{-1}\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)$$
$$\times\left[1+\left(\frac{k'v_{Te}}{\omega_{Le}}\right)^2\right]^2|k_i\Gamma_{is}(\omega-n\Omega_e)k_s'|^2. \tag{2.45}$$

The electron-cyclotron oscillations can interact also with the ion-cyclotron oscillations. Let $\omega''\approx l\Omega_i$ and let the spectrum $\omega''(\mathbf{k}'')$ be determined by (2.12); then

$$S=-i\frac{e}{m}\frac{\omega_{Le}^2}{kk'k''}\left(\frac{k_z''}{\omega''}\right)^2\frac{k_i\Gamma_{is}(\omega-n\Omega_e)k_s'}{\omega-n\Omega_e}$$
$$\times\int_0^\infty\frac{v_\perp dv_\perp}{v_{Te}^2}\exp(-v_\perp^2/2v_{Te}^2)$$
$$\times J_n\left(\frac{k_\perp v_\perp}{\Omega_e}\right)J_n\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right). \tag{2.46}$$

With the aid of (2.46) we easily obtain the kernel of the equation

$$Q=\frac{1}{4\pi N_e mc^2}\left(\frac{k_z''v_{Te}}{\omega''}\right)^4\frac{M}{m}\left(\frac{cv_{Ti}e}{n\Omega_e e_i}\right)^2\left[1+\left(\frac{k'v_{Te}}{\omega_{Le}}\right)^2\right]^2$$
$$\times A_n\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)A_n^{-1}\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)A_l\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_i^2}\right)$$
$$\times|k_i\Gamma_{is}(\omega-n\Omega_e)k_s'|^2$$
$$\times\left\{1+\left(\frac{k''v_{Ti}}{\omega_{Li}}\right)^2+\frac{|e|}{e_i}\frac{T_i}{T_e}\left[1-A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}$$
$$\times\left\{\int_0^\infty\frac{v_\perp dv_\perp}{v_{Te}^2}\exp(-v_\perp^2/2v_{Te}^2)J_n\left(\frac{k_\perp v_\perp}{\Omega_c}\right)\right.$$
$$\left.\times J_n\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right)\right\}^2. \tag{2.47}$$

14. In perfect analogy with item 13, we can also consider the formation of an ion-cyclotron harmonic $\omega\approx n\Omega_i$ (spectrum (2.12)), resulting from the interaction of a similar harmonic $\omega'\approx n\Omega_i$ and a long-wave oscillation with spectrum (2.3) ($\omega''\ll\Omega_i$). Such a process is described by the contraction

$$S=-i\frac{e}{m}\frac{\omega_{Le}^2}{\omega\omega'\omega''^2}\frac{k_z k_z' k_z''^2}{kk'k''}\exp\left[-\frac{1}{2}\frac{v_{Te}^2}{\Omega_e^2}(k_\perp^2+k_\perp'^2)\right]$$
$$\times I_0\left(k_\perp k_\perp'\frac{v_{Te}^2}{\Omega_e^2}\right). \tag{2.48}$$

From this we obtain for the kernel Q the expression

$$Q=\frac{1}{8\pi N_e mc^2}\left(\frac{M}{m}\right)^2\left(\frac{k_z k_z' k'' cv_{Ti}^2}{(n\Omega_i)^2\omega_{Le}}\right)^2\left[1+\left(\frac{k_\perp''\omega_{Li}}{k''\Omega_i}\right)^2\right]$$
$$\times A_n\left(\frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}\right)A_n\left(\frac{k_\perp'^2 v_{Ti}^2}{\Omega_i^2}\right)\exp\left[-\frac{v_{Te}^2}{\Omega_e^2}(k_\perp^2+k_\perp'^2)\right]$$
$$\times I_0^2\left(k_\perp k_\perp'\frac{v_{Te}^2}{\Omega_e^2}\right)\left\{1+\frac{k^2 v_{Ti}^2}{\omega_{Li}^2}+\frac{T_i|e|}{T_e e_i}\right.$$
$$\left.\times\left[1-A_0\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}\left\{1+\left(\frac{k'v_{Ti}}{\omega_{Li}}\right)^2\right.$$
$$\left.+\frac{T_i|e|}{T_e e_i}\left[1-A_0\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}. \tag{2.49}$$

15. If the frequency of the long-wave oscillation $\omega'-\Omega_i\sim\Omega_i$ is of the order of the ion gyrofrequency, then this oscillation coalesces with the low-frequency long-wave oscillation ($\omega''\ll\Omega_i$)

with spectrum (2.3) and is transformed into an oscillation with the same spectrum as before $\omega - \Omega_i \sim \Omega_i$ [see (2.6) and (2.7)]. Such a process is determined by a contraction

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega\omega'\omega''^2}\frac{k_z k_z' k_z''^2}{kk'k''} \qquad (2.50)$$

and by a kernel

$$Q = (32\pi N_e mc^2)^{-1}(ck''/\omega_{Le})^2[1 + (k_\perp''\omega_{Li}/k''\Omega_i)^2], \quad (2.51)$$

where $\omega$ and $\omega'$ are functions of the wave vectors **k** and **k'**, in accordance with (2.6) and (2.7).

16. The long-wave oscillations (2.7) with intermediate frequency ($\Omega_i \ll \omega \ll \Omega_e$) can result also from an interaction of the same oscillations with long-wave oscillations having frequencies $\omega'' \ll \Omega_i$ (spectrum (2.3)) and $\omega'' - \Omega_i \sim \Omega_i$ (spectrum (2.6) and (2.7)), as well as with short wave ion-cyclotron oscillations $\omega'' \approx n\Omega_i$ (spectrum (2.12)). These interactions correspond to the contraction (2.50) and a kernel

$$Q = (32\pi N_e mc^2)^{-1}(ck''/\omega_{Le})^2[1 + (k_\perp''\omega_{Li}/k''\Omega_i)^2], \qquad (2.52)$$

if $\omega'' \ll \omega_i$, or

$$Q = (32\pi N_e mc^2)^{-1}(ck_z''/\omega'')^2, \qquad (2.53)$$

if $\omega'' - \Omega_i \sim \Omega_i$, and the spectrum $\omega''(\mathbf{k}'')$ is given by (2.6) and (2.7), or

$$Q = \frac{1}{16\pi N_e mc^2}\left(\frac{e}{e_i}\right)^2\frac{T_i}{T_e}A_n\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right)\left\{1 + \left(\frac{k'' v_{Ti}}{\omega_{Li}}\right)^2 + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}, \qquad (2.54)$$

if $\omega'' \approx n\Omega_i$ (spectrum (2.12)).

17. In addition to the decay processes considered in items 6—16 of the present section, processes in which three longitudinal waves with three different spectra participate are also possible. Let one of the interacting waves be an electron-cyclotron wave with frequency $\omega' \approx \Omega_e$ (spectrum (2.15)). Interaction between this wave and a wave of frequency $\omega'' < \Omega_e$ can result in a long-wave oscillation with frequency of the order of $\Omega_e$ ($\omega - \Omega_e \sim \Omega_e$, spectrum (2.9)). We list below different variants of such interactions.

Let $\omega'' \ll \Omega_i$ and let the spectrum $\omega''(\mathbf{k}'')$ be defined by (2.3). Then

$$S = -\frac{i}{2}\sqrt{\frac{\pi}{2}}\frac{e}{m}\left(\frac{\omega_{Le}}{\omega''}\right)^2\frac{k_z k_z' k_z''^2}{kk'k''}\frac{1}{\omega(\omega' - \Omega_e)}\left(\frac{k_\perp' v_{Te}}{4\Omega_e^2}\right) \times \exp\left[-\frac{1}{4}\left(\frac{k_\perp' v_{Te}}{\Omega_e}\right)^2\right] \times \left[I_0\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right) - I_{3/2}\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right)\right], \qquad (2.55)$$

$$Q = \frac{1}{128 N_e mc^2}\left(\frac{k_z k_z' c v_{Te}}{\Omega_e^2}\right)^2\frac{k''^2}{\omega_{Le}^2}\left[\frac{k_\perp^2\omega^2}{(\omega^2 - \Omega_e^2)^2} + \frac{k_z^2}{\omega^2}\right]^{-1} \times A_1^{-1}\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)\exp\left[-\frac{1}{2}\left(\frac{k_\perp'^2 v_{Te}}{\Omega_e}\right)^2\right]\left(\frac{k_\perp' v_{Te}}{\Omega_e}\right)^2 \times \left[I_0\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right) - I_{3/2}\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right)\right]^2\left[1 + \left(\frac{k_\perp''\omega_{Li}}{k''\Omega_i}\right)^2\right]. \qquad (2.56)$$

If $\omega'' - \Omega_i \gtrsim \Omega_i$ (spectra (2.6) and (2.7)), then the contraction retains as before the form (2.55), and the kernel Q takes the form

$$Q = \frac{1}{128 N_e mc^2}\frac{(k_z k_z' k_z'' c v_{Te})^2}{\Omega_e^4\omega''^2}\left[\frac{k_\perp^2\omega^2}{(\omega^2 - \Omega_e^2)^2} + \frac{k_z^2}{\omega^2}\right]^{-1} \times A_1^{-1}\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)\exp\left[-\frac{1}{2}\left(\frac{k_\perp' v_{Te}}{\Omega_e}\right)^2\right]\left(\frac{k_\perp' v_{Te}}{\Omega_e}\right)^2 \times \left[I_0\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right) - I_{3/2}\left(\frac{k_\perp'^2 v_{Te}^2}{4\Omega_e^2}\right)\right]^2. \qquad (2.57)$$

The oscillation with frequency $\omega''$ can also be short-wave, for example, an ion-cyclotron harmonic of (2.12), $\omega'' \approx n\Omega_i$. Then

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega' - \Omega_e}\frac{k_z k_z' k_z''^2}{kk'k''\omega\omega''^2}\int_0^\infty\frac{v_\perp dv_\perp}{v_{Te}^2} \times \exp\left[-\frac{v_\perp^2}{2v_{Te}^2}\right]J_1\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right), \qquad (2.58)$$

$$Q = (8\pi N_e mc^2)^{-1}\left(\frac{k_z k_z' k_z''^2 v_{Te} v_{Ti} c}{\omega\Omega_e\omega''^2}\right)^2\frac{Me^2}{me_i^2} \times \left[\frac{k_\perp^2\omega^2}{(\omega^2 - \Omega_e^2)^2} + \frac{k_z^2}{\omega^2}\right]^{-1}A_n\left(\frac{k_\perp''^2 v_{Ti}^2}{\Omega_i^2}\right)A_1^{-1}\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right) \times \left\{1 + \frac{k''^2 v_{Ti}^2}{\omega_{Li}^2} + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp''^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2} \times \left\{\int_0^\infty\frac{v_\perp dv_\perp}{v_{Te}^2}\exp\left(-\frac{v_\perp^2}{2v_{Te}^2}\right)J_1\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right)\right\}^2. \qquad (2.59)$$

On the other hand, if one of the interacting waves is an ion-cyclotron wave with spectrum (2.12), the interaction results in a long-wave oscillation with frequency $\omega - \Omega_i$ (spectra (2.6) and (2.7)), and the role of the third interacting wave is played by the long-wave oscillation (2.3), then

$$S = -i\frac{e}{m}\frac{\omega_{Le}^2}{\omega\omega'\omega''^2}\frac{k_z k_z' k_z''^2}{kk'k''} + S_i. \qquad (2.60)$$

The ionic term in (2.60) can be neglected, and we obtain the kernel of the equation in the form of

$$Q = \frac{1}{16\pi N_e mc^2}\left(\frac{k_z' k_z'' v_{Ti} c}{\omega'\omega''}\right)^2\frac{Me^2}{me_i^2}A_1\left(\frac{k_\perp'^2 v_{Ti}^2}{\Omega_i^2}\right) \times \left\{1 + \left(\frac{k' v_{Ti}}{\omega_{Li}}\right)^2 + \frac{|e|T_i}{e_i T_e}\left[1 - A_0\left(\frac{k_\perp'^2 v_{Te}^2}{\Omega_e^2}\right)\right]\right\}^{-2}. \qquad (2.61)$$

## 3. ILLUSTRATIONS AND DEDUCTIONS

Just as in an isotropic plasma, the non-linear interaction of electromagnetic waves in a magnetoactive plasma can be characterized, generally speaking, by some characteristic time $\tau$. Owing to the strong angular dependence of the kernels Q and to the presence of two terms in the right side of the Eq. (1.11) that describes the coalescence of the longitudinal waves, only a very approximate estimate of the characteristic time can be obtained in this case. In some cases $\tau$ does not exist. An example of such a process is decay under the condition

$$\frac{d}{dt} W_l(\mathbf{k}) = \text{const},$$

which does not depend on the time t. It is therefore necessary to specify clearly the conditions under which such an estimate is made. Actually, the distinctions between such conditions correspond to different formulations of problems in the integration of (1.11). In view of the complexity of the latter procedure, we confine ourselves to qualitative deductions that follow from the inequalities between the energies of the interacting waves. If $W_l(\mathbf{k}')$ and $W_l(\mathbf{k}'')$ are small in magnitude compared with $W_l(\mathbf{k})$, then the characteristic time of the process is determined by the second term in the right curly bracket of (1.11). We can therefore write the following approximate relation

$$W_l(\mathbf{k}, t) \sim \exp\left[+t/\tau(\mathbf{k}, \mathbf{k}', \mathbf{h})\right], \tag{3.1}$$

where

$$\tau(\mathbf{k}, \mathbf{k}', \mathbf{h}) \equiv \tau = \pi\left[\int d\mathbf{k}' Q W_l(\mathbf{k}')\,\delta(\mathbf{k}'-\mathbf{k}_0')\,\omega(\mathbf{k})\,\omega''(\mathbf{k}'')\right.$$
$$\left.\times\left|\frac{\partial\mu}{\partial\mathbf{k}'}\right|^{-1}\right]^{-1} \approx \left[\omega(\mathbf{k})\,\omega''(\mathbf{k}'')\left|\frac{\partial\mu}{\partial\mathbf{k}'}\right|^{-1} Q W_l(\mathbf{k}_0')\right]^{-1},$$

$$\mu = \mu(\mathbf{k}, \mathbf{k}') = \omega(\mathbf{k}) \mp \omega'(\pm\mathbf{k}') \mp \omega''(\pm\mathbf{k} \mp \mathbf{k}'), \tag{3.2}$$

and $\tau$ does not depend on t during the time interval in which $W_l(\mathbf{k}) \gg W_l(\mathbf{k}'), W_l(\mathbf{k}'')$. If $W_l(\mathbf{k})$ and $W_l(\mathbf{k}')$ are large compared with $W_l(\mathbf{k}'')$, then we can again neglect the first term of (1.11) in the estimate of $\tau$, but we can no longer state that the "characteristic time" is a constant, since $W_l(\mathbf{k}') = W_l(\mathbf{k}', t)$ is large and

$$\frac{d}{dt}\ln W_l(\mathbf{k}) \neq \text{const}.$$

At any rate, the estimates (3.1) and (3.2) become more approximate in such a formulation of the problem.

Finally, if the energy $W_l(\mathbf{k})$ of the coalesced wave is small compared with $W_l(\mathbf{k}')$ and $W_l(\mathbf{k}'')$, then it is necessary to retain in (1.11) only the first term of the first curly bracket. Eliminating in this case the integral with respect to $d\mathbf{k}'$ by using the mean-value theorem, we can define the characteristic time as the time scale of the quantity

$$\left\{\frac{\omega^2(\mathbf{k})}{\pi} W_l^2(t)\, Q\left|\frac{\partial\mu}{\partial\mathbf{k}'}\right|^{-1}\right\}^{-1}$$

The first term plays the dominating role in the estimate of the time if the energy of all three interacting waves is of the same order of magnitude: $W_l(\mathbf{k}') \sim W_l(\mathbf{k}'') \sim W_l(\mathbf{k})$, and the frequency $\omega''$ of one wave is small compared with two others: $\omega'' \ll \omega$, $\omega'' \ll \omega'$. This approach is particularly convenient for estimates of the characteristic times of decays of unequal type.

In conclusion we present a few estimates of the characteristic times, using one of the described formulations of the problem in each case.

The interaction between three long-wave electron-cyclotron oscillations with spectrum (2.9) (see item 4 of Sec. 2) is described by Eq. (1.11) with a kernel

$$Q \approx (32\pi N_e mc^2)^{-1}(ck/\Omega_e)^2 \tag{3.3}$$

and a characteristic time

$$\tau \sim 3\cdot 10^2 \frac{N_e mc^2}{W_l}\frac{\Omega_e}{(ck)^2}\frac{1}{k^3}. \tag{3.4}$$

The decay of three short-wave electron-cyclotron oscillations with spectrum (2.16) (item 6 of Sec. 2)

$$\omega \approx (l+n)\Omega_e, \quad \omega' \approx n\Omega_e, \quad \omega'' \approx l\Omega_e, \quad l \sim n \sim l+n$$

corresponds to a time

$$\tau \sim 50\frac{N_e mc^2}{W_l}\frac{1}{k^3}\frac{(n\Omega_e)^7}{(\omega_{Le} c v_{Te}^2 k^3)^2}\left[1+\left(\frac{kv_{Te}}{\omega_{Le}}\right)^2\right]^{-4} \tag{3.5}$$

and to a kernel

$$Q \approx \frac{(\omega_{Le} c v_{Te} k^3)^2}{4\pi(n\Omega_e)^8}\frac{1}{N_e mc^2}\left[1+\left(\frac{kv\tau_e}{\omega_{Le}}\right)^2\right]^4 A_n^{-6}\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right)$$
$$\times\left\{\int_0^\infty \frac{v_\perp dv_\perp}{v_{Te}^2}\exp\left(-\frac{v_\perp^2}{2v_{Te}^2}\right) J_n\left(\frac{k_\perp' v_\perp}{\Omega_e}\right)\right.$$
$$\left.\times J_l\left(\frac{k_\perp'' v_\perp}{\Omega_e}\right) J_{l+n}\left(\frac{k_\perp v_\perp}{\Omega_e}\right)\right\}^2. \tag{3.6}$$

The process described at the end of item 12 of the Sec. 2 is characterized by an approximate value of the kernel

$$Q \approx \frac{1}{16\pi}\left(\frac{k^2 c v_{Te}}{\Omega_e^2}\right)^2 \frac{1}{N_e mc^2} A_1^{-2}\left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right) \tag{3.7}$$

and by a time

$$\tau \sim 10^2 \frac{1}{\Omega_e}\left(\frac{\Omega_e^2}{k^2 c v_{Te}}\right)^2 \frac{N_e m c^2}{W_l} \frac{1}{k^3} A_1^2 \left(\frac{k_\perp^2 v_{Te}^2}{\Omega_e^2}\right). \quad (3.8)$$

For greater clarity we present a numerical example. In a plasma with electron density $N_e = 10^{10}$ cm$^{-3}$, situated in an external magnetic field $B_0 = 5 \times 10^4$G, the decay interaction of three long-wave electron-cyclotron oscillations (2.9) has a relaxation time

$$\tau \sim 10^9 k^{-5} / W_l \text{ sec}, \quad (3.4a)$$

where k is in cm$^{-1}$ and $W_l$ is in electron volts.

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[1] V. D. Shafranov, Electromagnetic Waves in a Plasma, Coll. Voprosy teorii plazmy (Problems in Plasma Theory), Atomizdat, 3, 8 (1963).

[2] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and N. K. Stepanov, Kollektivnye kolebaniya v plazme (Collective Oscillations in Plasma), Atomizdat, 1964. K. N. Stepanov, ZhtF **33**, 246 (1963), Soviet Phys. Tech. Phys. **8**, 177 (1963). D. G. Lominadze and K. N. Stepanov, ibid. **34**, 823 (1964), transl. **8**, 618 (1965).

[3] R. E. Peierls, Quantum Theory of Solids, Oxford, 1955.

[4] A. A. Vedenov, Introduction to Theory of Weakly-turbulent Plasma, op. cit.[1] **3**, 235 (1963).

[5] V. P. Silin, PMTF (Appl. Mech. and Tech. Phys.) No. 1, 32 (1964).

[6] L. M. Al'tshul' and V. I. Karpman, JETP **47**, 1552 (1964), Soviet Phys. JETP **20**, 1043 (1965).

[7] L. M. Gorbunov, V. V. Pustovalov, and V. P. Silin, ibid. **47**, 1437 (1964), transl. **20**, 967 (1965).

[8] A. P. Kropotkin and V. V. Pustovalov, Preprint FIAN (Phys. Inst. Acad. Sci.) A-26, 1965.

[9] V. I. Karpman, PMTF (Appl. Mech. and Tech. Phys.) No. 6, 36 (1963).

[10] V. N. Oraevskii, Atomnaya énergiya **16**, 441 (1964).

[11] Yu. F. Filippov, Izv. Vuzov Radiofizika **6**, 932 (1963).

[12] N. P. Giorgadze, ZhTF **34**, 1580 (1964), Soviet Phys. Tech. Phys. **9**, 1224 (1965).

[13] N. L. Tsintsadze, ibid. **34**, 1807 (1964), transl. **9**, 1397 (1965).

[14] N. P. Giorgadze and N. L. Tsintsadze, Izv. Vuzov Radiofizika **7**, 262 (1964).

[15] V. N. Tsytovich and A. Shvartsburg, JETP **49**, 797 (1965), Soviet Phys. JETP **22**, 554 (1966).

[16] V. N. Tsytovich and A. Shvartsburg, ZhTF, in press.

[17] B. B. Kadomtsev, Coll. Voprosy teorii plazmy (Problems in Plasma Theory), Atomizdat, v. 4, 1964, pp. 227, 264, 266.

[18] M. Camac, A. R. Kantrowitz, M. M. Litwak, R. M. Patrick, and H. E. Petschek, Nucl. Fus. Suppl. **2**, 423 (1962).

[19] M. Sumi, J. Phys. Soc. Japan **15**, 1086 (1960).

[20] A. A. Galeev, V. I. Karpman, and R. Z. Sagdeev, Preprint, Nucl. Phys. Res. Inst., Novosibirsk.

[21] J. Allison and G. S. Kino, Appl. Phys. Lett. **5**, 129 (1964).

[22] V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media) Atomizdat, 1961, p. 138.

[23] L. M. Gorbunov and V. P. Silin, JETP **47**, 200 (1964), Soviet Phys. JETP **20**, 135 (1965).

[24] A. A. Rukhadze, Dissertation, Phys. Inst. Acad. Sci. 1964.

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