MAGNETIC AND ACOUSTIC EXCITATION OF COUPLED MAGNETOELASTIC OSCILLATIONS IN A THIN MAGNETIC FILM

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The amplitudes and half-widths of the resonance peaks of the magnetic and acoustic components of magnetoelastic oscillations in a thin magnetic film are calculated, within the framework of the phenomenological theory, both for magnetic and for acoustic microwave excitation. The optimal conditions for use of a thin film as a linear magnetoelastic transducer are analyzed.

THE discrete spectrum of characteristic frequencies of a thin magnetic film, determined by exchange and magnetoelastic interactions, was calculated earlier.[1] The general form of the coupled magnetoelastic boundary conditions on the surface of the film was later[2] made somewhat more precise. The preliminary results of a calculation of the amplitudes and line widths of magnetoelastic oscillations excited in a thin magnetic film by a uniform microwave field were communicated in[3]. The aim of the present work is to calculate the amplitudes and effective relaxation parameters (line widths) of magnetoelastic oscillations in a thin magnetic film both in the case of magnetic and in the case of acoustic excitation.

1. SYSTEM OF EQUATIONS; GENERAL SOLUTION

Consider a magnetically uniaxial ferrodielectric, isotropic with respect to its elastic and magnetoelastic properties, and having the form of a thin film (Fig. 1). For oscillations with time-varying factor $e^{i\omega t}$, uniform in the plane of the film, the linearized system of equations, with neglect of the change of volume, has the form

$$L_s^{\pm} m^{\pm} - \gamma g M_0^2 \frac{du^{\pm}}{dz} = - g M_0 h^{\pm}(z),$$

$$L_a u^{\pm} + \frac{\gamma M_0}{\rho}\frac{dm^{\pm}}{dz} = -\frac{f^{\pm}(z)}{\rho}; \qquad (1.1)$$

$$L_s^{\pm} = \alpha g M_0 \frac{d^2}{dz^2} \mp [(1 \pm i\xi_s)\omega \pm \omega_0],$$

$$L_a = (1 + 2i\xi_a) s_t^2 \frac{d^2}{dz^2} + \omega^2, \qquad (1.1')$$

Here $m^{\pm} = m_x \pm i m_y$ is the deviation of the magnetic moment from the equilibrium state; $u^{\pm} = u_x \pm i u_y$ are the circular components of the elastic displacement vector; $h^{\pm}(z) = h_x(z) \pm i h_y(z)$ is the external exciting magnetic field; $f^{\pm}(z) = f_x(z) \pm i f_y(z)$ is the density of external exciting force; α is an exchange constant, g the magnetomechanical ratio, γ a magnetoelastic constant, ω_0 the frequency of uniform ferromagnetic resonance, s_t the transverse speed of sound, ρ the density of the material; and ξ_s and ξ_a are, respectively, the magnetic and acoustic damping parameters,

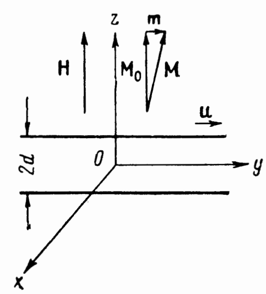

FIG. 1

$$\xi_s = 1/\omega\tau_s = 1/2Q_s, \qquad \xi_a = 1/\omega\tau_a = 1/2Q_a, \qquad (1.2)$$

where τ_s and τ_a are the relaxation times of the spin system and of the acoustic system, and Q_s and Q_a are their quality factors. We remark that the linearized system will have the same form for a crystal of cubic symmetry; in this case, $\gamma = \gamma_2 = B_2/M_0^2$ and $s_t = (c_{44}/\rho)^{1/2}$.

We shall consider the system (1.1) with the boundary conditions

$$m^{\pm}|_{z=\pm d} = 0, \qquad \left.\frac{du^{\pm}}{dz}\right|_{z=\pm d} = 0, \qquad (1.3)$$

which are a limiting case of the general boundary conditions,[2] and which correspond to rigid pinning of the spins and to absence of elastic stresses on the surface. Such idealized boundary conditions permit a clearer exposition of the basic dependence of the characteristics of the resonance peaks on the parameters of the magnetoelastic system; solution of the problem with boundary conditions of general form is complicated, and that analysis will not be made here.

The characteristic functions of the stated problem are plane standing waves. To the boundary conditions there correspond two sets of characteristic functions:

a) for a solution symmetric with respect to $m^\pm$ and antisymmetric with respect to $u^\pm$,

$$m^\pm \sim \cos k_p z, \quad u^\pm \sim \sin k_p z;$$

$$k_p = p\pi / 2d, \quad p = 1, 3, 5, \ldots; \tag{1.4}$$

b) for a solution antisymmetric with respect to $m^\pm$ and symmetric with respect to $u^\pm$,

$$m^\pm \sim \sin k_n z, \quad u^\pm \sim \cos k_n z;$$

$$k_n = n\pi / d, \quad n = 1, 2, \ldots \tag{1.5}$$

In both cases, the characteristic values of the operators $L_s^\pm$ and L_a are, respectively,

$$\Delta_s^\pm = -[(\omega_0 + \alpha g M_0 k_\gamma^2) \pm (1 \pm i\xi_s)\omega],$$

$$\Delta_a = \omega^2 - (1 + 2i\xi_a) s_t^2 k_\gamma^2. \tag{1.6}$$

The dispersion relations for this problem have the form

$$\Delta^\pm \equiv \mp (\Delta_s^\pm \Delta_a - \eta g M_0 s_t^2 k_\gamma^2), \tag{1.7}$$

where $\eta = \gamma^2 M_0^2/\rho s_t^2$ is the magnetoelastic coupling parameter. The dispersion relation for this problem in the usual components is the product of the dispersion relations (1.7). For righthand polarization, (1.7) describes a weakly modified elastic wave; this will hereafter be ignored. The real part of Δ^-, to terms quadratic in ξ_a and ξ_s, agrees with the dispersion relation for a magnetoelastic system without damping,[4]

$$[\omega - (\omega_0 + \alpha g M_0 k^2)][\omega^2 - s_t^2 k^2] - \eta g M_0 s_t^2 k^2 = 0. \tag{1.8}$$

As also in[1], it is convenient to take ω = const, and to regard as the variable a quantity dependent on the applied constant magnetic field, namely ω_0. Accordingly, we rewrite (1.8) in the form

$$\sigma = 1 - \sigma_e x - \eta \sigma_M \frac{x}{x_c - x}; \quad \sigma = \omega_0 / \omega, \quad \sigma_M = g M_0 / \omega,$$

$$\sigma_e = \alpha \sigma_M / d^2, \quad x = k^2 d^2, \quad x_c = \omega^2 d^2 / s_t^2, \tag{1.9}$$

where d is the half-thickness of the film. For the discrete set of values $x = x_\gamma$, (1.9) describes the discrete spectrum of magnetic resonance fields studied in detail in[1].

For left-hand polarization (the index "minus" will be omitted hereafter), we seek a solution of (1.1) in the form of a superposition of characteristic functions:

for case a),

$$m = \sum_p (-1)^{(p+1)/2} \hat{m}_p \cos k_p z,$$

$$u = \sum_p (-1)^{(p+1)/2} \hat{u}_p \sin k_p z; \tag{1.10}$$

for case b),

$$m = \sum_n (-1)^n \hat{m}_n \sin k_n z, \quad u = \sum_n (-1)^n \hat{u}_n \cos k_n z. \tag{1.10'}$$

In the case of excitation by a magnetic microwave field uniform in z, h = const, and by an elastic force acting only on one surface (z = d) of the film, $f(z) \to f\delta(z - d)$, where f is the surface force density, the amplitudes of the excited oscillations are determined by the following expressions:

for case a),

$$\hat{m}_p = \frac{2gM_0 h}{\sqrt{x_p}} \frac{\Delta_a^{(p)}}{\Delta^{(p)}} + \gamma \frac{gM_0^2 f}{\rho d^2} \frac{\sqrt{x_p}}{\Delta^{(p)}} = \hat{m}_p^{(h)} + \hat{m}_p^{(f)},$$

$$\hat{u}_p = \frac{f}{\rho d} \frac{\Delta_s^{(p)}}{\Delta^{(p)}} + 2\gamma \frac{gM_0^2 h}{\rho d} \frac{1}{\Delta^{(p)}} = \hat{u}_p^{(f)} + \hat{u}_p^{(h)}; \tag{1.11}$$

for case b),

$$\hat{m}_n = \gamma \frac{gM_0^2 f}{\rho d^2} \frac{x_n^{1/2}}{\Delta^{(n)}} = \hat{m}_n^{(f)}, \quad \hat{u}_n = \frac{f}{\rho d} \frac{\Delta_s^{(n)}}{\Delta^{(n)}} = \hat{u}_n^{(f)}. \tag{1.12}$$

It is evident that the type of oscillation corresponding to case b) is not excited by a uniform magnetic field.

2. AMPLITUDES OF SPIN WAVES WITH MAGNETIC EXCITATION

The complex amplitudes of the magnetic components of the magnetoelastic oscillations (for brevity, "spin waves"), with magnetic (h) excitation, are determined by the expression

$$\hat{m}_p^{(h)} = \frac{2\sigma_M h}{\sqrt{x_c}} \frac{(1 + 2A_p)^{1/2}}{D_p^2 + \Gamma_p^2} \{[S_p(A_p^2 + \xi_a^2) - \lambda A_p] + i[\xi_s(A_p^2 + \xi_a^2) + \lambda \xi_a]\};$$

$$S_p = 1 - \sigma_e x_p - \sigma, \quad A_p = (x_c - x_p) / 2x_p,$$

$$\lambda = \eta \sigma_M / 2, \tag{2.1}$$

$$D_p = S_p A_p - (\lambda + \xi_a \xi_s), \quad \Gamma_p = \xi_s A_p + \xi_a S_p. \tag{2.2}$$

The imaginary part of (2.1) and the modulus of this expression, upon change of the constant magnetic field, reach a maximum at the same value of σ, corresponding to the vanishing of the real part of (2.1):

$$\sigma_p = 1 - \sigma_e x_p - 2\lambda \frac{x_p (x_c - x_p)}{(x_c - x_p)^2 + 4\xi_a^2 x_p^2}. \qquad (2.3)$$

This expression is exhibited graphically in Fig. 2; the turning points α_1 and α_2 are determined by the expressions

$$S_{1,2} = \pm\lambda / 2\xi_a, \qquad A_{1,2} = \pm\xi_a \qquad (2.4)$$

or

$$\sigma_{1,2} \approx \sigma_c \pm \lambda / 2\xi_a, \qquad \sigma_c = 1 - \sigma_e x_c + 2\lambda.$$

In the absence of acoustical damping, (2.3) coincides with the dispersion relation (1.9); when $\xi_a \neq 0$, it differs appreciably from (1.9) only in the neighborhood of the point $x = x_c$; with increase of ξ_a, (2.3) goes over to the pure spin branch $S = 0$. When (2.3) is satisfied,

$$|\hat{m}_p^{(h)}|_{\sigma=\sigma_p} = \frac{2\sigma_M h}{\xi_s \sqrt{x_c}} (2A_p + 1)^{1/2} \frac{A_p^2 + \xi_a^2}{A_p^2 + \xi_a^2 + \lambda\xi_a / \xi_s}$$

$$= \frac{2\sigma_M h}{\xi_s \sqrt{x_c}} M_p^{(h)}. \qquad (2.5)$$

This expression describes a discrete set of resonance amplitudes of spin waves. If in (2.5) we omit the index p and consider x a continuous function of σ, determined by the expression (2.3), then (2.5) will represent the envelope of the resonance amplitudes as a function of the external constant magnetic field. The difficulty connected with the fact that (2.3) is a cubic equation in x can be evaded by restricting oneself to the practically most interesting case $\lambda \gg \xi_a \xi_s$; then, as far as the turning points, (2.3) is described approximately by the relation (1.9), and

$$x_\pm \approx \frac{1}{2}\left(\frac{1 + 2\lambda - \sigma}{\sigma_e} + x_c\right)$$

$$\pm \frac{1}{2}\left[\left(\frac{\sigma - \sigma_c}{\sigma_e}\right)^2 + 4\varkappa x_c^2\right]^{1/2}, \qquad (2.6)$$

where $\kappa = 2\lambda / x_c\sigma_e = \eta s_t^2 / \alpha\omega^2$.

On substituting (2.6) in (2.5), we get the envelope of the resonance amplitudes as a function of σ. In parts s corresponding to a weakly modified spin branch and at the points $\sigma = \sigma_c$ and $\sigma = \sigma_{1,2}$, the expression $M^{(h)}$ takes the following form (the indices $\pm$ correspond to the $x_\pm$ branches):

$$M_\pm^{(h)} \approx \begin{cases} \left(\frac{1-\sigma_c}{1-\sigma}\right)^{1/2} \left[1 + \lambda \frac{1 - 2\sigma_c + \sigma}{(1-\sigma_c)}\right] \left[1 + \delta\left(\frac{1-\sigma}{\sigma - \sigma_c}\right)^2\right]^{-1}, & s \\ (1 \pm \varkappa^{1/2})^{-1/2} \left[1 + \frac{\delta}{\varkappa}(1 \pm \varkappa^{1/2})^2\right]^{-1}, & \sigma = \sigma_c, \\ (1 \pm 2\xi_a)^{1/2} \frac{2\xi_a\xi_s}{\lambda + 2\xi_a\xi_s}, & \sigma = \sigma_{1,2} \end{cases} \qquad (2.7)$$

where $\delta = 4\lambda\xi_a/\xi_s$.

The behavior of these functions is largely determined by the parameter δ. In the limiting case $\delta = 0$ ($\xi_a = 0$), the function $M_+^{(h)}$ takes a value $\approx (x_c\sigma_e)^{1/2}$ at $\sigma = 0$ and increases monotonically with increase of σ, asymptotically approaching unity (Fig. 3). For $\delta \neq 0$, the value of $M_+^{(h)}$ has a maximum at $\sigma = \sigma_{m1}$; depending on the value of δ, the point σ_{m1} can lie either to the right or to the left of σ_c; for $\delta \approx \frac{1}{4}\kappa^{3/2}$, we have $\sigma_{m1} = \sigma_c$; for $\delta \lessgtr \frac{1}{4}\kappa^{3/2}$, $\sigma_{m1} \gtrless \sigma_c$. The function $M_-^{(h)}$ is monotonic and increasing in the interval $(\sigma_2, 1)$. When $\sigma = \sigma_c$, $M_-^{(h)} > M_+^{(h)}$ always; and for sufficiently small δ, the amplitudes of the spin waves that correspond to the x_- branch are larger than the

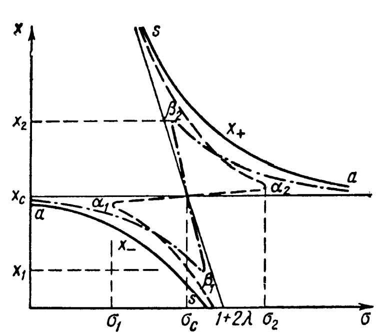

FIG. 2. Essential form of the relations (2.3), (3.3), and (4.3), corresponding to the vanishing of the real parts of the amplitudes of spin waves (dashed line), acoustic waves (dashes and dots), and cross-amplitudes (solid line).

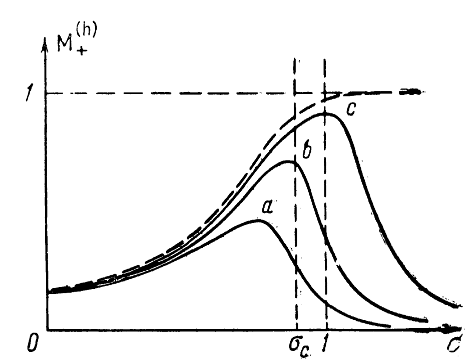

FIG. 3. Normalized envelope of resonance amplitudes of spin waves $M_+^{(h)}$: curve a, $\delta > \frac{1}{4}\kappa^{3/2}$; curve b, $\delta \sim \frac{1}{4}\kappa^{3/2}$; curve c, $\delta < \frac{1}{4}\kappa^{3/2}$.

amplitudes of the spin waves on the x_+ branch; the spin waves corresponding to the x_- branch, however, can be excited only in sufficiently thick films:[1] $2d > \pi s_t/\omega$. The amplitudes of the spin waves that correspond to the section (α_1, α_2), if the condition $\lambda \gg \xi_a \xi_s$ is satisfied, are of no interest because of their smallness.

In numerical estimates, the parameter values used have been those corresponding to nickel: $M_0 \sim 5 \times 10^2$ G, $B_2 \sim 9 \times 10^7$ erg/cm^3, $\alpha \sim 6 \times 10^{-12}$ cm^2, $\omega \sim 6 \times 10^{10}$ sec^{-1}, $s_t = 3 \times 10^5$ cm/sec, $\eta \sim 2.7 \times 10^{-2}$, $\kappa \sim 0.1$, $2d \sim 4 \times 10^{-5}$ cm, and $x_c \sim 16$.

In study of the absorption and dispersion of a microwave magnetic field by a magnetoelastic film, the harmonics of the sums (1.10) must be averaged over the thickness of the film.

3. AMPLITUDES OF ACOUSTIC WAVES WITH ACOUSTIC EXCITATION

The complex amplitudes of the elastic components of the magnetoelastic oscillations (for brevity, "acoustic waves"), with acoustic (f) excitation, are determined by the expression

$$u_\gamma^{(f)} = \frac{f}{2\rho d\omega^2} \frac{2A_\gamma + 1}{D_\gamma^2 + \Gamma_\gamma^2} \{[A_\gamma (S_\gamma^2 + \xi_s^2) - \lambda S_\gamma] + i[\xi_a (S_\gamma^2 + \xi_s^2) + \lambda \xi_s]\}. \tag{3.1}$$

The modulus of this expression, upon change of the constant magnetic field, reaches a maximum on fulfillment of the condition

$$A_\gamma (S_\gamma^2 - \xi_s^2) - (\lambda + 2\xi_a \xi_s) S_\gamma = 0, \tag{3.2}$$

whereas the real part vanishes when

$$A_\gamma (S_\gamma^2 + \xi_s^2) - \lambda S_\gamma = 0. \tag{3.3}$$

The relation (3.3) is exhibited graphically in Fig. 2; the turning points β_1, β_2 are determined by the expressions

$$A_{1,2} = \pm\lambda / 2\xi_s, \qquad S = \pm\xi_s \tag{3.4}$$

or

$$x_{1,2} = x_c \xi_s / (\xi_s \pm \lambda).$$

In the absence of spin damping, (3.3) coincides with (1.9); when $\xi_s \neq 0$, it differs appreciably from (1.9) only in the neighborhood of S = 0; with increase of ξ_s, (3.3) goes over to the pure acoustic branch A = 0. When (3.3) is satisfied,

$$|\hat{u}_\gamma^{(f)}| = \frac{f}{2\rho d\omega^2 \xi_a} |2A_\gamma + 1| \frac{S_\gamma^2 + \xi_s^2}{S_\gamma^2 + \xi_s^2 + \lambda\xi_s / \xi_a} = \frac{f}{2\rho d\omega^2 \xi} U_\gamma^{(f)}. \tag{3.5}$$

When $\lambda \gg \xi_a \xi_s$, the expression (3.2), like (1.9), agrees approximately with (3.3) all the way to the turning points β_1 and β_2. Thus over this range, (3.5) describes the discrete spectrum of resonance amplitudes of acoustic waves.

As in Sec. 2, if we omit the index γ and consider x a continuous function of σ, we may regard (3.5) as the envelope of the resonance amplitudes. In the sections a that correspond to a weakly modified acoustic branch, at the point $\sigma = \sigma_c$, and at the turning points $\beta_{1,2}$, the expression for $U^{(f)}$ takes the following forms:

$$U_\pm^{(f)} \approx \begin{cases} \left[1 + \varkappa \dfrac{1-\sigma_c}{\sigma_c - \sigma}\right]\left[1 + \dfrac{\varkappa^2}{\sigma}\left(\dfrac{1-\sigma_c}{\sigma_c - \sigma}\right)^2\right]^{-1}, & a \\ (1 \pm \varkappa^{1/2}) \left[(1 \pm \varkappa^{1/2})^2 + \dfrac{\varkappa}{\varepsilon}\right]^{-1}, & \sigma = \sigma_c. \\ \left|1 \pm \dfrac{\lambda}{2\xi_s}\right| \dfrac{2\xi_a \xi_s}{\lambda + 2\xi_a \xi_s}, & x = x_{1,2} \end{cases} \tag{3.6}$$

The functions $U_\pm^{(f)}$ approach unity (which corresponds to purely acoustic waves) on withdrawal of σ to the right and to the left, respectively, from the point σ_c; they decrease on approach to the turning points $\beta_{1,2}$. The change of $U_+^{(f)}$ is monotonic, but $U_-^{(f)}$ attains a not very pronounced maximum at $A = \frac{1}{4}\delta$, taking there the value $(1 + 2\delta)/(1 + \delta)$. The amplitudes of acoustic waves corresponding to the interval (β_1, β_2) are of no interest if the condition $\lambda \gg \xi_a \xi_s$ is satisfied.

4. CROSS-AMPLITUDES OF MAGNETOELASTIC OSCILLATIONS

The greatest interest attaches to the magnetic components of magnetoelastic oscillations under acoustic (f) excitation and to the elastic components of magnetoelastic oscillations under magnetic (h) excitation. From the expressions (1.11) and (1.12) it is evident that the amplitudes of such components (for brevity, "cross-amplitudes") have much in common with each other:

$$\hat{u}_\gamma^{(h)} = \frac{\gamma\sigma_M M_0 h}{\rho d\omega^2} (2A_\gamma + 1) \frac{D_\gamma + i\Gamma_\gamma}{D_\gamma^2 + \Gamma_\gamma^2}, \tag{4.1}$$

$$\hat{m}_\gamma^{(f)} = \frac{\gamma\sigma_M M_0 f \sqrt{x_c}}{2\rho\omega^2 d^2} (2A_\gamma + 1)^{1/2} \frac{D_\gamma + i\Gamma_\gamma}{D_\gamma^2 + \Gamma_\gamma^2}. \tag{4.2}$$

In these expressions, the index γ can take the values p and n; it must be understood that when γ = n, the value of $\hat{u}_\gamma^{(h)} \equiv 0$. Upon change of the constant magnetic field, the moduli of these expressions reach maxima on satisfaction of condition (2.3), and the real part vanishes when

$$S_\gamma A_\gamma - (\lambda + \xi_a \xi_s) = 0. \tag{4.3}$$

The relation (4.3) is also exhibited graphically in Fig. 2; when $\lambda \gg \xi_a\xi_s$, (4.3) agrees with (1.9), and as far as the turning points with (2.3). When (4.3) is satisfied,

$$|\hat{u}_\gamma^{(h)}| = \frac{2\gamma\sigma_M M_0 h}{\rho d\omega^2\xi_s}\frac{\xi_s}{2}\frac{2A_\gamma+1}{|\xi_s A_\gamma + \xi_a S_\gamma|} \equiv \frac{2\gamma\sigma_M M_0 h}{\rho d\omega^2\xi_s}U_\gamma^{(h)}, \tag{4.4}$$

$$|\hat{m}_\gamma^{(f)}| = \frac{\gamma\sigma_M M_0 f\sqrt{x_c}}{\rho\omega^2 d^2\xi_s}\frac{\xi_s}{2}\frac{(2A_\gamma+1)^{1/2}}{|\xi_s A_\gamma + \xi_a S_\gamma|} \equiv \frac{\gamma\sigma_M M_0 f\sqrt{x_c}}{\rho\omega^2 d^2\xi_s}M_\gamma^{(f)}. \tag{4.5}$$

If we omit the index γ and consider x a continuous function of σ, we may regard these expressions as envelopes of the resonance amplitudes. In the sections s, at the point $\sigma = \sigma_c$, and at the turning points, the expressions for $U^{(h)}$ and $M^{(f)}$ take the following form (Figs. 3 and 4):

$$U_\pm^{(h)} \approx \begin{cases} \dfrac{1-\sigma_c}{|\sigma-\sigma_c|}\left[1-\varkappa\dfrac{(1-\sigma_c)(1-\sigma)}{(\sigma-\sigma_c)^2}\right]\times\left[1+\delta\left(\dfrac{1-\sigma}{\sigma-\sigma_c}\right)^2\right]^{-1}, & s\\ \left|1\pm\dfrac{\lambda}{\xi_s}\right|\dfrac{\xi_s}{\lambda+2\xi_a\xi_s}, & \beta_{1,2}\\ \varkappa^{1/2}(1\pm\varkappa^{1/2})^{1/2}[\varkappa+\delta(1\pm\varkappa^{1/2})^2]^{-1}, & \sigma=\sigma_c;\\ |1\pm 2\xi_a|\dfrac{\xi_s}{\lambda+2\xi_a\xi_s}, & \alpha_{1,2}\end{cases} \tag{4.6}$$

$$M_\pm^{(f)} \approx \begin{cases} \dfrac{(1-\sigma_c)^{1/2}(1-\sigma)^{1/2}}{|\sigma-\sigma_c|}\times\left[1+\lambda\dfrac{(1-2\sigma_c+\sigma)(2-3\sigma_c+\sigma)}{(1-\sigma_c)(\sigma-\sigma_c)^2}\right]\times\left[1+\delta\left(\dfrac{1-\sigma}{\sigma-\sigma_c}\right)^2\right]^{-1}, & s\\ \left|1\pm\dfrac{\lambda}{\xi_s}\right|^{1/2}\dfrac{\xi_s}{\lambda+2\xi_a\xi_s}, & \beta_{1,2}\\ \varkappa^{1/2}(1\pm\varkappa^{1/2})^{1/2}[\varkappa+\delta(1\pm\varkappa^{1/2})^2]^{-1}, & \sigma=\sigma_c\\ (1\pm 2\xi_a)^{1/2}\dfrac{\xi_s}{\lambda+2\xi_a\xi_s}, & \alpha_{1,2}.\end{cases} \tag{4.7}$$

The function $U_+^{(h)}$ has a maximum when σ (Fig. 4) is equal to

$$\sigma_{m2} = 1-\sigma_c x_c\left[1+\left(\frac{\delta}{1+\delta}\right)^{1/2}\right]+2\lambda\left[1+\left(\frac{1+\delta}{\delta}\right)^{1/2}\right], \tag{4.8}$$

taking there the value

$$U_m^{(h)} \approx [\sqrt{\delta(\delta+1)}+\delta]^{-1}. \tag{4.9}$$

For $\delta = \kappa/(1-\kappa)$, the maximum is located at the point σ_c; for $\delta \gtrless \kappa/(1-\kappa)$, at a point $\sigma_{m2} \lessgtr \sigma_c$. The function $U_-^{(h)}$ is approximately a mirror image of the function $U_+^{(h)}$ with respect to $\sigma = \sigma_c$. The general form of the functions $M_\pm^{(f)}$ is close to that of the functions $U_\pm^{(h)}$. The points of maximum $M_\pm^{(f)}$ coincide with σ_c when $\delta = \kappa/(1 \pm \kappa^{1/2})$, respectively.

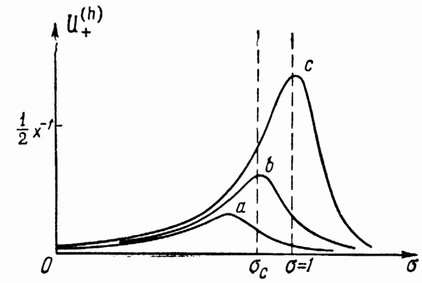

FIG. 4. Normalized envelope of resonance peaks of cross-amplitudes $U_+^{(h)}$: curve a, $\delta > \kappa$; curve b, $\delta \approx \kappa$; curve c, $\delta < \kappa$.

5. RELAXATIONAL CHARACTERISTICS OF MAGNETOELASTIC OSCILLATIONS

The relaxational characteristics, in contrast to the amplitudes, are, in the linear approximation, independent of the boundary conditions; consequently, they do not reflect what is special to a thin film. Thus the results of this section still hold for a different shape of the magnetoelastic crystal.

The complex dispersion relation (1.7), after the substitution $\omega = \omega' + i\omega'' \equiv \omega' + i\xi\omega'$, separates into a system of two equations, which (on neglect of ξ^2 in comparison with unity) can be put into the form

$$A\{S(A^2+\xi_a^2)-\lambda A\}+S(2A+1)^2\{A(S^2+\xi_s^2)-\lambda S\} + 2AS(2A+1)\{AS-(\lambda+\xi_a\xi_s)\}=0,$$

$$\xi = (S\xi_a + A\xi_s)/[S(1+2A)+A]. \tag{5.1}$$

The first equation of the system is satisfied if all three of the expressions in curly brackets vanish simultaneously; that is, if the relations (2.3), (3.3), and (4.3) are simultaneously satisfied. These correspond to the vanishing of the real parts of the amplitudes of the spin waves and of the cross-amplitudes. Thus the dispersion relation for a system with damping is described by a many-valued function, all branches of which are represented in Fig. 2 (this function has the simplest form in a rectangular system of coordinates AOS). For $\xi_a, \xi_s \to 0$, all the branches of this function fuse with branches of the dispersion relation (1.9).

The second equation of the system (5.1) gives the value of the effective relaxation parameter $\xi = \omega''/\omega'$ of the magnetoelastic system on any of the branches in Fig. 2. It is clear that on approach to the pure spin branch ($S \to 0$), $\xi \to \xi_s$; and that for $A \to 0$, $\xi \to \xi_a$. In the sections s and a and at the point $\sigma = \sigma_c$, ξ as a function of σ has the form

$$\xi_\pm \approx \begin{cases} \xi_s\left[1 - 2\lambda\dfrac{1}{\sigma_c - \sigma} + \varkappa\left(\dfrac{1-\sigma_c}{\sigma_c - \sigma}\right)^2\right] + 4\xi_a\lambda\left(\dfrac{1-\sigma}{\sigma_c-\sigma}\right)^2, & s \\ [\xi_s + \xi_a 2\sigma_e x_c(1 \pm \varkappa^{1/2})^2][1 + 2\sigma_e x_c(1\pm\varkappa^{1/2})]^{-1}, & \sigma = \sigma_c. \\ \xi_a\left[1 + 2\lambda\dfrac{\sigma - 1}{(\sigma-\sigma_c)^2} - \dfrac{1}{2}\varkappa\dfrac{2\sigma_c - 1}{\sigma - \sigma_c}\right] + \dfrac{1}{2}\xi_s\varkappa\dfrac{1-\sigma_c}{(\sigma-\sigma_c)^2}, & a \end{cases} \tag{5.2}$$

These functions reach extrema at certain points $\sigma_{m4}^{\pm}$, which coincide with σ_c if the condition

$$\xi_a / \xi_s = 1 \mp {}^1/_2\varkappa^{1/2}(1 \mp 2\sigma_e x_c) \approx 1 \tag{5.3}$$

is satisfied. If $\xi_a/\xi_s \lessgtr 1$, then $\sigma_{m4}^{+} \lessgtr \sigma_c$ and $\sigma_{m4}^{-} \gtrless \sigma_c$.

The line widths, on the frequency and magnetic field scales, are determined by the expeessions

$$\Delta\omega = 2\xi\omega, \qquad \Delta\sigma = 2\xi\partial\omega_0/\partial\omega. \tag{5.4}$$

On the sections s and a and at the point $\sigma = \sigma_c$, the functions $\Delta\sigma(\sigma)$ have the form

$$^1/_2\Delta\sigma_\pm \approx \begin{cases} \xi_s + 4\lambda\xi_a\left(\dfrac{1-\sigma}{\sigma_c - \sigma}\right)^2, & s \\ \xi_s + 2\xi_a\sigma_e x_c(1\pm\varkappa^{1/2})^2 & \sigma = \sigma_c. \\ \xi_s + 4\lambda\xi_a\left(1 + \dfrac{1}{\varkappa}\dfrac{\sigma-\sigma_c}{1-\sigma}\right)^2, & a \end{cases} \tag{5.5}$$

It is evident that on approach to the acoustic branch, $\Delta\sigma_\pm$ rises sharply.

DISCUSSION OF RESULTS

Both with magnetic and with acoustic excitation (when $\eta \neq 0$), there should be observed in a thin magnetic film a discrete spectrum of resonance peaks of coupled magnetoelastic oscillations, determined by exchange and magnetoelastic interactions. With acoustic excitation, in contrast to excitation by a uniform microwave magnetic field, even modes (satisfying the boundary conditions) are also excited, i.e., the number of resonance peaks is doubled; since uniform ferromagnetic resonance should not be observed with f-excitation, the resonance spectrum has a clearer structure in this case.

The distributions of the resonance-peak amplitudes (as dependent on σ, that is, on the value of the constant magnetic field) for the m-components of the oscillations under h- and f-excitation and for the u-components under h-excitation have much in common with one another (if $2d \leq \pi s_t/\omega$): in all three cases, a resonance peak of maximum amplitude should be observed at a certain value $\sigma = \sigma_{mi}$ (different for each of the cases); a resonance peak of maximum amplitude may be observed either to the left or to the right of σ_c, depending on the nonlinear relations (different for each of the cases) between the dimensionless parameters $\delta = 2\eta\sigma_M\xi_a/\xi_s$ and $\kappa = \eta s_t^2/\alpha\omega^2$; in all three cases resonance peaks can be observed also at values of the constant magnetic field greater than the field for uniform ferromagnetic resonance. The ratio ξ_a/ξ_s can be calculated from the measured value of σ_{mi}.

If $2d > \pi s_t/\omega$, then resonance peaks corresponding to the x_--branch are also excited; the amplitudes of such peaks can under certain conditions be greater than the amplitudes of the peaks that correspond to the x_+-branch.

The effective relaxation parameters ξ of the system, far from σ_c, go over to ξ_s (on the modified spin branch) or to ξ_a (on the modified acoustic branch); they reach maxima near σ_c; ξ may lie either to the left or to the right of σ_c, depending on the relation between ξ_s and ξ_a. The resonance line widths of the m-components, measured in magnetic-field units, behave differently in their dependence on σ: they are weakly modified on the modified spin branch and rise sharply on transition to the modified acoustic branch (even when $\xi_a < \xi_s$).

The calculation made permits the establishment of optimal conditions for most effective use of a thin magnetic film as an element for converting microwave oscillations of different types from one to another: (h) → (m), (h) → (u), and (u) → (m). For each type of conversion the optimal conditions are different; in particular, for given system parameters the values of the external constant magnetic field that correspond to the most effective conversion are different for each type.

A thin magnetic film permits the bringing about of a linear conversion of a uniform microwave magnetic field to acoustic oscillations (generation of hypersound); in bulk material the conversion (h) → (u) is possible only in the nonlinear range (either with nonuniform h or with nonuniform H_0).

Ferroacoustic resonance in a thin film was observed in[5,6]. In the work of Seavey[6] acoustic oscillations, excited by a pulse of microwave magnetic field, were led from the thin film into a quartz rod and, after reflection from its free end, again excited spin waves in the film; the microwave magnetic field produced by these latter pulses was registered with a superheterodyne receiver. Thus in this research, conversion was accomplished according to the scheme (h) → (u) → (m) → (h). The envelope of resonance amplitudes in this case had a form corresponding to large δ (curves of type a in Figs. 3 and 4). It must be kept in mind, however, that with such large film thicknesses (~ 4500 Å), there may be observed, between the peaks corresponding to the x_+-branch, peaks corresponding to the x_--branch; the resonance-amplitude envelope of the latter is appreciably different from the envelope of peaks of the x_+-branch.

A detailed comparison of theory with experiment is possible only when there is sure identification of the resonance peaks. In this respect the most suitable experiments would be ones conducted on films of identical composition and of different, sufficiently smoothly varying thicknesses.[1]

[1] V. A. Ignatchenko and E. V. Kuz'min, JETP **47**, 1814 (1964), Soviet Phys. JETP **20**, 1223 (1965).

[2] V. A. Ignatchenko and E. V. Kuz'min, FTT **7**, 1962 (1965), Soviet Phys. Solid State **7**, 1585 (1966).

[3] V. A. Ignatchenko, E. V. Kuz'min, and L. M. Gorenko, Izv. AN SSSR ser. fiz. (in press).

[4] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, JETP **35**, 228 (1958), Soviet Phys. **8**, 157 (1959).

[5] M. Pomerantz, Phys. Rev. Letters **7**, 312 (1961).

[6] M. H. Seavey, Jr., IEE Transactions on Ultrasonics Engineering, **UE-10**, 49 (1963).

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106