CONTRIBUTION TO THE THEORY OF HOT ELECTRONS IN AN ANISOTROPIC SEMICONDUCTOR

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The stationary state and fluctuations of the system of electrons in an anisotropic semiconductor in a strong electric field are considered for the case of scattering by acoustic phonons within the limits of a single energy minimum with a nondegenerate bottom. The calculation is performed within the framework of the deformation-potential theory without introduction of a relaxation time. The results are expressed in terms of the weak field electric conductivity tensor.

1. INTRODUCTION

THE principles of the theory of hot electrons were laid down in a paper of Davydov [1], who obtained the stationary nonequilibrium distribution function of the carriers and the current in a semiconductor in a strong electric field. It was assumed that the scattering is from acoustic phonons, and that the electron and phonon spectra and the transition probabilities are isotropic. In the case of a nonequilibrium system it is of interest, in addition to finding the stationary state, to calculate the fluctuation deviations from this state, for there are no general relations of the Kallen-Welton type in the nonequilibrium case. This problem, under the same assumptions concerning the mechanism of scattering and the spectrum, was solved by Price [2] (case of low-frequency fluctuations) and by one of the authors [3] (general case).

Real semiconductors differ in their properties from the simplest model on which Davydov's theory was based in at least two respects: a) the scattering is not only by the acoustic phonons but also by the optic phonons and by the phonons that give rise to intervalley transitions; b) the electron and phonon spectra and the probabilities of the electron-phonon scattering are anisotropic. Many papers have been devoted to an account of these circumstances (see the review by Page [4] and also [5-7]). The anisotropy, however, has been taken into account so far only to the degree to which it admits of introduction of the time or the relaxation-time tensor.

The purpose of this paper is to make the analysis more general, within the framework of the deformation-potential theory, by taking into consideration the anisotropy of the crystal for the case of scattering by acoustic phonons within one energy valley with nondegenerate bottom.

We shall show that owing to the quasielasticity of the scattering, the hot electrons in an anisotropic crystal are described, in spite of the apparent complexity of the picture, by the same characteristic equations as in the isotropic model. The anisotropy, on the other hand, is manifest in the dependence of the corresponding parameters in these equations on the electric conductivity tensor σ in a weak electric field, and on the material constant—a certain combination of the components of the reciprocal effective mass tensors and the deformation potential. In particular, the anisotropy does not influence the form of the energy distribution function $F(\epsilon)$, but makes the parameter that characterizes the degree of heating dependent on the orientation of the field relative to the principal axes of the ellipsoid:

$$F = F(a\sigma_{\parallel}E^2, \varepsilon), \qquad (1.1)$$

where a is proportional to the indicated material constant, and $\sigma_{\parallel}$ is the electric conductivity in the weak field in the direction in which a strong field $\mathbf{E}$ has now been produced. The dependence of the current density on the magnitude and direction of the field is given by the formula[1)]

$$J_i = \varkappa(a\sigma_{\parallel}E^2)\,\sigma_{ik}E_k. \qquad (1.2)$$

[1)]When the electric conductivity tensor σ_{ik} is proportional to the tensor of the reciprocal effective mass $(1/m)_{ik}$, expression (1.2) goes over into the corresponding formulas of [5,7]. Price[7] also constructed a fluctuation theory for this case.

The analytic form of the functions F and κ, as already mentioned, is the same as in the isotropic model. The appearance of the parameter a in the formulas makes it possible in principle to determine experimentally the corresponding combination of the effective masses and the constants of the deformation potential.

The conclusion that the nature of the equations and the form of the energy distribution function are independent of the anisotropy, deduced for scattering by acoustic phonons, is in general true for such a quasielastic scattering mechanism, in which the transition probability W is homogeneous as a function of the variable p_i and $p_{i'}$ ($i = x, y, z$). If the relaxation with respect to the momenta is determined, for example, by two scattering mechanisms, which depend differently on the energy, then the dependence of the distribution function and of the current on the orientation of the electric field is in general more complicated in the anisotropic case. On the other hand, in a real situation the momentum relaxation is determined only by the acoustical phonons, and the energy loss by the electron is noticeably influenced also by the contribution of the optical phonons. We can generalize our analysis to such a case.

2. DISTRIBUTION FUNCTION AND CURRENT IN THE STATIONARY CASE

Let the electron and the phonon dispersion laws be

$$\varepsilon_{\mathbf{p}} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} = \sum_\alpha \frac{p_\alpha^2}{2m_\alpha},$$

$$\omega_{\mathbf{q}j} = w_{\mathbf{n}j} q, \quad \mathbf{n} = \frac{\mathbf{q}}{q}. \tag{2.1}$$

Then the transition probability is

$$W_{\mathbf{pp}'} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}j} |c_{\mathbf{q}j}|^2 [N_{\mathbf{q}j}\delta_{\mathbf{p}',\,\mathbf{p}+\hbar\mathbf{q}}\,\delta(\varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}} - \hbar\omega_{\mathbf{q}j})$$
$$+ (N_{\mathbf{q}j} + 1)\,\delta_{\mathbf{p}',\,\mathbf{p}-\hbar\mathbf{q}}\,\delta(\varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{q}j})], \tag{2.2}$$

$$|c_{\mathbf{q}j}|^2 = \frac{\hbar}{2\rho\omega_{\mathbf{q}j}}(\Lambda_{\alpha\beta}\, e_j^\beta q_\alpha)^2. \tag{2.3}$$

Here $\Lambda_{\alpha\beta}$ is the deformation-potential tensor and $\mathbf{e}_j$ are the orthonormal polarization vectors.

The stationary distribution function $F_{\mathbf{p}}$ of the electrons in the electric field $\mathbf{E}$ is described by the kinetic equation

$$eE_i \frac{\partial F_{\mathbf{p}}}{\partial p_i} = IF_{\mathbf{p}}, \tag{2.4}$$

where I is the collision operator

$$IF_{\mathbf{p}} = \sum_{\mathbf{p}'}(W_{\mathbf{p}'\mathbf{p}}F_{\mathbf{p}'} - W_{\mathbf{pp}'}F_{\mathbf{p}}). \tag{2.5}$$

The manipulations required to solve (2.4) are similar to those given in a paper by Gantsevich and one of the authors [8], the method of which will now be employed. Resolving $F_{\mathbf{p}}$ into even and odd parts:

$$F_{\mathbf{p}} = F_{\mathbf{p}}^+ + F_{\mathbf{p}}^-, \quad F_{\mathbf{p}}^\pm = \frac{1}{2}(F_{\mathbf{p}} \pm F_{-\mathbf{p}}),$$
$$F_{-\mathbf{p}}^\pm = \pm F_{\mathbf{p}}^\pm \tag{2.6}$$

and using the fact that the operator I conserves parity (since $W_{-\mathbf{p},-\mathbf{p}'} = W_{\mathbf{p},\mathbf{p}'}$ if $|c_{-\mathbf{q}}|^2 = |c_{\mathbf{q}}|^2$ and $\omega_{-\mathbf{q}} = \omega_{\mathbf{q}}$), we obtain the equations

$$eE_i \frac{\partial F^-}{\partial p_i} = IF^+, \tag{2.7}$$

$$eE_k \frac{\partial F^+}{\partial p_k} = IF^- \tag{2.8}$$

or

$$F^- = eE_k I^{-1} \frac{\partial F^+}{\partial p_k}. \tag{2.9}$$

In the function F^+ we separate in turn the part that depends only on the energy

$$F_{\mathbf{p}}^+ = F(\varepsilon_{\mathbf{p}}) + \Delta F_{\mathbf{p}}, \tag{2.10}$$

$$F(\varepsilon_{\mathbf{p}}) = \sum_{\mathbf{p}'}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})F_{\mathbf{p}'}^+ \Big/ \sum_{\mathbf{p}'}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'}),$$

$$\sum_{\mathbf{p}'}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})\Delta F_{\mathbf{p}'} = 0. \tag{2.11}$$

The scattering by the acoustic phonons is almost elastic

$$\hbar\omega_{\mathbf{q}j}/\bar{\varepsilon} \sim (mw^2/\bar{\varepsilon})^{1/2} \ll 1, \tag{2.12}$$

and therefore the energy relaxation is much slower than the quasi-momentum relaxation. We introduce for estimating purposes the characteristic times τ_ϵ and τ_p such that in order of magnitude we have

$$If(\varepsilon) \sim \tau_\varepsilon^{-1} f(\varepsilon), \quad If(\mathbf{p}) \sim \tau_p^{-1} f(\mathbf{p}) \tag{2.13}$$

for the function $f(\epsilon)$ that depends only on the energy, and for the function $f(\mathbf{p})$ with vanishing mean value on the surface $\epsilon_{\mathbf{p}} = \text{const}$, respectively. Neglecting inelasticity, we have

$$If(\mathbf{p}) = \frac{2\pi}{\hbar}\sum_{\mathbf{q}j}|c_{\mathbf{q}j}|^2(2N_{\mathbf{q}j} + 1)\,\delta(\varepsilon_{\mathbf{p}+\hbar\mathbf{q}} - \varepsilon_{\mathbf{p}})$$
$$\times [f(\mathbf{p} + \hbar\mathbf{q}) - f(\mathbf{p})], \tag{2.14}$$

so that $If(\epsilon_{\mathbf{p}}) = 0$ in the elastic approximation; in the first nonvanishing approximation

$$\frac{If(\varepsilon)}{If(\mathbf{p})} \sim \frac{mw^2}{T}\frac{f(\varepsilon)}{f(\mathbf{p})} \tag{2.15}$$

or

$$\tau_p / \tau_\varepsilon \sim mw^2 / T. \quad (2.16)$$

Let us estimate the ratio of the functions $\Delta F_p / F(\epsilon_p)$ in (2.10). We substitute (2.9) and (2.10) in (2.7); we get

$$e^2 E_i E_k \frac{\partial}{\partial p_i} I^{-1} \frac{\partial}{\partial p_k} (F(\varepsilon_p) + \Delta F) = IF(\varepsilon_p) + I\Delta F \quad (2.17)$$

or

$$\frac{e^2 E^2 \tau_p}{p^2} (F(\varepsilon_p) + \Delta F) \sim \frac{1}{\tau_\varepsilon} F(\varepsilon_p) + \frac{1}{\tau_p} \Delta F \quad (2.18)$$

($I^{-1} \partial F(\epsilon_p)/\partial \mathbf{p} \sim \tau_p \partial F(\epsilon_p)/\partial \mathbf{p}$). Since $\tau_p \ll \tau_\epsilon$, the electron loses within a time τ_p an energy $\Delta\epsilon \sim (\tau_p/\tau_\epsilon)$; on the other hand, during this time the electron receives from the field an energy $\Delta\epsilon \sim eEV\tau_p$, where V is the average drift velocity, determined by the condition $mV \sim eE\tau_V$. Comparing both values of $\Delta\epsilon$, we obtain

$$e^2 E^2 \tau^2_p / m\varepsilon \sim \tau_p / \tau_\varepsilon. \quad (2.19)$$

Then, from (2.18)

$$\Delta F / F(\varepsilon_p) \sim \tau_p / \tau_\varepsilon \sim mw^2 / T. \quad (2.20)$$

This, however, denotes that both terms in the right side of (2.17) are of the same order. To make the second term small, we average (2.17) over the constant-energy surface

$$eE_i \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) \frac{\partial F^-}{\partial p_i} = \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) IF(\varepsilon_{\mathbf{p}}) + \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) I\Delta F_{\mathbf{p}}. \quad (2.21)$$

Since

$$\bar{I}f(\mathbf{p}) \equiv \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) If(\mathbf{p}) = \sum_{\mathbf{pp'}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) [W_{\mathbf{p'p}} f(\mathbf{p'}) - W_{\mathbf{pp'}} f(\mathbf{p})] = \sum_{\mathbf{pp'}} W_{\mathbf{p'p}} f(\mathbf{p'}) (\delta(\varepsilon - \varepsilon_{\mathbf{p}}) - \delta(\varepsilon - \varepsilon_{\mathbf{p'}})), \quad (2.22)$$

$\bar{I}f(\mathbf{p})$ vanishes in the elastic approximation, unlike $If(\mathbf{p})$: at the same time the averaging does not change the order of magnitude of the term $IF(\epsilon_p)$; thus

$$\bar{I}\Delta F / \bar{I}F(\varepsilon_p) \sim \sqrt{mw^2 / T}. \quad (2.23)$$

The terms with ΔF in (2.9) and (2.21) are therefore small. Discarding them and integrating the left side of (2.21) by parts, we get

$$eE_i \frac{d}{d\varepsilon} \Big(\sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) v_i F^- \Big) = \bar{I}F(\varepsilon_{\mathbf{p}}), \quad (2.24)$$

$$F^- = eE_k I^{-1} v_k \frac{dF(\varepsilon_{\mathbf{p}})}{d\varepsilon_{\mathbf{p}}} \quad \Big(v_i \equiv \frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i} \Big). \quad (2.25)$$

In the elastic approximation, as can be seen from (2.14)

$$If(\varepsilon_{\mathbf{p}}) f(\mathbf{p}) = f(\varepsilon_{\mathbf{p}}) If(\mathbf{p}). \quad (2.26)$$

This property is possessed also by the inverse operator I^{-1}, so that

$$F^- = eE_k \frac{dF(\varepsilon_{\mathbf{p}})}{d\varepsilon_{\mathbf{p}}} I^{-1} v_k. \quad (2.27)$$

Substituting (2.27) in (2.24) we obtain the balance equation

$$\frac{1}{2} E_i E_k \frac{d}{d\varepsilon} \Big(\frac{dF(\varepsilon)}{d\varepsilon} \sigma_{ik}(\varepsilon) \Big) = \bar{I}F(\varepsilon). \quad (2.28)$$

The quantity

$$\sigma_{ik}(\varepsilon) = 2e^2 \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) v_i I^{-1} v_k, \quad (2.29)$$

turns out to depend on ϵ only as a factor [8]

$$\sigma_{ik}(\varepsilon) = \sigma_{ik}^0 \cdot \varepsilon. \quad (2.30)$$

To verify this, let us examine (2.14) in greater detail. It contains the expression

$$\sum_j |c_{\mathbf{q}j}|^2 (2N_{\mathbf{q}j} + 1); \quad (2.31)$$

Substituting $N_{qj} \approx T/\hbar\omega_{qj}$ and using the explicit form of (2.3) for $|c_{qj}|^2$, we see that (2.31) is a linear combination of the quantities $q_\alpha q_\beta / q^2$, that is, a homogeneous function of q_i of zeroth order. Further, in the coordinates

$$P_\alpha = (m / m_\alpha)^{1/2} p_\alpha, \quad Q_\alpha = (m / m_\alpha)^{1/2} q_\alpha, \quad m = (m_1 m_2 m_3)^{1/3}:$$

$$\delta(\varepsilon_{\mathbf{p}+\hbar\mathbf{q}} - \varepsilon_{\mathbf{p}}) = \delta\Big(\sum_\alpha \Big(\frac{\hbar p_\alpha q_\alpha}{m_\alpha} + \frac{\hbar^2 q_\alpha^2}{2m_\alpha}\Big)\Big) = \delta\Big(\frac{\hbar QP\cos\theta}{m} + \frac{\hbar^2 Q^2}{2m}\Big) = \frac{m}{\hbar PQ} \delta\Big(\cos\theta + \frac{\hbar Q}{2P}\Big), \quad (2.32)$$

where θ is the angle between $\mathbf{P}$ and $\mathbf{Q}$. In addition, recalling that $d\mathbf{q} = d\mathbf{Q}$, we go over from $\Sigma_{\mathbf{q}}$ to the integral over the angles and over the dimensionless quantity $x = \hbar Q/P$ within the limits defined by the δ-function (2.32):

$$\sum_q \to \frac{1}{(2\pi)^3} \int d\mathbf{q} \to \frac{1}{(2\pi)^3} \int d\mathbf{Q} \to \frac{1}{(2\pi)^3} \int_0^{2P/\hbar} Q^2 dQ \int d\Omega \to \frac{1}{(2\pi\hbar)^3} P^3 \int_0^2 x^2 dx \int d\Omega, \quad (2.33)$$

so that

$$If(\mathbf{p}) = P \int_0^2 dx \int d\Omega \psi(\Omega, x) [f(\mathbf{p} + \hbar\mathbf{q}) - f(\mathbf{p})]. \quad (2.34)$$

Let now $f(\mathbf{p})$ depend only on the direction of $\mathbf{p}$, but not on its absolute value. Then

$$If(\mathbf{n}) = \sqrt{\varepsilon}\varphi(\mathbf{n}), \quad (2.35)$$

or, in particular, $I^{-1}v_i$ is a homogeneous function of p_k of zeroth order. The remaining factors and the summation over p in (2.29) give as a net result precisely the factor ϵ. Thus, (2.30) is correct. We note that the property $\sigma_{ik}(\epsilon) = \nu_{ik}\mu(\epsilon)$ is not general. If two quasielastic scattering mechanisms, with different energy dependences, are influential and if one cannot introduce the relaxation time, this relation does not hold true in general.

The quantity $\overset{0}{\sigma}_{ik}$ can be expressed in terms of the electric conductivity tensor in the weak electric field σ_{ik}

$$\sigma_{ik} = 2e^2 \sum_{\mathbf{p}} v_i I^{-1} v_k \frac{dF_0(\varepsilon)}{d\varepsilon} = \int_0^\infty d\varepsilon \, \frac{dF_0}{d\varepsilon} 2e^2 \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) v_i I^{-1} v_k = \sigma^0_{ik} \int_0^\infty \frac{dF_0}{d\varepsilon} \varepsilon \, d\varepsilon = -\sigma^0_{ik} \int_0^\infty F_0 d\varepsilon, \qquad (2.36)$$

where $F_0(\epsilon)$ is the equilibrium distribution function. The left side of (2.28) has thus been transformed into

$$-\sigma_{ik}E_iE_k\left(2\int_0^\infty F_0 d\varepsilon\right)^{-1} \frac{d}{d\varepsilon}\left(\varepsilon \frac{dF}{d\varepsilon}\right). \qquad (2.37)$$

The right side is[5,8]

$$\bar{I}F(\varepsilon_{\mathbf{p}}) \equiv \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) IF(\varepsilon_{\mathbf{p}}) = \frac{2\pi}{\hbar} \sum_{\mathbf{pq}j} |c_{\mathbf{q}j}|^2 \delta(\varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{q}j} - \varepsilon_{\mathbf{p}+\hbar\mathbf{q}}) \times [\delta(\varepsilon_{\mathbf{p}} - \varepsilon) - \delta(\varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{q}j} - \varepsilon)] \times [(N_{\mathbf{q}j} + 1)F(\varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{q}j}) - N_{\mathbf{q}j}F(\varepsilon_{\mathbf{p}})]. \qquad (2.38)$$

Expanding in terms of the inelasticity, we obtain (cf. the results of Davydov [1])

$$\bar{I}F(\varepsilon_{\mathbf{p}}) = \frac{d}{d\varepsilon}\left[A(\varepsilon)\left(1 + T\frac{d}{d\varepsilon}\right)F(\varepsilon)\right], \qquad (2.39)$$

$$A(\varepsilon) = 2\pi \sum_{\mathbf{q}j} |c_{\mathbf{q}j}|^2 \omega_{\mathbf{q}j} \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}+\hbar\mathbf{q}}). \qquad (2.40)$$

Calculation in terms of the coordinates P_α and Q_α yields

$$A(\varepsilon) = \frac{1}{(2\pi\hbar)^3} \frac{16m^4E_0^2}{\rho\hbar^4}\varepsilon^2,$$

$$E_0^2 = \frac{1}{3m}\sum_\alpha (\Lambda^2)_{\alpha\alpha} m_\alpha, \qquad (2.41)$$

so that

$$\frac{\bar{I}F(\varepsilon_{\mathbf{p}})}{\sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}})} = \frac{1}{\tau_s(\varepsilon)} \frac{1}{\varepsilon} \frac{d}{d\varepsilon}\left[\varepsilon^2\left(1 + T\frac{d}{d\varepsilon}\right)F(\varepsilon)\right], \qquad (2.42)$$

$$\tau_s(\varepsilon) = \tau_{s0}\frac{T^{1/2}}{\varepsilon^{1/2}}, \quad \tau_{s0} = \frac{\pi\hbar^4\rho}{2m^3E_0^2}\frac{m^{1/2}}{2^{1/2}T^{1/2}}. \qquad (2.43)$$

The quantity $\tau_s(\epsilon)$ plays the role of the characteristic energy relaxation time, and $\tau_{s0} = \tau_s(T)$.

Equation (2.28) assumes finally the form

$$\mathscr{D}_E F(\varepsilon) \equiv \varepsilon_E^2 \frac{d}{d\varepsilon}\varepsilon\frac{dF}{d\varepsilon} + \frac{d}{d\varepsilon}\varepsilon^2\left(1 + T\frac{d}{d\varepsilon}\right)F = 0, \qquad (2.44)$$

where

$$\varepsilon_E^2 = \frac{\sqrt{\pi}\,T\tau_{s0}}{2\,n_0}\sigma_{ik}E_iE_k = a\sigma_{ik}E_iE_k = a\sigma_\parallel E^2, \qquad (2.45)$$

$$a = \frac{\sqrt{\pi}}{2}\frac{T\tau_{s0}}{n_0} = \frac{\pi^{3/2}\hbar^4\rho T^{1/2}}{2^{5/2}n_0}\frac{1}{m^{5/2}E_0^2}. \qquad (2.46)$$

Here n_0 is the electron concentration and $\sigma_\parallel$ is the electric conductivity parallel to the weak electric field. Equation (2.44) is the same as in the isotropic model; its solution is

$$F = N(\varepsilon T + \varepsilon_E^2)^{\varepsilon_E^2/T^2} e^{-\varepsilon/T}, \qquad (2.47)$$

where N is a normalizing factor.

Let us calculate the current density

$$J_i = 2e\sum_{\mathbf{p}} v_i F_{\mathbf{p}}^- = 2e^2E_k \sum_{\mathbf{p}} v_i I^{-1} v_k \frac{dF(\varepsilon_{\mathbf{p}})}{d\varepsilon_{\mathbf{p}}} = E_k\int_0^\infty \frac{dF}{d\varepsilon}\sigma_{ik}(\varepsilon)\,d\varepsilon = -\sigma_{ik}^0 E_k \int_0^\infty F(\varepsilon)\,d\varepsilon$$
$$= \pi^{1/2}T^{1/2}\int_0^\infty F\,d\varepsilon\left(2\int_0^\infty F\varepsilon^{1/2}d\varepsilon\right)^{-1}\sigma_{ik}E_k = \frac{\pi^{1/2}}{2}\mathfrak{A}(\varepsilon_E^2)\sigma_{ik}E_k, \qquad (2.48)$$

$$\mathfrak{A}(\varepsilon_E^2) = T^{1/2}\int_0^\infty F\,d\varepsilon \Big/ \int_0^\infty F\varepsilon^{1/2}\,d\varepsilon. \qquad (2.49)$$

Expression (2.48) shows that the direction of the current J (from one ellipsoid!), for a specified direction of E is the same in strong and weak fields. The expression (2.48) for the current takes the form

$$J_i = \sigma_{ik}^{\mathbf{E}} E_k, \qquad (2.50)$$

where the "electric conductivity" is

$$\sigma_{ik}^{\mathbf{E}} = \frac{\pi^{1/2}}{2}\mathfrak{A}(a\sigma_\parallel E^2)\sigma_{ik}. \qquad (2.51)$$

In the case when $\epsilon_E \gg T$, when $F \approx \text{const}\cdot\exp(-\epsilon^2/2\epsilon_E^2)$ we have

$$\sigma_{ik}^{\mathbf{E}} = 2^{1/4}\Gamma\left(\frac{5}{4}\right)\left(\frac{T}{\varepsilon_E}\right)^{1/2}\sigma_{ik} = 2^{1/4}\Gamma\left(\frac{5}{4}\right)\frac{T^{1/2}}{a^{1/4}E^{1/2}}\frac{\sigma_{ik}}{\sigma_\parallel^{1/4}}; \qquad (2.52)$$

In the opposite limiting case $\epsilon_E \ll T$ ("warm electrons") we have

$$\sigma_{ik}^{\mathbf{E}} = \left[1 - 2(1 - \ln 2)\frac{a\sigma_\parallel E^2}{T^2}\right]\sigma_{ik}. \qquad (2.53)$$

These formulas make it possible to determine experimentally the value of the parameter a. According to (2.46), this yields a quantity

$$m^{5/2}E_0^2 = \frac{1}{3}(m_1m_2m_3)^{1/2}\sum_\alpha(\Lambda^2)_{\alpha\alpha}m_\alpha \qquad (2.54)$$

which is a combination of the components of the deformation tensor and of the effective-mass tensor.

3. CURRENT FLUCTUATIONS

The distribution function F_p and the current density J fluctuate about their mean values, assuming at each instant of time certain values $F_p = \overline{F}_p + \delta F_p(t)$ and $J = \overline{J} + g(r, t)$. We are interested in the current density fluctuations averaged over a certain selected volume V_0 of the order of the sample volume (in particular, in the fluctuation of the total current flowing in the circuit), that is, fluctuations of the quantity

$$g_i(t) = \frac{1}{V_0}\int_{V_0} d^3r g_i(\mathbf{r}, t). \qquad (3.1)$$

We are considering the case of total ionization of the impurities, when there is no generation and recombination of the carriers. The deviations of the number of particles in the selected volume V_0 from the mean, $\overline{N} = V_0 n_0$, would in the case of an ideal gas be $(\overline{\Delta N^2})_{id} = \overline{N^2} - \overline{N}^2 \sim \overline{N}$. We shall show that in the case of an equilibrium state in a gas of charged particles, the deviations are smaller by a factor $(R_D/L)^2$, where R_D is the Debye radius and $L \sim V_0^{1/3}$. A change ΔN in the number of electrons in the volume V_0 changes the charge on it by $\Delta Q = e\Delta N$. The minimum work necessary to bring this change about in reversible fashion is $\Delta Q^2/2C \sim e^2\Delta N^2/L$, where C is the capacitance of the selected region of the sample, and $C \sim L$. But then the mean value $(\overline{\Delta N^2})_{ch}$ is determined in the case of fluctuations by the condition $e^2(\overline{\Delta N^2})_{ch}/L \sim T$, hence

$$\frac{(\overline{\Delta N^2})_{\text{ch}}}{(\overline{\Delta N^2})_{\text{id}}} \sim \frac{LT}{e^2}\frac{1}{\overline{N}} \sim \left(\frac{R_D}{L}\right)^2,$$

since $TV_0/e^2\overline{N} \sim R_D^2$. Thus, if the dimensions L are much larger than the Debye radius, we can assume that the concentration of the electrons does not fluctuate:

$$\sum_{\mathbf{p}}\delta F_{\mathbf{p}}(t) = 0, \qquad (3.2)$$

and we can consider only fluctuations in velocity space.

Our problem is to calculate the Fourier components of the correlators $\overline{g_i(t+\tau)g_k(t)} \equiv \overline{g_i(\tau)g_k}$ (the superior bar denotes averaging over t for fixed value of τ) in the stationary nonequilibrium state considered in the preceding section. We begin with an examination of the correlator $\overline{\delta F_p(\tau)g_k}$. When $\tau > 0$ it describes the dissolution of a fluctuation that has a definite value at $\tau = 0$, and obeys an equation of the kinetic type:

$$\frac{\partial}{\partial\tau}\overline{\delta F_{\mathbf{p}}(\tau)g_k} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\overline{\delta F_{\mathbf{p}}(\tau)g_k} - I\overline{\delta F_{\mathbf{p}}(\tau)g_k} = 0. \qquad (3.3)$$

For the quantity

$$\gamma_{\mathbf{p}}^k(\omega) = \frac{1}{2\pi}\int_0^\infty d\tau e^{i\omega\tau}\overline{\delta F_{\mathbf{p}}(\tau)g_k} \qquad (3.4)$$

it takes the form

$$-i\omega\gamma_{\mathbf{p}}^k(\omega) + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\gamma_{\mathbf{p}}^k(\omega) - I\gamma_{\mathbf{p}}^k(\omega) = \frac{1}{2\pi}\overline{\delta F_{\mathbf{p}}g_k}. \qquad (3.5)^*$$

From the condition (3.2) follows a corresponding condition for γ:

$$\sum_{\mathbf{p}}\gamma_{\mathbf{p}}^k(\omega) = 0. \qquad (3.6)$$

The sought Fourier component of the correlator $\overline{g_i(\tau)g_k}$ is expressed in terms of γ_p^k in the following manner:

$$(g_ig_k)_\omega = \frac{1}{2\pi}\int_{-\infty}^\infty e^{i\omega\tau}\overline{g_i(\tau)g_k}\,d\tau = \frac{1}{2\pi}\int_0^\infty e^{i\omega\tau}\overline{g_i(\tau)g_k}\,d\tau + \frac{1}{2\pi}\int_0^\infty e^{-i\omega\tau}\overline{g_k(\tau)g_i}\,d\tau = 2e\sum_{\mathbf{p}}(v_{\mathbf{p}}^i\gamma_{\mathbf{p}}^k(\omega) + v_{\mathbf{p}}^k\gamma_{\mathbf{p}}^i(-\omega)). \qquad (3.7)$$

Equation (3.5) was derived in a paper by one of the authors [3] (formula (6)), but an error has crept into that paper in the subsequent transformation of the right side. We shall repeat the transformation here (see [7]). In order to calculate the mean value of $\overline{\delta F_p g_k}$, which characterizes the correlation of the fluctuations at the same instant of time, we represent it in the form

$$\overline{\delta F_{\mathbf{p}}g_k} = \frac{e}{V_0}\sum_{\mathbf{p}'\nu'}\overline{\delta F_{\mathbf{p}\nu}\delta F_{\mathbf{p}'\nu'}}v_{\mathbf{p}'}^k, \qquad (3.8)$$

where F is designated explicitly by the index ν, which marks the spin state of the electron.

We denote the combination of indices $\mathbf{p}$ and ν by λ, and obtain the mean value

$$\overline{\delta F_\lambda\delta F_{\lambda'}} = \overline{F_\lambda F_{\lambda'}} - \overline{F}_\lambda\overline{F}_{\lambda'}. \qquad (3.9)$$

It is obvious that $\overline{F_\lambda^2} = \overline{F}_\lambda^2$. On the other hand, we cannot set $\overline{F_\lambda F_{\lambda'}}$ $(\lambda \neq \lambda')$ equal to $\overline{F}_\lambda\overline{F}_{\lambda'}$, as may seem reasonable at first glance. As noted above,

*$[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

the number of particles $N = n_0V_0$ in the volume V_0 is assumed fixed. The mean value $\overline{F_\lambda F_{\lambda'}}$ is the probability that the state λ' is occupied if it is known with assurance that the state λ is occupied, multiplied by the probability that the state λ is occupied. Assume that the state λ is occupied. Then the remaining $N - 1$ particles are distributed over the states in accordance with the distribution function $\overline{F}_\lambda$. All that changes is the normalization of this function, so that the probability of the state λ' being occupied is equal in this case to $((N-1)/N)\overline{F}_{\lambda'}$. Then

$$\overline{F_\lambda F_{\lambda'}} = ((N-1)/N)\overline{F}_\lambda \overline{F}_{\lambda'};$$

$$\overline{\delta F_\lambda \delta F_{\lambda'}} = -\frac{1}{n_0V_0}\overline{F}_\lambda \overline{F}_{\lambda'} \quad (\lambda \neq \lambda'). \tag{3.10}$$

Thus,

$$\overline{\delta F_\lambda \delta F_{\lambda'}} = \overline{F}_\lambda\left(\delta_{\lambda\lambda'} - \frac{1}{n_0V_0}\overline{F}_{\lambda'}\right), \tag{3.11}$$

$$\overline{\delta F_{\mathbf{p}} g_k} = \frac{e}{V_0}\overline{F}_{\mathbf{p}}\left(v_{\mathbf{p}}{}^k - \frac{1}{n_0V_0}\sum_{\mathbf{p}'\nu'}\overline{F}_{\mathbf{p}'\nu'}v_{\mathbf{p}'}{}^k\right)$$

$$= \frac{e}{V_0}\overline{F}_{\mathbf{p}}(v_{\mathbf{p}}{}^k - V_k); \tag{3.12}$$

where

$$V_k = \sum_{\mathbf{p}'\nu'}\overline{F}_{\mathbf{p}'}v_{\mathbf{p}'}{}^k \Big/ \sum_{\mathbf{p}'\nu'}\overline{F}_{\mathbf{p}'} = J_k/en_0 = \pi^{1/2}\sigma_{kl}E_l\,\mathfrak{A}/2en_0 \tag{3.13}$$

is the drift velocity of the electrons (2.48). Assuming for simplicity that $V_0 = 1$, we obtain (3.5) in final form

$$-i\omega\gamma_{\mathbf{p}}{}^k(\omega) + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\gamma_{\mathbf{p}}{}^k(\omega) - I\gamma_{\mathbf{p}}{}^k(\omega)$$

$$= \frac{e}{2\pi}(v_{\mathbf{p}}{}^k - V_k)\overline{F}_{\mathbf{p}}. \tag{3.14}$$

We proceed further in the same manner as when solving the equation for $\overline{F}_{\mathbf{p}}$. We break up the sought function $\gamma_{\mathbf{p}}$ into even and odd parts

$$\gamma_{\mathbf{p}} = \gamma_{\mathbf{p}}{}^+ + \gamma_{\mathbf{p}}{}^-, \tag{3.15}$$

and obtain from (3.14) the following equation ($H = 0$)

$$-i\omega\gamma_k{}^+ + eE_i\frac{\partial}{\partial p_i}\gamma_k{}^- - I\gamma_k{}^+$$

$$= \frac{e}{2\pi}v_kF_{\mathbf{p}}{}^- - \frac{e}{2\pi}V_kF(\varepsilon), \tag{3.16}$$

$$-i\omega\gamma_k{}^- + eE_i\frac{\partial}{\partial p_i}\gamma_k{}^+ - I\gamma_k{}^- = \frac{e}{2\pi}v_kF(\varepsilon) \tag{3.17}$$

(in the approximation in which $mW^2/T \ll 1$ it is necessary to neglect V_kF^- compared with $v_kF(\epsilon)$); the function $\gamma_{\mathbf{p}}^+ \approx \gamma(\epsilon_{\mathbf{p}})$, and we obtain from (3.16) and (3.17), in the same manner as used to derive (2.24) and (2.25),

$$-i\omega\gamma_k(\varepsilon)\sum_{\mathbf{p}}\delta(\varepsilon - \varepsilon_{\mathbf{p}}) + eE_i\frac{d}{d\varepsilon}\sum_{\mathbf{p}}\delta(\varepsilon - \varepsilon_{\mathbf{p}})v_i\gamma_k{}^- - \overline{I\gamma_k}(\varepsilon)$$

$$= \frac{e}{2\pi}\sum_{\mathbf{p}}\delta(\varepsilon - \varepsilon_{\mathbf{p}})v_kF_{\mathbf{p}}{}^- - \frac{e}{2\pi}V_kF(\varepsilon)\sum_{\mathbf{p}}\delta(\varepsilon - \varepsilon_{\mathbf{p}}), \tag{3.18}$$

$$\gamma_k{}^- = (I + i\omega)^{-1}\left[eE_i\frac{d\gamma_k(\varepsilon_{\mathbf{p}})}{d\varepsilon_{\mathbf{p}}}v_i - \frac{e}{2\pi}F(\varepsilon_{\mathbf{p}})v_k\right]. \tag{3.19}$$

We substitute γ^- in (3.18). Then there appears in this equation the quantity

$$2e^2\sum_{\mathbf{p}}\delta(\varepsilon - \varepsilon_{\mathbf{p}})v_i(I + i\omega)^{-1}v_k. \tag{3.20}$$

In the general case it is impossible to separate an energy factor in this equation. Therefore the results that follow are valid only if

$$\omega\tau_p \ll 1 \tag{3.21}$$

(see (2.13)), or more accurately, if we can assume that

$$(I + i\omega)^{-1}v_i \approx I^{-1}v_i. \tag{3.22}$$

The final form of (3.18) is

$$i\omega\tau_{s0}T^{1/2}\varepsilon^{1/2}\gamma_k(\varepsilon) + \mathscr{D}_E\gamma_k(\varepsilon)$$

$$= \frac{1}{2\pi}a\sigma_{ik}E_i\left[2\frac{d}{d\varepsilon}(\varepsilon F) - F\left(1 - \mathfrak{A}\frac{\varepsilon^{1/2}}{T^{1/2}}\right)\right], \tag{3.23}$$

where a is defined in (2.46). Thus, the quantity $\gamma_k(\epsilon)$, connected with the fluctuations of the symmetrical part of the distribution function, is proportional to $a\sigma_{ik}E_i$

$$\gamma_\omega{}^k(\varepsilon) = a\sigma_{ik}E_iK_\omega(\varepsilon_E^2, \varepsilon)/2\pi, \tag{3.24}$$

where $K\omega(\epsilon_E^2, \epsilon)$ is the solution of the equation

$$i\omega\tau_{s0}T^{1/2}\varepsilon^{1/2}K + \mathscr{D}_EK = 2\frac{d}{d\varepsilon}(\varepsilon F)$$

$$-F\left(1 - \mathfrak{A}\frac{\varepsilon^{1/2}}{T^{1/2}}\right), \tag{3.25}$$

and, according to (3.6),

$$\int_0^\infty K(\varepsilon)\varepsilon^{1/2}d\varepsilon = 0. \tag{3.26}$$

Equation (3.25) and the form of the function K are the same in the isotropic and anisotropic cases. However, just as in the case of $F(\epsilon_E^2, \epsilon)$, in the anisotropic case $K(\epsilon_E^2, \epsilon)$ depends on the direction of the electric field, since the latter governs also ϵ_E.

Let us calculate $(g_ig_k)_\omega$ as given by (3.7). Using (3.19), (3.22), (2.30), (2.36), and (3.24) we get

$$(g_i g_k)_\omega = T^{1/2}\left[2\pi\frac{2}{\pi^{1/2}}\int_0^\infty F(\varepsilon)\varepsilon^{1/2}d\varepsilon\right]^{-1}\left[2\sigma_{ik}\int_0^\infty F(\varepsilon)\varepsilon\, d\varepsilon\right.$$

$$\left.+ a\sigma_{il}E_l\sigma_{kl'}E_{l'}\int_0^\infty (K_\omega(\varepsilon)+K_{-\omega}(\varepsilon))\,d\varepsilon\right]. \tag{3.27}$$

For the isotropic case $\sigma_{ik} = \sigma\delta_{ik}$ we obtain the known result ($E \parallel z$):

$$(g_i g_k)_\omega = \delta_{ik}\left[2\pi\frac{2}{\pi^{1/2}}\int_0^\infty F(\varepsilon)\varepsilon^{1/2}d\varepsilon\right]^{-1}\left[2\int_0^\infty F(\varepsilon)\varepsilon\, d\varepsilon\right.$$

$$\left.+\delta_{kz}\,\varepsilon_E^2\int_0^\infty (K_\omega(\varepsilon)+K_{-\omega}(\varepsilon))\,d\varepsilon\right]\sigma T^{1/2}. \tag{3.28}$$

In the case of low-frequency fluctuations, when $\omega\tau_{S0} \ll 1$, we discard the first term in (3.25); taking (3.26) into account, we obtain

$$\int_0^\infty K_0(\varepsilon)\,d\varepsilon = \int_0^\infty \frac{\varkappa(\varkappa - 2\varepsilon F)\,d\varepsilon}{F\varepsilon(\varepsilon_E^2 + T\varepsilon)},$$

$$\varkappa(\varepsilon) = \int_0^\varepsilon (1-(\varepsilon'/T)^{1/2}\mathfrak{A})\,F(\varepsilon')\,d\varepsilon'. \tag{3.29}$$

It is easy to verify that $2F\epsilon \geq \kappa(\epsilon) \geq 0$ for all ϵ, so that

$$\int_0^\infty K_0(\varepsilon)\,d\varepsilon < 0. \tag{3.29a}$$

In the limiting cases

$$\varepsilon_E \gg T:\quad (g_i g_k)_0 = \frac{2^{3/4}\Gamma\left(\frac{5}{4}\right)}{\pi^{1/2}}\left(\frac{\varepsilon_E}{T}\right)^{1/2}$$

$$\times\frac{T}{\pi}\left(\sigma_{ik} - D_1\frac{\sigma_{il}E_l\sigma_{kl'}E_{l'}}{\sigma_{ll'}E_lE_{l'}}\right),\quad D_1 = 0.51. \tag{3.30}$$

$$\varepsilon_E \ll T:\quad (g_i g_k)_0 = \frac{T}{\pi}\left\{\sigma_{ik}\left[1 + (2\ln 2 - 1)\frac{a}{T^2}\sigma_{ll'}E_lE_{l'}\right]\right.$$

$$\left. - D_2\frac{a}{T^2}\sigma_{il}E_l\sigma_{kl'}E_{l'}\right\},\quad D_2 = 0.83. \tag{3.31}$$

Let us compare the fluctuations with the response of a system in the same stationary nonequilibrium state to the action of a weak alternating field $\mathscr{E}^k = \mathscr{E}_0^k e^{-i\omega t}$. The equation for $\mathscr{G}_p^k(\omega)$ (where $\mathscr{G}_p^k(\omega)e^{-i\omega t}$ is the correction to the stationary function $\bar{F}_p$) differs for $\mathscr{E}_0^k = 1$ from Eq. (3.14) (which defines γ_p^k) in the right side, which is equal to

$$-e\partial\bar{F}_p/\partial p_k \tag{3.32}$$

(see (12) in [3]). For the tensor

$$\tilde{\sigma}_{ik}(\omega) = 2e\sum_p v_i\mathscr{G}_p{}^k(\omega), \tag{3.33}$$

which determines the response current

$$j_i = \tilde{\sigma}_{ik}(\omega)\mathscr{E}_k, \tag{3.34}$$

derivations similar to those just presented lead to the expression

$$\sigma_{ik}(\omega) = T^{1/2}\left(\frac{2}{\pi^{1/2}}\int_0^\infty F(\varepsilon)\varepsilon^{1/2}d\varepsilon\right)^{-1}\left[\sigma_{ik}\int_0^\infty F(\varepsilon)\,d\varepsilon\right.$$

$$\left.+ a\sigma_{il}E_l\sigma_{kl'}E_{l'}\int_0^\infty K_\omega^1(\varepsilon)\,d\varepsilon\right], \tag{3.35}$$

where K_ω^1 is a solution of the equation

$$i\omega\tau_{s0}T^{1/2}\varepsilon^{1/2}K^1 + \mathscr{D}_E K^1 = 2\frac{d}{d\varepsilon}\left(\varepsilon\frac{dF(\varepsilon)}{d\varepsilon}\right), \tag{3.36}$$

which again differs from (3.25) in the right side. Thus, the ratio usually called the noise temperature is given by the formula

$$2\pi\frac{(g_i g_k)_\omega}{\tilde{\sigma}_{ik}(\omega) + \tilde{\sigma}_{ki}{}^*(\omega)} = \frac{2\sigma_{ik}\int_0^\infty F\varepsilon d\varepsilon + a\sigma_{il}E_l\sigma_{kl'}E_{l'}\int_0^\infty (K_\omega(\varepsilon)+K_{-\omega}(\varepsilon))\,d\varepsilon}{2\sigma_{ik}\int_0^\infty F d\varepsilon + a\sigma_{il}E_l\sigma_{kl'}E_{l'}\int_0^\infty (K_\omega{}^1(\varepsilon)+K_{-\omega}{}^1(\varepsilon))\,d\varepsilon} \tag{3.37}$$

At low frequencies it is easiest to obtain $\tilde{\sigma}_{ik}(0)$ by differentiating (2.48). since it is equal to $\partial J_i/\partial E_k$. In the limiting cases we have

$$\varepsilon_E \gg T:\quad \tilde{\sigma}_{ik}(0) = 2^{1/4}\Gamma\left(\frac{5}{4}\right)\left(\frac{T}{\varepsilon_E}\right)^{1/2}$$

$$\times\left(\sigma_{ik} - \frac{1}{2}\frac{\sigma_{il}E_l\sigma_{kl'}E_{l'}}{\sigma_{ll'}E_lE_{l'}}\right); \tag{3.38}$$

$$\varepsilon_E \ll T:\quad \tilde{\sigma}_{ik}(0) = \sigma_{ik}[1 - 2(1-\ln 2)\varepsilon_E^2T^{-2}]$$

$$- 4(1-\ln 2)aT^{-2}\sigma_{il}E_l\sigma_{kl'}E_{l'} \tag{3.39}$$

and expression (3.37) takes the form

$$\varepsilon_E \gg T:\ \left(\frac{2}{\pi}\right)^{1/2}\varepsilon_E\frac{\sigma_{ik} - D_1\sigma_{il}E_l\sigma_{kl'}E_{l'}/\sigma_{ll'}E_lE_{l'}}{\sigma_{ik} - \sigma_{il}E_l\sigma_{kl'}E_{l'}/2\sigma_{ll'}E_lE_{l'}}; \tag{3.40}$$

$$\varepsilon_E \ll T,\ i = k:\quad T\left\{1 + \frac{\varepsilon_E^2}{T^2} + \left[-D_2 + 4(1-\ln 2)\right]\right.$$

$$\left.\times\frac{a}{T^2}\frac{(\sigma_{il}E_l)^2}{\sigma_{ii}}\right\}. \tag{3.41}$$

The first term in expression (3.27) for $(g_i^2)_\omega$ is connected with the fluctuations of the antisymmetrical part of the distribution function and does not depend on ω (so long as $\omega \ll \tau_p^{-1}$, as assumed before). The second term is determined by the fluctuations of the symmetrical part of the distribution function and exhibits dispersion when $\omega \sim \tau_S^{-1}$. It is equal to zero in directions perpendicular to the stationary current; the fluctuations of the symmetrical part of the distribution function influence only the fluctuations of the longitudinal current [3], partially suppressing them in the low-frequency case. The latter circumstance is connected with the neutrality condition (3.2).

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