DOUBLE BREMSSTRAHLUNG IN COLLIDING BEAM EXPERIMENTS

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The differential cross section of double bremsstrahlung in e-e collisions in the case of large energies and small angles between the momenta of initial and final particles in the c.m.s. is obtained with the aid of an electronic computer.

THE emission of two photons in electron-electron or electron-positron collisions

$$e^- + e^- \to e^- + e^- + \gamma_1 + \gamma_2, \tag{1}$$

$$e^- + e^+ \to e^- + e^+ + \gamma_1 + \gamma_2 \tag{2}$$

can serve as a convenient standard process in colliding-beam experiments, if one detects the photons traveling in almost opposite directions at small angles to the collision line.

Let us consider process (1). Let $p_1(\mathbf{p}_1, \epsilon_1)$, $p_2(\mathbf{p}_2, \epsilon_2)$, $p_3(\mathbf{p}_3, \epsilon_3)$, $p_4(\mathbf{p}_4, \epsilon_4)$, $k_1(\mathbf{k}_1, \omega_1 = \epsilon_5)$, and $k_2(\mathbf{k}_2, \omega_2 = \epsilon_6)$ be the four-momenta of the initial and final electrons and photons, respectively. In accordance with the conditions of the proposed experiment, we shall assume the following: a) $\mathbf{p}_1 = -\mathbf{p}_2$, and b) θ_1 and $\theta_2 \ll 1$ (see Fig. 1); c) the energies of all particles are much smaller than the electron rest mass m. In addition, we impose a supplementary limitation d) θ_3 and $\theta_4 \ll 1$, the motivation for which will be given below. Thus, the small parameters in our problem are θ_k^2 and the ratios $(m/\epsilon_k)^2$, k = 1, 2, 3, 4, 5, 6.

Process (1) is described by forty Feynman diagrams. Ten of these are shown in Fig. 2. The matrix elements corresponding to the remaining 30 diagrams can be obtained from the relations

$$M_{10+i} = M_i(k_1 \leftrightarrow k_2), \qquad M_{20+i} = M_i(p_3 \leftrightarrow p_4),$$
$$M_{30+i} = M_i(k_1 \leftrightarrow k_2,\ p_3 \leftrightarrow p_4), \quad i = 1, 2, \ldots, 10.$$

We introduce the quadratic expressions $M_{ij} = \langle M_i \overline{M}_j \rangle$, averaged over the initial and summed over the final spin states of all the particles. We denote by μ_i the order of smallness of the denominator of the matrix element M_i, by ν_{ij} the order of smallness of the numerator of M_{ij}, and by λ_{ij} the order of smallness of all of M_{ij}, so that

$$\lambda_{ij} = \nu_{ij} - \mu_i - \mu_j. \tag{3}$$

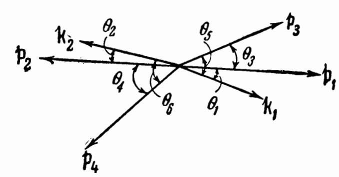

FIG. 1.

It is advantageous to separate all matrix elements into three groups A_1, A_2, and A_3. Group A_1 contains matrix elements M_2, M_3, M_4, and M_5, for which $\mu = 3$. Belonging to A_2 are M_1, M_{11}, M_{21}, M_{22}, M_{35}, M_6, M_{16}, M_{36}, M_7, M_{18}, M_{19}, and M_{20}, for which $\mu = 2$. All the remaining matrix elements belong to A_3, and for them $\mu = 1$ or $\mu = 0$.

We introduce symbols of the type A_{12} for the contribution made to the differential cross section by the interference of the diagrams from A_1 and A_2, and $\lambda(A_{12})$ for the order of smallness of this contribution:

$$A_{12} = \sum_{i,j} M_{ij}, \quad \lambda(A_{12}) = \max \lambda_{ij}; \qquad M_i \in A_1, \quad M_j \in A_2 \tag{4}$$

etc. Direct calculation showed that $\nu_{ij} = 2$ for all M_i and M_j from A_1 and that consequently $\lambda(A_{11}) = -4$. We then investigated expressions of the type M_{kk} for M_k from A_2. It turned out that for these $\nu_{kk} \geq 2$, and consequently $\lambda_{kk} \geq -2$. Hence we can conclude with the aid of the Schwartz inequality that $\lambda(A_{22}) \geq -2$ and $\lambda(A_{12}) \geq -3$. As to the diagrams A_3, it is clear that $\lambda(A_{33}) \geq -2$, and consequently $\lambda(A_{23}) \geq -2$ and $\lambda(A_{13}) \geq -3$.

It follows from the foregoing that with a relative error whose order of magnitude does not exceed max $(\theta_k^2, m^2/\epsilon_k^2)$, we can retain in the expression for the differential cross section of process (1) only the contribution of the diagrams A_1. With this accuracy, the square of the amplitude of the process, S, takes the form

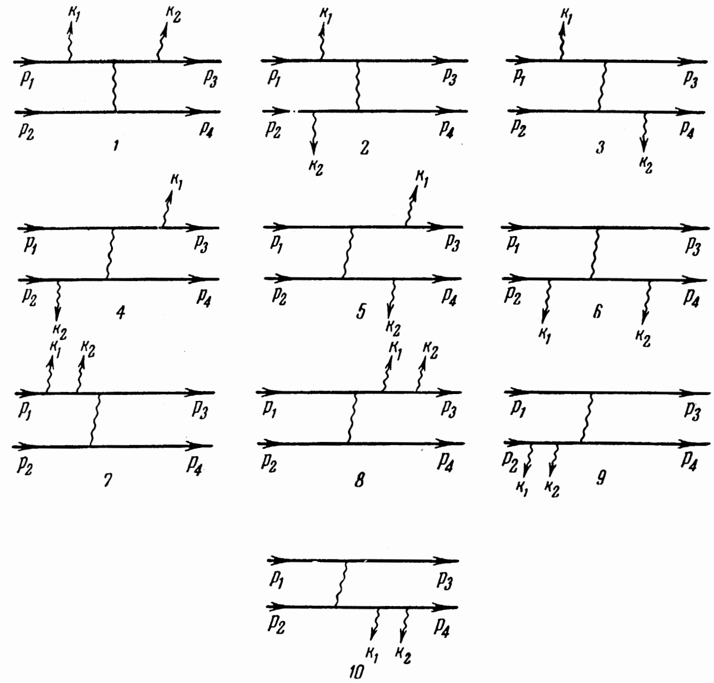

FIG. 2.

$$S=\frac{16\alpha^4(2\pi)^{12}}{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4\omega_1\omega_2}\left[a_1(a_1+\omega_1)+a_3(a_3+\omega_1)+\frac{b_1+c_1-c_3}{(k_1p_1)(k_1p_3)}\right]$$
$$\times\left[a_2(a_2'+\omega_2)+a_4(a_4'+\omega_2)+\frac{b_2+c_2-c_4}{(k_2p_2)(k_2p_4)}\right]\cdot\frac{1}{q^4};$$
$$a_1=\varepsilon_3/(k_1p_1),\quad a_2=\varepsilon_4/(k_2p_2),\quad a_3=\varepsilon_1/(k_1p_3),$$
$$a_4=\varepsilon_2/(k_2p_4),\quad a_2'=(\varepsilon_2-\omega_2)/(k_2p_2),$$
$$a_4'=(\varepsilon_4+\omega_2)/(k_2p_4),\quad b_1=(\varepsilon_1^2+\varepsilon_3^2)(p_3p_1)+\omega_1^2,$$
$$b_2=[\varepsilon_2(\varepsilon_4+\omega_2)+\varepsilon_4(\varepsilon_2-\omega_2)](p_4p_2)+\omega_2^2,$$
$$c_1=\varepsilon_3(\varepsilon_1+\varepsilon_3)(k_1p_1),\quad c_2=\varepsilon_4(\varepsilon_2+\varepsilon_4)(k_2p_2),$$
$$c_3=\varepsilon_1(\varepsilon_1+\varepsilon_3)(k_1p_3),\quad c_4=\varepsilon_2(\varepsilon_2+\varepsilon_4)(k_2p_4);$$
$$q=p_3-p_1-k;\quad \alpha=1/137; \tag{5}$$

$\hbar = c = 1$. All the energies are expressed in units of m; the metric is chosen in the form $ab = \mathbf{a}\cdot\mathbf{b} - a_0b_0$; the wave functions are normalized to unity. For comparison with experiment it is necessary to multiply the right side of (5) by 4. This is connected with the possibility of the relabeling $k_1 \leftrightarrow k_2$ and $p_3 \leftrightarrow p_4$.

In order to obtain the square of the amplitude of the probability of process (2), it is necessary to replace in (5) p_2 by $-p_{1+}$ and p_4 by $-p_{2+}$, where p_{1+} and p_{2+} are the four-momenta of the initial and final positrons, respectively. We note that expression (5) for process (1) becomes somewhat simpler if we take into account that, by virtue of the conservation laws, the relation $\epsilon_4 = \epsilon_2 - \omega_2$ is satisfied at the assumed accuracy.

Generally speaking, it is not clear beforehand whether the region of large final-fermion emission angles can make an appreciable contribution to the total cross section (especially at photon energies comparable with the energy of the initial particles). Therefore analogous calculations were made with condition d) discarded. The main contribution is made in this case to the differential cross section by the matrix elements M_2 and M_{22}, and the result has an order of smallness with respect to m^2/ϵ_k^2 or θ_1^2 and θ_2^2 which is at least larger by two than (5).

The analysis presented and the derivation of the formulas were carried out with the electronic computer of the Computational Center of the Siberian Division of the USSR Academy of Sciences with the aid of a procedure previously developed by the author.[1] The computer time amounted to approximately 30 minutes.

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[1] V. S. Synakh, Nuclear Physics (in press).

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