TEMPERATURE DEPENDENCE OF THE MEAN FREE PATH OF ELECTRONS IN TIN AT LOW TEMPERATURES

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The temperature dependence of the amplitude of the size effect was measured at the limiting points in tin. The mean free path between two elementary acts of interaction with phonons was determined, from the experimental data, for electrons located near the investigated point on the Fermi surface.

1. To obtain detailed data on the mean free path of electrons in metals, we must have a method for investigating individual groups of electrons, i.e., certain small regions of the Fermi surface. The method must satisfy additional requirements if we deal with the scattering of electrons which depends on temperature. If the mean free path is understood to be the average distance traveled by electrons with a given momentum between two single acts of interaction with other quasiparticles or with lattice inhomogeneities, then the method of measuring the mean free path so defined should be equally sensitive, in the investigated temperature range, to all types of scattering processes. We must bear in mind that when an electron interacts with a phonon at $T \ll \Theta_D$ (Θ_D is the Debye temperature), the electron is scattered through only a small angle proportional to T (of the order of p_{ph}/R, where p_{ph} is the average absolute value of the phonon momentum at temperature T, R is the radius of curvature of the Fermi surface), while scattering on impurities produces deviations through large angles at all temperatures.

In our investigation of the temperature dependence of the mean free path of electrons in tin, we have attempted to use the method proposed in [1], which can satisfy the stated requirements. The method is based on the measurement of the relative line intensity in the size effect, discovered by Gantmakher and Kaner. In the presence of a constant magnetic field inclined at a small angle to the surface of a plane-parallel sample, the dependence of the radio-frequency impedance on the field intensity shows a peak when the electrons in the region of an elliptic limiting point on the Fermi surface are able to make an integral number of turns moving from one surface of the sample to the other along an elongated helix. The intensity of the effect is governed by the number of electrons in the region considered which, moving in the skin layer parallel to the surface of the sample, are accelerated in that layer and then reach the skin layer on the opposite surface, again moving along the surface, so that they make a considerable additional contribution to the surface current. The selection of electrons by their velocity direction is thus made twice: at the beginning and at the end of the trajectory. Under such conditions, as shown by Azbel' and Kaner, [2] the role of the scattering processes reduces to the removal of some electrons from among the effective electrons (there is practically no replenishment at the expense of the remaining electrons).

Since the skin layer thickness δ is small, it is sufficient to scatter an electron through a small angle to make it ineffective. The value of this angle ϑ depends directly on the ratio of δ to the thickness of the sample d, angle of inclination of the magnetic field φ and the line number n, which gives the number of turns executed by the electrons. An estimate of ϑ may be obtained, for example, as follows. In the isotropic case, the longest path L which an effective electron can travel in the skin layer has an order of magnitude of $\delta^{1/3}d^{2/3}\varphi^{-1}n^{-2/3}$. The scattering through the angle $\vartheta = \delta/L$ obviously causes a considerable reduction in the electron path in the skin layer. In our case, we obtain $\vartheta \sim 10^{-3}$ for $\delta \sim 10^{-4}$ cm, $d = 4 \times 10^{-2}$ cm, $\varphi \sim 10^{-1}$, and $n = 2$.

At helium temperatures, the interaction of an electron with a phonon deflects the electron by an angle of the order of 10^{-2}. The difference between ϑ and this deflection angle, amounting to one order of magnitude, is obviously small, but according to

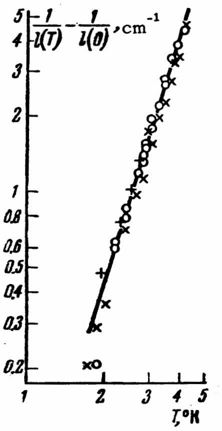

FIG. 1. Temperature dependence of 1. Thickeness of sample d = 0.39 mm, normal to the surface n ∥ [010], E ∥ [100], H ∥ (001); measuring field frequency 3 Mc; symbols representing the different angles of inclination of the magnetic field: × – 4°02', + – 2°45', ○ – 3°10', ● – 4°15', The systematic discrepancies between the series of points are evidently related to the inaccurate determination of the angle of inclination φ.

these estimates, we may assume that even a single interaction with a phonon makes an electron ineffective.

Gantmakher and Kaner [1] have shown that the amplitude A depends on the length of the electron trajectory $s \sim d/\varphi$, as $\exp(-s/l)$. We may consequently assume that the value of l obtained in this way represents the path between two elementary scattering acts.

One observation may be made in this connection. Since, at high values of n (i.e., in strong fields), ϑ increases, then, if $n \gg 1$, several collisions may be needed to make an electron ineffective. Thus, the effective mean free path may, in general, depend on the magnetic field intensity, which must be allowed for in the theoretical calculations.

2. The value of A was measured as a function of temperature for the elliptic limiting point I in the notation used in [1], located on the Fermi surface near the [100] direction, on the side surface of a tube joining two "corrugated planes" in the fourth zone. We also investigated the temperature dependence of several lines of the size effect for closed extremal orbits.

In the first case, the function $(\varphi/d)\ln A$ was equal to the quantity $1/l(T)$, to within a constant term. The limiting value of this function at $T \to 0$ may be determined quite accurately by extrapolation. The temperature dependence of the quantity $1/l(T) - 1/l(0)$, found in this way, is shown in Fig. 1 on the logarithmic scale. The straight line drawn through the points represents a power dependence $T^{3.3}$, close to the cubic law which should be obeyed (according to Bloch's theory) by the scattering from phonons.

Since the electron-electron collisions in the investigated range of temperatures make no marked contribution to the scattering, the experimental results obtained may be considered as confirmation that, under our conditions, the mean free path between single scattering acts was indeed measured. In that case, the quantity plotted along the ordinate axis represents directly $1/l_{ph}$, which is the probability of an interaction with a phonon, per centimeter of the electron path. The observed small deviation from the cubic law is obviously associated with the insufficiently small value of the angle ϑ. In the case of the static electrical conductivity of a bulk sample, when the scattering through small angles makes only a small contribution to the resistance (which is proportional to the square of the angle), tin exhibits—in accordance with the theory—a faster rise of the resistance with temperature, proportional to T^5, in the same range of temperatures. [3]

On the other hand, attention should be drawn to the considerable quantitative discrepancy obtained if we attempt to compare our results in their order of magnitude with the average mean free path which may be calculated from the electrical resistance data, using the free electron model. If we denote by ρ_{ph}, the temperature-dependent part of the electrical resistivity (excluding the residual resistivity), we can determine the value of the effective mean free path $l_{ph'}$, using the well known formula $\rho_{ph} = p_F/Ne^2 l_{ph'}$ (p_F is the Fermi momentum, N is the number of electrons per cm^3, e is the electron charge). Assuming that the number of free electrons per atom is of the order of unity, we find that the value of $l_{ph'}$ for tin at 2°K is of the order of several centimeters, i.e., it is of the same order as l_{ph} determined from our experiments; according to the free electron model estimates, these quantities should differ by a factor (p_F/p_{ph}), i.e., by a factor of 10^4.

Similar discrepancies, although not so large, were reported earlier by Klemens who made a comparison with the data obtained from the temperature dependence of the electrical conductivity and thermal conductivity. [4] The reason for this discrepancy lies, according to Klemens, in the considerable role played in collisions with long-wavelength phonons by the umklapp processes, which are possible in metals with an open Fermi surface, such as tin.

Our data are best compared with the estimates of the electron—phonon path l_T, obtained from the data on the electronic thermal conductivity of a metal. For tin, it follows from Zavaritskiĭ's data [5] that l_T is proportional to T^{-3} and that at 2°K the value is $l_T \sim 10^{-1}$ cm, which is much closer to our data. The difference is possibly due to the fact that, at the point on the Fermi surface investigated by us, the phonon scattering is somewhat weaker than the average scattering over the whole Fermi sur-

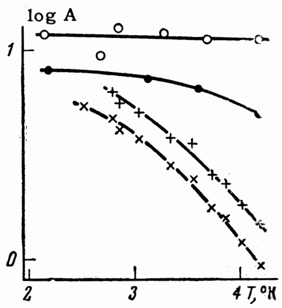

FIG. 2. Temperature dependence of the line amplitudes in the size effect: ○ – line 1_1 for 18° between **H** and [001] axis, ● – line 1_1 for 30° between **H** and [001] axis, + – line 2_1 for **H** ∥ [001], × – line 3_1 for **H** ∥ [001] (line notation in accordance with[6]).

face. It should also be mentioned that the value of l_{ph} found by us agrees with the theoretical estimate of this quantity $l_{ph}^{theor} \sim (v\hbar/k\Theta_D)(\Theta_D/T)^3 \sim 10$ cm (v is the Fermi velocity).

3. We also carried out observations of the temperature dependence of the amplitude of the size effect lines at extremal trajectories, as described in [6]. Some typical results are shown in Fig. 2. It is evident from this figure that, in addition to the lines whose intensity changes by one order of magnitude in the investigated range of temperatures, there is also a line with an amplitude which is independent of temperature within the limits of experimental error (10–15%). This line corresponds to the extremal trajectory enveloping the cylinder in the fourth zone near the surface of the sample in the (100) plane. The independence of temperature was also observed in the same region of the Fermi surface of a sample with its surface parallel to (110) and in another case relating to a portion of the Fermi surface again of approximately cylindrical shape. If we rotate the magnetic field so that the orbit proceeds along the edges of the cylindrical region in the fourth zone, the temperature dependence reappears (black points in Fig. 2).

Thus, the absence of the temperature dependence is evidently associated with the cylindrical shape of the investigated part of the Fermi surface, although we were unable to explain qualitatively this connection. The fact that the scattering of electrons by those phonons whose momentum is directed along the cylinder axis does not render the affected electrons ineffective is obviously unimportant because the number of such phonons is small. We also recall that the limiting point I considered here lies on the lateral surface of the same cylinder in the fourth zone. Thus, the electrons, in fact, experience phonon scattering over a considerable part of their extremal trajectory. The problem of the temperature dependence of the effect for an extremal trajectory requires a more detailed theoretical study in which allowance must be made for the possible repeated return of electrons to the skin layer. The number of returns cannot be regarded as large, as assumed in [7], which complicates the calculations even further.

In the size effect in an inclined field, the repeated return of electrons to the skin layer does not occur and the observed temperature dependence of the effect makes it possible to determine l_{ph} sufficiently reliably.

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[1] V. F. Gantmakher and É. A. Kaner, JETP **45**, 1430 (1963), Soviet Phys. JETP **18**, 988 (1964).

[2] M. Ya. Azbel' and É. A. Kaner, JETP **32**, 896 (1957), Soviet Phys. JETP **5**, 730 (1957).

[3] V. B. Zernov and Yu. V. Sharvin, JETP **36**, 1038 (1959), Soviet Phys. JETP **9**, 737 (1959).

[4] P. G. Klemens, Handbuch der Physik **14**, 198 (1956); Russ. Transl. in Sbornik: Fizika nizkikh temperatur (Collection: Physics of Low Temperatures), IIL, 1959.

[5] N. V. Zavaritskiĭ, JETP **39**, 1571 (1960), Soviet Phys. JETP **12**, 1093 (1961).

[6] V. F. Gantmakher, JETP **44**, 811 (1963), Soviet Phys. JETP **17**, 549 (1963).

[7] É. A. Kaner and M. Ya. Azbel', JETP **33**, 1461 (1957), Soviet Phys. JETP **6**, 1126 (1958).

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