CYCLOTRON ABSORPTION AND HEATING IN A PLASMA

A. A. ANDRONOV

Radiophysics Institute, Gorkiĭ State University

Submitted to JETP editor May 17, 1965

J. Exptl. Theoret. Physics (U.S.S.R.) **49**, 1293-1303 (October, 1965)

We consider the nonlinear theory of cyclotron resonance in a uniform low-density plasma. The kinetic instability of the quasistationary state of the plasma in a monochromatic plane wave field is discussed. The possibility is indicated of using this instability for turbulent cyclotron heating. An example is given of resonance (continuous) acceleration of a nonrelativistic particle in the electromagnetic field produced by two plane waves.

INTRODUCTION

THE effectiveness of cyclotron heating of a low-density plasma, which derives from the resonance absorption of electromagnetic energy at harmonics of the gyro-frequency of the plasma,[1] is reduced when a steady-state situation is reached—this is the so-called plateau.[2-4] In this state the cyclotron absorption is reduced to very low values and the plasma is not heated. For this reason it is of interest to investigate cases in which the time required for the establishment of this plateau is large or in which the plateau is not formed at all.

These questions are considered in the present work. On the basis of an analysis of the motion of particles in a specified electromagnetic field it is shown that the time in which the cyclotron absorption is reduced can be made large if the electromagnetic wave propagates along the magnetic field with the velocity of light. The absorption of electromagnetic waves under these conditions is considered in the "quasilinear" approximation. A criterion is found for the kinetic instability of the quasi-stationary plasma state in the field of a slow monochromatic plane wave with respect to a wave propagating along the fixed magnetic field H_0. This criterion is found to be the same as the stability criterion for the plateau obtained in quasilinear theory.[5] An instability of this kind can increase the efficiency of cyclotron heating by providing a mechanism for the continuous transfer of energy to the plasma (cf.[5]). The latter situation is illustrated using the example of the nonrelativistic motion of a charged particle in a field produced by two monochromatic plane waves.

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

In this section we treat the motion of a charged particle in the field produced by a plane electromagnetic wave propagating along a fixed magnetic field $\mathbf{H}_0$. This problem has been studied by a number of authors.[6-9] However, since the solution of the problem will be used later in this paper, we present it here in a form suitable for our purposes. Let a plane homogeneous linearly polarized wave*

$$\mathbf{E}_1 = \mathbf{E}_0 \cos\omega(t - \mathbf{n}_0\mathbf{r}/c), \qquad \mathbf{H}_1 = [\mathbf{n}_0\mathbf{E}_1]$$

propagate along the magnetic field:[1)]

$$\mathbf{H}_0 = \{0, 0, H_0\}, \qquad \mathbf{E}_0 = \{E_0, 0, 0\}.$$

In the variables $s = \omega(t - n_0 z/c)$, $\mathbf{u} = \mathbf{p}/m_0 c$ where $\mathbf{p}$ is the momentum, while m_0 is the particle rest mass and c is the velocity of light, the particle equation of motion in the field ($\mathbf{E}_1$, $\mathbf{H}_0 + \mathbf{H}_1$) is

$$\frac{d\mathbf{u}}{ds} = \frac{\lambda}{1 - n_0 u/\varkappa}\left\{\boldsymbol{\mu}\left(1 - \frac{\mathbf{n}_0\mathbf{u}}{\varkappa}\right) + \frac{\mathbf{n}_0}{\varkappa}(\boldsymbol{\mu}\mathbf{u}) + \frac{[\mathbf{u}\mathbf{n}_0]}{\varkappa n_0}\right\}. \quad (1)$$

Here, $\lambda = \omega_H/\omega$, $\omega_H = eH_0/m_0c$, e is the particle charge, $\kappa = (1 + u^2)^{1/2}$, $\mu = E_1/H_0 = \mu_1 \cos s$, $\mu_1 \ll 1$.

We now introduce the "slow" variables A, v, and φ ($u_x = A \sin(s + \varphi)$, $u_y = A\cos(s + \varphi)$, $v = u_z$). After taking averages over the variable s, which appears explicitly, the equations for A,

*$[\mathbf{n}_0 \mathbf{E}_1] \equiv \mathbf{n}_0 \times \mathbf{E}_1$.

1)The motion of a charged particle in the field of a plane wave propagating at an angle to the magnetic field can be studied in a similar fashion if $\mathbf{E}_0 \perp \mathbf{H}_0$ (cf.[9]).

v, and φ are found to be (the dot denotes differentiation with respect to s)

$$\dot{A} = \mu_0 \sin\varphi,$$
$$\dot{\varphi} = \frac{\lambda}{Q} - 1 + \frac{\mu_0}{A}\cos\varphi, \quad \dot{v} = \frac{\mu_0 n_0 A}{Q}\sin\varphi, \tag{2}$$

where $\mu_0 = \mu_1\lambda/2$ (for a circularly polarized wave $\mu_0 = \lambda\mu_1$ or $\mu_0 = 0$ depending on the direction of rotation of the vector $\mathbf{E}_1$ in the wave), $Q = \kappa - n_0 v$.

From (2) we have [2)]

$$q = n_0\varkappa - v = \text{const}; \tag{3}$$

$$\dot{\varphi} = \frac{1}{Q}\frac{\partial H}{\partial v}, \quad \dot{v} = -\frac{1}{Q}\frac{\partial H}{\partial \varphi}. \tag{4}$$

Here

$$H = p(n_0\Delta/p + v)^2/2n_0 + An_0\mu_0\cos\varphi = \text{const},$$
$$p = n_0^2 - 1, \quad \Delta = \lambda - q/n_0. \tag{5}$$

The equations in (4) are the Pfaff equations. The qualitative nature of the trajectories on the phase plane is shown in Figs. 1—3 ($-\pi < \varphi < \pi$). When $n_0^2 \gg 1$ (this case can be obtained from the nonrelativistic equations of motion) the phase trajectories $v = v_1$ (cf. Fig. 1) are similar to the trajectories of a charged particle in an electrostatic field. The width of the trapping region Δv and the oscillation frequency Ω_0 close to the state of equilibrium $v = v_0$ are given approximately by ($\mu_0 \ll 1$)

$$\Delta v \approx 4\sqrt{\mu_0 A_0}, \quad \Omega_0 \approx n_0\sqrt{\mu_0 A_0}/\lambda; \quad v_0 \approx v_1 - \mu_0\lambda/A_0,$$
$$v_1 = (\lambda - 1)/n_0, \quad A_0 = A_{v=v_1}. \tag{6}$$

In many ways the case $n_0^2 < 1$ is similar to the case $n_0^2 > 1$. However, the complete analysis requires taking account of relativistic effects.

Finally, when $n_0^2 = 1$ (the "autoresonance" motion[7] cf. Figs. 2 and 3) the equations for A and φ are

$$\dot{A} = \mu_0\sin\varphi, \quad \dot{\varphi} = \Delta + \mu_0 A^{-1}\cos\varphi \tag{7}$$

and coincide with the averaged equations for a harmonic oscillator with a sinusoidal external force. The parameter Δ characterizes the deviation between the frequency of the external force

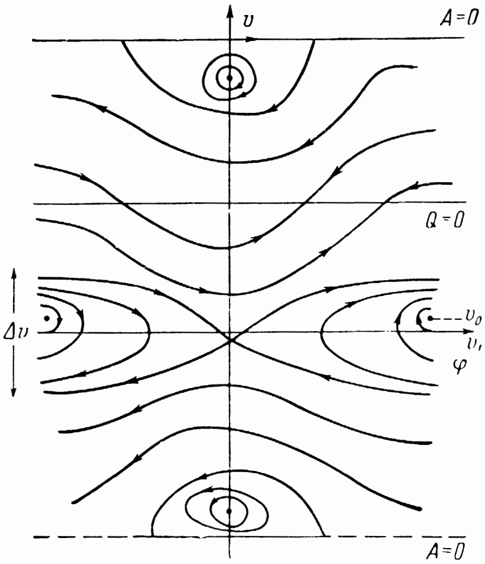

FIG. 1

and the characteristic frequency of the oscillator: $\Delta = 0$ corresponds to the resonance case and $\Delta \neq 0$ to the nonresonance case ($A_0 = -\mu_0/\Delta$, $\Delta < 0$).

GYRO-RESONANCE ABSORPTION OF ELECTROMAGNETIC WAVES

The damping of a wave in a plasma can be determined from the change in particle energy. If the wave damping is small, so that the particle energy does not change greatly because of the change in wave amplitude, the nature of the wave absorption in the plasma can be understood from an analysis of the particle motion in a specified electromagnetic field. The particle energy only changes significantly when the Doppler condition is satisfied:

$$v \approx v_p = (\varkappa - \lambda)/n_0.$$

When $n_0 \gg 1$ the particle energy does not increase continuously. The characteristic time for the reduction of the absorption τ_0 (the time required to establish the plateau) can be estimated from the time required for the dephasing of particles located on different phase trajectories near $v = v_1$ (cf. Fig. 1). This time is determined by the width of the spectrum $\Delta\omega$ of the particle oscillation frequencies. For the slow variables the frequency vanishes on the separatrix. Hence we can write $\Delta\omega \sim \omega_H\Omega_0$, while

$$\tau_0 \sim 1/\Delta\omega \sim 1/n_0\omega\sqrt{\mu_0 A_0}. \tag{8}$$

This relation is similar to the one used in the theory of damping of longitudinal waves and has an analog in the quasilinear theory (cf.[10]).

---

2)From the quantum-mechanical point of view the change in the momentum of the charged particle in the plane wave is along a line determined by the momentum increment due to the radiation of a photon (cf.[5]):

$$\frac{dA}{dv} = \frac{\hbar\omega_H m_0}{p_\perp} \bigg/ \frac{\hbar\omega n_0}{c} = \frac{\lambda}{An_0}.$$

Using the Doppler condition it is easy to show that the function q(A, v) = constant satisfies this equation.

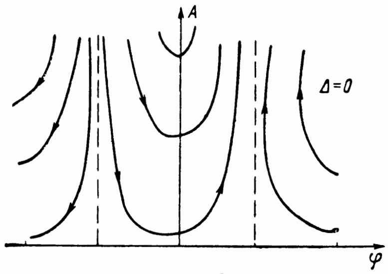

FIG. 2

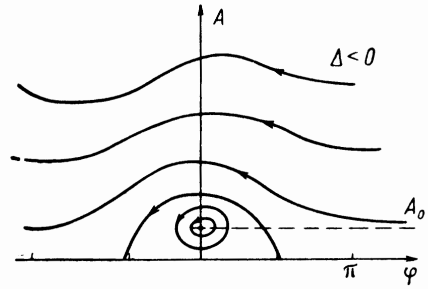

FIG. 3

When $n_0 = 1$ it is possible to have trajectories which vanish at infinity ($\Delta = 0$, Fig. 2). The change in the energy of the plasma particles is computed in similar fashion from the change in the energy of a harmonic oscillator under the effect of a force whose spectral peak is close to the characteristic frequency of the oscillator. The width of the spectrum $\Delta\omega$ is determined by the quantity $\Delta_0 = \langle\Delta\rangle$ the mean value of Δ in the initial particle distribution function (cf. Eq. (7) and Fig. 3). In the nonrelativistic case $\Delta_0 \sim \beta_T$, while

$$\tau_0 \sim 1/\Delta\omega \sim 1/\omega_H\beta_T, \qquad (8')$$

where $\beta_T^2 = T/m_0c^2$, T is the plasma temperature. Thus, when $n_0 = 1$ the time τ_0 is independent of wave amplitude.

A similar conclusion can be drawn from the quasilinear theory. Consider a packet with all possible polarizations propagating along a magnetic field with the velocity of light. The electromagnetic energy (per second) radiated by the charged particle in this wave is [(cf.[11])]

$$dW = \frac{e^2}{8\pi}\frac{p_\perp^2}{m^2}\delta(\omega-\omega_1)k^2\,dk, \qquad (9)$$

where $\mathbf{p} = \{p_\perp, p_z\}$ is the particle momentum, $\omega_1 = \Omega_H - kp_z/m$ (the Doppler condition) $\Omega_H = \omega_H m_0/m$, $m = m_0\kappa$, $k = \omega/c$ is the modulus of the wave vector.

The quasilinear equation for the particle momentum distribution function $f = f(p_\perp, p_z, t)$ is (cf.[5]):

$$\frac{\partial f}{\partial t} = \frac{m_0 e^2\pi^2\omega_H}{p_\perp}\left\{\frac{\partial}{\partial p_\perp}\left[\frac{p_\perp^2}{m^2}\left(\frac{m_0\omega_H}{p_\perp}\frac{\partial f}{\partial p_\perp}\times\int k^2\,dk\frac{\varepsilon_k\delta(\omega-\omega_1)}{\omega^2}+\frac{1}{c}\frac{\partial f}{\partial p_z}\int k^2\,dk\frac{\varepsilon_k\delta(\omega-\omega_1)}{\omega}\right)\right]+\frac{p_\perp}{m_0c\omega_H}\frac{\partial}{\partial p_z}\left[\frac{p_\perp^2}{m^2}\left(\frac{m_0\omega_H}{p_\perp}\frac{\partial f}{\partial p_\perp}\int k^2\,dk\frac{\varepsilon_k\delta(\omega-\omega_1)}{\omega}+\frac{1}{c}\frac{\partial f}{\partial p_z}\int k^2\varepsilon_k\delta(\omega-\omega_1)\,dk\right)\right]\right\} \qquad (10)$$

($(2\pi)\int\epsilon_k k^2dk$ is the electromagnetic energy of the packet per unit volume). In the variables $q = mc - p_z$, $p = p_z$ (diffusion line) the quantity $\omega_1 = \omega_1(q)$ while

$$\frac{\partial f}{\partial t} = \frac{2\pi^3\varepsilon(q)m_0^3\omega_H^2e^2}{c^3q^3}\left\{\frac{\partial}{\partial p}\left(D\frac{\partial f}{\partial p}\right)+\frac{D}{mc}\frac{\partial f}{\partial p}\right\}. \qquad (11)$$

The damping factor γ_0 is

$$\gamma_0 = -4\pi^3\frac{e^2q^2}{m_0\omega_H}\int_{p_1}^{\infty} f\,dp, \qquad p_\perp^2(p_1, q) = 0. \qquad (12)$$

It follows from (11) that the total number of particles on a line q = const is a constant:

$$\int_{p_1}^{\infty}\frac{m}{m_0}f\,dp = \text{const.} \qquad (13)$$

Thus, the damping factor γ_0 in the nonrelativistic case ($m \sim m_0$) does not change in the process of particle diffusion in momentum space and is reduced when the mean particle mass exceeds the rest mass.

Features similar to those considered above must also hold for "oblique" waves with $n_z = k_zc/\omega = 1$ and for waves propagating across the magnetic field $\mathbf{H}_0$ (in the nonrelativistic case). This follows from the fact that the plasma particles behave like harmonic oscillators under these conditions. In this case the damping of the wave is due to the momentum derivative of the intensity of particle radiation and an instability is possible only if the dipole approximation does not hold; the condition for establishment of the plateau is then modified considerably. This result provides another interpretation of the conclusions of Mazitov and Fridman[12] regarding the damping of plasma waves propagating across a magnetic field $\mathbf{H}_0$.

The results given above for a monochromatic wave have been obtained under the assumption that the wave amplitude remains constant. Changes in wave amplitude can be neglected if the motion of the image points along the phase trajectories can be regarded as adiabatic in a wave with variable amplitude. Under these conditions the increase in energy of plasma particles due to the change in wave amplitude is small compared with the increase in the energy of the plasma particles due to an electromagnetic wave with fixed amplitude.

If γ is the wave damping factor in the equilibrium plasma the wave amplitude can be regarded as constant if

$$\gamma \ll 1/\tau_0. \qquad (14)$$

Thus, the wave amplitude must be large enough so that the plateau is established in a time short compared with the time required for a significant change in wave amplitude.

CYCLOTRON HEATING OF A PLASMA

The efficiency of cyclotron heating of a plasma is determined not only by the characteristic time τ_0, but also by the magnitude of the damping factor γ which, in general, increases with increasing n_0. The use of a wave characterized by $n_0 \gg 1$ for plasma heating is also desirable because the development of the instability in the plateau serves as an effective means for heating the plasma if the plateau is unstable. The analogous problem in the quasilinear approximation has been treated by Trakhtengertz and the author.[5] We discuss this problem here for a monochromatic wave.

1. We first consider (in the nonrelativistic approximation) the kinetic instability of the quasi-stationary state (plateau) in the field of a slow ($n_0 \gg 1$) monochromatic circularly polarized wave propagating along the magnetic field. The plateau is established in a time $t \gtrsim \tau_0$ and it can be assumed[3)] that when $t \gg \tau_0$ the distribution function on the plateau is a function only of the integrals of motion of the particles and the field (cf. 2—5) i.e., $f = f_0 = f_0(q, H)$. We assume, as is usually done in the quasilinear theory, that the establishment of the plateau only changes the wave growth rates in the plasma. We now determine the linear response of the plateau state to a plane electromagnetic wave with specified ω and k. An expression for the perturbation current can be written if one knows the unperturbed motion of the particles in the plateau state (cf. for example[11,13]).

For small amplitudes the trajectory close to $v = v_1$ (cf. Fig. 1) can be approximated by

$$s = \omega(t - n_0 z/c) = \omega_H t.$$

The unperturbed trajectories averaged over the gyro-frequency are then given by the equations:

$$\dot{A} = \mu_0(1 - n_0 v)\sin\varphi, \quad \dot{\varphi} = \lambda - 1 + n_0 v + (\mu_0/A)\cos\varphi,$$

$$\dot{v} = \mu_0 n_0 A \sin\varphi, \quad \dot{\xi} = v, \quad \xi = z\omega_H/c. \qquad (15)$$

Whence

$$q = A^2 + \left(\frac{1}{n_0} - v\right)^2 = \text{const}, \quad \dot{\varphi} = \frac{\partial H}{\partial v}, \quad v = -\frac{\partial H}{\partial \varphi},$$
$$H = {}^1/_2 n_0 (v - v_1)^2 + \mu_0 A \cos\varphi = \text{const}. \qquad (16)$$

Let

$$v_\gamma = v_\gamma(\tau, \tau', \mathbf{v}'), \qquad \mathbf{r} = \mathbf{r}' + \int_{\tau'}^{\tau} \mathbf{v}(\tau_0, \tau', \mathbf{v}')\,d\tau_0$$

be the unperturbed velocity and radius vector of the charged particle at time τ as functions of the "initial" velocity $\mathbf{v}'$ and radius vector $\mathbf{r}'$ at time τ'. The solution of the linearized kinetic equation for the correction f_1 to the distribution function $f = f_1 + f_0$ (due to the electromagnetic perturbation field $\mathbf{E} = \mathbf{E}_0 \exp[i\omega t - i\mathbf{k}\cdot\mathbf{r}]$, $\mathbf{H} = (c/\omega)[\mathbf{k}\times\mathbf{E}]$) is of the form (cf.[13])

$$f_1 = -\frac{e}{m}\sum_{\alpha,\beta}\frac{\partial f_0}{\partial \xi_\alpha'}\int_{-\infty}^{\tau'} F_{1\beta}\frac{\partial \xi_\alpha'}{\partial v_\beta}\,d\tau, \qquad (17)$$

where ξ_α' represents an arbitrary function of the initial velocity and coordinates

$$F_{1\beta} = \left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta_{\beta\sigma} + \frac{k_\beta v_\sigma}{\omega}\right]$$
$$\times E_{0\sigma}\exp(i\omega\tau - i\mathbf{kr}(\tau, \tau')) = S_{\beta\sigma}E_\sigma.$$

The ξ_α' are taken to be the integrals of the motion q and H. If the wave amplitude is small enough in the plateau regime we can assume that $f = f(q)$ and is independent of H for the trapped particles whose phase trajectories encompass the equilibrium state $v = v_0$ (cf. Fig. 1). This follows from the fact that the number of particles on each phase trajectory is the same for small μ_0 and a smooth initial distribution function $f = f(v, A)$. Hence the current j in the perturbation wave is given by

$$j_i = e\int v_i' f_1\,d^3\mathbf{v}' = \sigma_{ik}E_k, \qquad \sigma_{ik} = \sigma_{ik}' + i\sigma_{ik}'',$$

$$\sigma_{ik}' = -\operatorname{Re}\frac{e^2}{m}\int d^3\mathbf{v}' v_i' \frac{\partial f_0}{\partial q}\int_{-\infty}^{\tau'} S_{\beta k}\frac{\partial q}{\partial v_\beta}\,d\tau. \qquad (18)$$

Here it is assumed that σ_{ik}' contains a contribution due to the trapped particles only. The conditions for which this statement is valid are indicated below.

We limit our analysis of the plateau instability to waves propagating along the magnetic field. As shown below, in this case the criterion for plateau instability coincides with the criterion for the plateau instability in the quasilinear theory.[5] The latter is valid for an arbitrary direction of

---

3)In the field of a specified electromagnetic wave the particle distribution function f does not approach the stationary distribution function $f = f_0(q, H)$ when $t \gg \tau_0$. However, the integral over f for any finite region of phase space tends toward the integral over f_0 for large t. Furthermore, the existence of collisions means that $f \to f_0$ (cf.[10]).

propagation both for the waves which form the plateau as well as for waves which are unstable at the plateau.

Wave absorption for arbitrary propagation is determined by the following components of the conductivity tensor: σ'_{xx}, σ'_{xy}, σ'_{yy}, σ'_{zz}. Let us consider σ'_{xx} and σ'_{zz} (the other elements can be treated in similar fashion):

$$\sigma_{xx}' = -\operatorname{Re}\frac{e^2}{m}\int d^3\mathbf{v}' v_x' \frac{\partial f_0}{\partial q}\exp[-i\omega\tau' + ikz(\tau')]$$
$$\times\int_{-\infty}^{\tau'}\exp[i\omega\tau - ikz(\tau)]\left\{\left(1-\frac{kv_z}{\omega}\right)\delta_{\beta x} + \frac{k_\beta v_x}{\omega}\right\}\frac{\partial q}{\partial v_\beta}d\tau$$
$$= 2\operatorname{Re}\frac{e^2}{m_0}\left(1-\frac{n}{n_0}\right)\int d^3\mathbf{v}' v_x' \frac{\partial f_0}{\partial q}\exp[i\omega\tau' + ikz(\tau')]$$
$$\times\int_{-\infty}^{\tau'}\exp[i\omega\tau - ikz(\tau)]\, v_x(\mathbf{v}', \tau', \tau)\, d\tau, \tag{19}$$

$n = ck/\omega$. Using the variables A', φ', and q we find

$$v_x = A(\tau-\tau', q, A', \varphi')\cos[\omega_H\tau + \varphi(\tau-\tau', q, A', \varphi')],$$
$$v_x' = A'\cos[\omega_H\tau' + \varphi'].$$

Here we have taken account of the fact that (14) is autonomous.

The tensor σ_{ik} can be time dependent. Hence, in evaluating the stability criterion we average the values of the tensor components over the period $\tau = 2\pi/\omega_H$:

$$\langle\sigma_{xx}'\rangle = -\operatorname{Re}\frac{e^2}{m_0}\left(1-\frac{n}{n_0}\right)\int d^3\mathbf{v}'\frac{\partial f_0}{\partial q}$$
$$\times\int_0^\infty A'A(\tau_0)\exp[i\omega\tau_0 - ikz(\tau)$$
$$+ ikz(\tau')]\cos(\omega_H\tau_0 + \varphi(\tau_0) - \varphi')\,d\tau_0,$$
$$\tau_0 = \tau - \tau'. \tag{20}$$

From (14) we have

$$z(\tau) - z(\tau') = \frac{c}{n_0\omega_H}[\varphi(\tau_0) - \varphi']$$
$$+ cv_1\tau_0 - c\mu_0\int_0^{\tau_0}\frac{(1-n_0v)}{A}\cos\varphi\,d\tau_0. \tag{21}$$

If $\mu_0 A \sim A'$ is small the harmonic components of the last term in (21) can be neglected. Then

$$\langle\sigma_{xx}'\rangle \approx -\operatorname{Re}\frac{e^2}{2m_0}\int d^3\mathbf{v}'\frac{\partial f_0}{\partial q}(A')^2\sum_\pm\int_0^\infty\exp\left[i(\omega\pm\omega_H)\tau_0\right.$$
$$\left.- ikcv_0\tau_0 - i\left(\frac{kc}{n_0\omega_H}\pm 1\right)(\varphi-\varphi')\right]d\tau_0. \tag{22}$$

The time dependence of φ, the phase in the trapping region, is represented by an infinite sum of harmonics of the rotational frequency $\Omega(H)$ along the phase trajectory; outside of this region, but close to it, there is a term $\Delta\omega t$ with $\Delta\omega \approx \omega_H n_0\Delta v/2$, while Δv is given by (6). It then follows that $\langle\sigma'_{xx}\rangle$ is determined by the trapped particles if

$$|\omega\pm\omega_H - kv_0| < {}^1/_2(kc/n_0\omega_H \pm 1)\,n_0\Delta v\omega_H. \tag{23}$$

The resonance condition for which absorption (amplification) of the electromagnetic wave occurs is written in the form

$$\omega\pm\omega_H - kcv_0/\omega_H + l\Omega(H)\omega_H = 0,$$
$$l = 0, \pm1, \ldots \tag{24}$$

Thus, $\langle\sigma'_{xx}\rangle$ is determined by a discrete series of trajectories in the trapping region. If we consider the variables q, H, and ψ, where ψ is an angular variable, it is easy to show that

$$\operatorname{Re}\left\{\int dv_z'\,d\varphi'\int_0^\infty\exp\left[i\Delta\omega_\pm\tau_0\right.\right.$$
$$\left.\left.+ i\left(\frac{kc}{n_0\omega_H}\pm1\right)(\varphi-\varphi')\right]d\tau_0\right\} > 0,$$
$$\Delta\omega_\pm = \omega\pm\omega_H - kcv_0. \tag{25}$$

Since we assume that the relation between ω and k is given and corresponds to an equilibrium plasma, the instability condition is given by $\langle\sigma'\rangle < 0$. If $\partial f_0/\partial q < 0$, is the case when a plateau is formed on the Maxwellian distribution function (cf.[5]), this relation yields

$$1 - n/n_0 < 0. \tag{26}$$

For longitudinal waves, with damping given by the σ_{zz} component, we have (17)

$$S_{\beta z}\frac{\partial q}{\partial v_\beta} = v_z - \frac{c}{n_0}. \tag{27}$$

If $|\omega - kv_0| < c\Delta v$ the absorption is determined only by the trapped particles and to accuracy of order $\sqrt{\mu_0}$ we find $v_z = \omega/k$.

Thus, $\langle\sigma'_{zz}\rangle$ contains a factor which is approximately equal to $1 - n/n_0$ and the stability condition (to this accuracy) again reduces to (26).

If we use the Doppler condition, the plateau stability criterion obtained in[5] can be written in the form

$$1 - n_z^1/n_z^0 < 0, \quad n_z^\alpha = k_z^\alpha c/\omega^\alpha, \quad \alpha = 0.1, \tag{28}$$

where $\mathbf{k}^0$ and $\mathbf{k}^1$ are the wave vectors respectively for the waves which form the plateau and the waves which are unstable at the plateau. For longitudinal propagation the criterion in (28) coincides with that in (26).

2. In a paper by Trakhtengertz and the author[5] it has been shown in the quasilinear approximation that the energy of a nonrelativistic particle increases continuously in the development of a plateau instability (in which case the velocity of the particles satisfies several (two) Doppler conditions for electromagnetic waves with different projections k_z). In order to understand this phenomenon let us consider the interaction of a nonrelativistic charged particle with two monochromatic waves. If the particle velocity simultaneously satisfies two Doppler conditions this means that there exists a coordinate system in which the frequencies of both waves are equal to ω_H. Assume that these waves propagate along the magnetic field $\mathbf{H}_0$ and that they see refractive indices n_0, $n_1 > 0$, and are polarized in one plane. If h_0 and h_1, the magnetic fields of these waves, satisfy $|h_0| = |h_1| = h$, the equation of motion of the particle in the electromagnetic field produced by these two waves together with the magnetic field $\mathbf{H}_0$ has solutions of the following kind:

$$\dot{A} = -\Delta n \frac{h}{H_0}\sin\varphi, \qquad \dot{\varphi} = -\Delta n \frac{h}{H_0 A}\cos\varphi,$$
$$v = 0, \quad \xi = \xi_+ \quad (h_1 = h_0), \qquad \xi = \xi_- \quad (h_1 = -h_0),$$
$$\xi_- = \frac{2\pi l}{\Delta n}, \qquad \xi_+ = \frac{(2l-1)\pi}{\Delta n},$$
$$\Delta n = n_1 - n_0, \qquad l = 1, 2, \ldots. \tag{29}$$

Similar solutions can be obtained for waves with different polarization. The coordinates $\xi = \xi_\pm$ define planes in which the time-averaged magnetic force of the waves vanishes but in which the time-averaged electric force does not vanish. For waves with the same polarization these are planes in which the variable magnetic field vanishes. Thus, under these conditions the transverse energy of the particle increases whereas the longitudinal velocity and the coordinate remain unchanged. The relations in (29) are similar to those considered above and the integral curves for (29) are the same as those in Fig. 2.

Let us now consider the work done by the electric fields of these waves on the particles. From the particle equation of motion we find

$$\frac{e}{m}(\mathbf{vE}_0) = \frac{c dv_z/dt - n_1 d(v^2/2)/dt}{n_0 - n_1},$$
$$\frac{e}{m}(\mathbf{vE}_1) = \frac{c dv_z/dt - n_0 d(v^2/2)/dt}{n_1 - n_0}, \tag{30}$$

where $\mathbf{E}_0$ and $\mathbf{E}_1$ are the electric fields of the waves while v is the particle velocity. Since $dv_z/dt = 0$, it is evident from (30) that as the transverse energy of the particle increases the work associated with the wave with the high refractive index is negative while that of the wave with the low refractive index is positive, in accordance with the relations between the electric fields at the point $\xi = \xi_\pm$. These results are in complete agreement with the results of the quasilinear theory.[5]

We note that in the relativistic case there is generally no reason to expect a continuous acceleration of the electron in the field associated with two plane waves (this also applies for the plateau instability) since two Doppler conditions can be satisfied simultaneously.

3. We now wish to discuss the role of the plateau instability considered above in connection with ion-cyclotron heating.[1] If one uses the cold plasma formula it is easy to show[13] that on the Alfvén branch of the plasma waves (which is usually used for cyclotron heating) the refractive index $n_z^A = n_z^A(p_z)$ is independent of the angle between the direction of propagation of the wave and the external magnetic field. For fast magnetoacoustic waves $n_z < n_z^A$. Thus, cyclotron heating is stable with respect to the cold plasma wave in the approximation being considered here. The instability of cyclotron heating with respect to the ion-acoustic branch is suppressed by Cerenkov absorption. However, the plateau instability can arise in a non-isothermal plasma since it is analogous to the instability due to the anisotropy in plasma temperature. It is well known that the latter instability can arise on the ion-acoustic branch in a nonisothermal plasma.[14]

Finally, if one uses oblique fast magnetosonic waves for cyclotron heating the plateau will be unstable with respect to the Alfvén branch. However, the cyclotron absorption factor is small for magnetosonic waves.

CONCLUSION

We have discussed the conditions under which there is effective transfer of energy to a plasma. This situation arises when no plateau is formed or when the plateau is unstable. We wish to emphasize that we have only considered a homogeneous plasma and even a weak plasma inhomogeneity can change the results given above. The condition for which an inhomogeneity does not effect the stability criterion (24) can be obtained by comparing the time for establishing the plateau τ_0 and the time τ_1 for the displacement of the plateau on the distribution function due to an inhomogeneity with magnitude of the order of the trapping region.[4] If the

[4] A similar situation occurs in the damping of plasma oscillations propagating across a magnetic field.[12]

effect of the magnetic field inhomogeneity on the particle motion is neglected,

$$c\Delta v \approx \frac{\partial}{\partial z}\left(\frac{\omega_0 - \omega_H}{k_0}\right)\tau_1 \frac{\omega_0 - \omega_H}{k_0} \quad \text{for} \quad \frac{\omega_0 - \omega_H}{k_0} \gg c\Delta v \tag{31}$$

and the inhomogeneity will be unimportant if

$$\mu_0 v_T \gg \frac{1}{8n_0\omega_0}\frac{\partial}{\partial z}\left(\frac{\omega_0 - \omega_H}{k_0}\right)^2, \quad v_T^2 = \frac{T}{m_0}, \quad k_0 = \frac{\omega_0 n_0}{c}. \tag{32}$$

If $(\omega_0 - \omega_H)/k_0 \sim v_T$, the condition in (29) is written in the form

$$\mu_0 kL / \beta_T \gg 1, \tag{32a}$$

where $\beta_T = v_T/c$ and L is the characteristic scale size of the plasma inhomogeneity.

Thus, a plateau can be established and can be unstable in an inhomogeneous plasma in the field of an electromagnetic wave with amplitude that satisfies (32). If the inverse inequality to (22) holds the plateau is not formed and the wave absorption is evidently described by the linear theory.

I wish to thank V. K. Yulpatov for valuable comments and N. G. Denisov for discussion of the manuscript.

[1] T. H. Stix and R. W. Palladino, Phys. Fluids **1**, 446 (1958); W. M. Hooke and M. A. Rothman, Nuclear Fusion **4**, 33 (1964).

[2] Vedenov, Velikhov, and Sagdeev, Nuclear Fusion **1**, 82 (1961).

[3] W. Drummond and D. Pines, 1st International Conference on Plasma Physics and Controlled Thermonuclear Fusion, Salzburg, 1961, No. 134.

[4] A. A. Vedenov, Atomnaya énergiya (Atomic Energy) **13**, 5 (1962); Voprosy teorii plazmy (Problems of Plasma Physics) Vol. 3, Atomizdat, 1963.

[5] A. A. Andronov and V. Yu. Trakhtengerts, JETP **45**, 1009 (1963), Soviet Phys. JETP **18**, 698 (1963).

[6] T. H. Stix, Theory of Plasma Waves, McGraw Hill, New York, 1962.

[7] A. A. Kolomenskiĭ and A. H. Lebedev, JETP **44**, 261 (1963), Soviet Phys. JETP **17**, 179 (1963).

[8] J. W. Dungey, J. Fluid Mech., **15**, 74 (1963).

[9] I. A. Gilinskiĭ and K. A. Ryazantsev, Izv. Vuzov, Radiofizika **7**, 838 (1964).

[10] V. E. Zakharov and V. I. Karpman, JETP **43**, 490 (1963), Soviet Phys. JETP **16**, 351 (1963); V. I. Karpman, PMTF (Appl. Math. and Theor. Phys.) No. 4, 3 (1964).

[11] V. D. Shafranov, Voprosy teorii plazmy (Problems of Plasma Physics) Vol. 3, Atomizdat (1963).

[12] R. K. Mazitov and A. M. Fridman, Preprint, Novosibirsk, 1965.

[13] M. I. Petelin, Izv. Vuzov, Radiofizika **5**, 736 (1965).

[14] M. I. Pistunovich and B. D. Shafranov, Nuclear Fusion **1**, 189 (1961).

[15] A. B. Timofeev, JETP **39**, 397 (1960), Soviet Phys. JETP **12**, 280 (1961).

Translated by H. Lashinsky
166