BEHAVIOR OF A PLASMA WITH LARGE RADIATION PRESSURE IN A STRONG ELECTRIC FIELD

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Electric-field heating of a completely ionized plasma where the photon heat capacity exceeds the electron heat capacity is considered. The attainment of a steady state of the system depends on the photon thermal conductivity. The photon distribution function is found for the case when heating of the photons is due mainly to Compton, not radiative, processes. This distribution is Maxwellian with a "chemical potential" determined by conservation of the number of photons. The possible instabilities in this plasma are considered: thermal instability due to unbounded (in the linear approximation) heating, acoustic instability, and instability associated with a descending current-voltage characteristic in a strong magnetic field.

1. INTRODUCTION

We have previously [1-3] investigated the kinetic properties associated with the drag and mutual drag of electrons and photons in a rarefied hot plasma with large radiation pressure. The present work considers the properties of the same plasma in a high electric field, which heats the plasma electrons to a temperature different from that of the ambient. Energy exchange between the electrons and ions is hindered by their large mass difference. Therefore, when the radiation pressure exceeds the gas pressure electrons transfer energy mainly to photons even when the radiation pressure is not very high; in this case electron momentum is still transferred mainly to ions.

Let us consider a plasma in which the energy transfer from electrons to photons occurs through Compton scattering. The electron concentration n must then satisfy the inequality

$$n < 10^2\left(\frac{T}{mc^2}\right)^{3/2}\left(\frac{T}{\hbar c}\right)^3 \approx 7\cdot 10^{60} T^{9/2}, \qquad (1.1)$$

where T is the plasma temperature in ergs. As in our earlier work, we shall also assume that the dimensions L of the system exceed the mean free photon path l_f for the considered processes:

$$L > l_f = 1/n\Sigma_f \qquad (1.2)$$

where $\Sigma_f \approx 7\times 10^{-25}$ cm^2 is the Compton scattering cross section. For $n \gtrsim 2\times 10^{21}$ cm^{-3}, L does not exceed 10^3 cm; this corresponds to $T \gtrsim 13 \times 10^6$ deg. In stellar interiors (1.1) is fulfilled for hot stars of the early spectral classes O—B, for Wolf-Rayet stars, and for hot supergiants.

A steady state of the system is attained through radiative thermal conductivity, the role of which we shall here evaluate by introducing the characteristic cooling time of the photon system. Our investigations lead to the following conclusions:

1. In a sufficiently hot rarefied plasma Compton scattering has the principal role. Since the total number of photons is here conserved a steady state is reached with a Bose-Einstein (not Planck) photon distribution and non-zero chemical potential. In a highly heated plasma the Maxwell-Boltzmann distribution $N_q = Ae^{-cq/T}$ is approximated.

2. The behavior of a plasma with large radiation pressure in a strong electric field differs greatly in low and high magnetic fields. In the first case critical electron heating occurs (almost independently of the rate of plasma heat transfer), above which no steady state exists and the plasma can be heated without limit (at least, in a linear approximation). This transition to an unsteady state will here be called thermal instability. An analytic relation between the electric field and electron temperature can be found near the threshold of this instability.

3. Thermal instability occurs for an electron drift velocity which, as a function of the system parameters, can be many times greater than the velocity of sound. Therefore acoustic instability can occur in this plasma, which thus differs from a plasma with small radiation pressure, where

thermal instability arises in a smaller field than that required for acoustic instability, so that the latter does not occur.

4. Thermal instability does not exist in a high magnetic field. Therefore steady-state solutions of the heat balance equation are possible in electric fields of any strength, so that acoustic instability is also possible.

5. The magnetic field dependence of electric conductivity of the plasma leads in a high magnetic field to a third type of instability, which is associated with a descending current-voltage characteristic.

2. KINETIC EQUATIONS

We consider a fully ionized plasma with large radiation pressure in a high electric field **E** and magnetic field **H**. Electrons can transfer the energy that they derive from the field to ions, to photons through Compton scattering, and to photons through bremsstrahlung. In the present work we shall confine ourselves to a plasma which is so highly rarefied and hot that photons interact with electrons mainly through Compton scattering. The last of the aforementioned processes is therefore very slow; we have the following ratio of its characteristic time $\tau_r \sim \tau_i \cdot 137 mc^2 \Lambda / T$ (where τ_i is the time constant for momentum transfer from electrons to ions [see Eq. (1.4) of [1]] and Λ is the Coulomb logarithm) to the characteristic time constant $\tau_{ei} \sim \tau_i M/m$ of the first process:

$$137\Lambda \frac{mc^2}{T}\frac{M}{m} \approx \frac{mc^2}{T} \gg 1$$

under the conditions of present interest. The ratio to the time for energy transfer through Compton processes, $\tau_{ef} \sim \tau_f$ [from Eq. (1.4) of [1]] is

$$\frac{\tau_i}{\tau_f} 137 \frac{mc^2}{T}\Lambda \gg 1$$

with

$$\gamma = \frac{\tau_f}{\tau_i} < 137 \frac{mc^2}{T}\Lambda,$$

i.e., at sufficiently large radiation pressure. Substituting γ from Eq. (1.10) of [1], we obtain our Eq. (1.1). We shall therefore neglect bremmstrahlung in the energy balance.

The system of kinetic equations for the electron distribution n_p and photon distribution N_q is then

$$\frac{\partial n_\mathbf{p}}{\partial t} + \left(e\mathbf{E} + \frac{e}{c}[\mathbf{vH}]\right)\frac{\partial n_\mathbf{p}}{\partial \mathbf{p}} = W_i(n_\mathbf{p}) + W_c(n_\mathbf{p}, N_\mathbf{q}), \quad (2.1)^*$$

$$W_c'(n_\mathbf{p}, N_\mathbf{q}) = 0.$$

Here

$$W_c(n_\mathbf{p}, N_\mathbf{q}) = \int d^3q\, d^3q'\, W_{\mathbf{qq}'}\{n_{\mathbf{p}'}N_{\mathbf{q}'}(1+N_\mathbf{q}) - n_\mathbf{p}N_\mathbf{q}(1+N_{\mathbf{q}'})\}\,\delta(\varepsilon_p + cq - \varepsilon_{p'} - cq'),$$

where W_i and W_c are the Coulomb and Compton collision integrals [see Eq. (1.1) of [1]] and $W_{qq'}$ is the Compton scattering probability.

If the distribution n_p is divided into two parts which are odd and even, respectively, with regard to the momentum, then as a result of electron interaction the first part becomes a Maxwell distribution with the electron temperature T_e during a time which is much shorter than the time for electron energy exchange with ions and photons. Under this condition the odd part is determined by an equation of the same form as in a low electric field. Therefore the electric conductivity σ is of the customary form, but with an electron temperature T_e determined from the energy balance equations.

It was shown in [1] that in the presence of photons and taking account of photon drag the electric conductivity is determined by the electron-ion relaxation time τ_i; therefore in a low magnetic field $\Omega\tau_i \ll 1$ (where $\Omega = eH/mc$ is the cyclotron frequency), we have

$$\sigma = \sigma_0 (T_e / T_0)^{3/2}, \qquad \sigma_0 \equiv \sigma(T_0). \quad (2.2)$$

An equilibrium distribution of plasma ions is also reached in a time shorter than that required for energy exchange with electrons, and a temperature T_i can also be assigned to the ions. Then, according to Braginskiĭ,[4] the time required for energy transfer from electrons to ions is

$$\tau_{ei} = \frac{\tau_i \sqrt{\pi}}{4}\frac{M}{m} = \frac{T^{3/2} m^{1/2}}{4\sqrt{2\pi}\, e^4 n \Lambda Z^2}\frac{M}{m}. \quad (2.3)$$

At the initial time the photons have the temperature T_0. In the case of small photon heating we shall assume $T_f = T_0$ and a Planck distribution. Then, multiplying $W_c(n_p, N_q)$ by $\epsilon_p d^3p$ and integrating over all momenta, we obtain the time for energy transfer from electrons to photons:

$$\tau_{ef} = \frac{45}{32\pi^3}\frac{mc^2}{T_0} \Big/ r_0^2 \left(\frac{T_0}{\hbar c}\right)^3 c. \quad (2.4)$$

Having obtained energy from the electrons, the photons ultimately reach a steady-state distribution, which cannot be of the Planck type in the presence of only Compton scattering. The Compton collision integral in the photon equation (4.1) vanishes for

$$N_\mathbf{q}(1+N_\mathbf{q})^{-1} e^{cq/T} = N_{\mathbf{q}'}(1+N_{\mathbf{q}'})^{-1} e^{cq'/T} = \text{const.}$$

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

Hence

$$N_q = \left[\exp\left(\frac{cq-\mu}{T_f}\right)-1\right]^{-1}. \qquad (2.5)$$

Since the number of photons is conserved in Compton scattering, the constant μ signifying the "chemical potential" of the conserved photons is obtained using the condition

$$2\int N_q \frac{d^3q}{(2\pi\hbar)^3} = N,$$

where N is the initial number of photons.

For small heating when $(T_f/T_0)^3 - 1 \ll 1$, we have

$$\mu / T_f = 1 - (T_f / T_0)^3.$$

In the opposite case of large heating, when $3 \ln (T_f/T_0) > 1$, we have

$$\mu / T_f \approx -3 \ln (T_f / T_0).$$

In this case we can neglect unity in the denominator of (2.5) and assume a Maxwell-Boltzmann photon distribution:

$$N_q = \exp\left(\frac{\mu - cq}{T_f}\right) = \left(\frac{T_0}{T_f}\right)^3 \exp\left(-\frac{cq}{T_f}\right). \qquad (2.6)$$

We now have

$$\tau_{ef}' = \frac{T_0}{T_f}\tau_{ef} = \frac{45}{32\pi^3}\frac{mc^2}{T_f}\frac{1}{r_0^2Nc}. \qquad (2.7)$$

During the interval between the initial time and the time when the distribution (2.5) is established we do not have an equilibrium distribution of the photons, and we can assign to them only an effective temperature T_f that characterizes qualitatively the rate of energy exchange between electrons and photons governed by

$$n\left(\frac{d\langle\varepsilon_p\rangle}{dt}\right)_{ef} = \frac{1}{\tau_{ef}}(T_f - T_e). \qquad (2.8)$$

To determine T_f and τ_{ef} we proceed, as previously, by multiplying (2.1) with the electron energy ϵ_p, and then integrating over d^3p. Substituting the Maxwell distribution with the temperature T_e for n_p, we obtain

$$n\left(\frac{d\langle\varepsilon_p\rangle}{dt}\right)_{ef} = \int n_p \frac{\varepsilon_p}{T_e} d^3p \int W_{\mathbf{qq}'} d^3q\, d^3q' N_{\mathbf{q}'}(1 + N_{\mathbf{q}})$$
$$\times\, \delta(\varepsilon_p + cq - \varepsilon_{p'} - cq')$$
$$\times\, T_e\left\{\exp\left(\frac{\varepsilon_p - \varepsilon_{p'}}{T_e}\right) - \frac{N_{\mathbf{q}}(1 + N_{\mathbf{q}'})}{(1 + N_{\mathbf{q}})N_{\mathbf{q}'}}\right\}. \qquad (2.9)$$

Since $c\,|\,q - q'\,| \ll T$, the exponential and $N_q(1 + N_{q'})/(1 + N_q)N_{q'}$ differ very little from unity and when expanded in a series yield

$$\left\{\frac{1}{T_e} + \frac{1}{Nq(1+N_q)}\frac{\partial N_q}{\partial cq}\right\} c(q' - q)$$

so that (2.9) becomes

$$\int n_p \frac{\varepsilon_p}{T_e} d^3p \int W_{\mathbf{qq}'} N_{q'}(1 + N_q)\, c\,(q' - q)$$
$$\times\, \delta(\varepsilon_p + cq - \varepsilon_{p'} - cq')$$
$$\times\left(1 + \frac{T_e}{N_q(1+N_q)}\frac{\partial N_q}{\partial cq}\right) = n(A - BT_e). \qquad (2.10)$$

This expression acquires the form of (2.8) through the substitutions

$$\tau_{ef}^{-1} = B, \qquad \tau_f = \tau_{ef} A = A / B. \qquad (2.11)$$

We note that a more detailed calculation for the Planck distribution gives $\tau_{ef}^{-1}(\epsilon_p) = \tau_{ef}^{-1}(\tfrac{2}{3}\epsilon_p/T_e - 1)$.

3. THERMAL INSTABILITY OF A PLASMA WITH LARGE RADIATION PRESSURE

In the presence of a magnetic field we have the current

$$\mathbf{j} = \sigma\mathbf{E} + \sigma'[\mathbf{Eh}] + \sigma''(\mathbf{Eh})\mathbf{h}, \qquad (3.1)$$

where $\mathbf{h} = \mathbf{H}/H$. The heating produced by an electric field in unit time and unit volume is

$$\mathbf{jE} = \sigma_0(T_e) E_{\parallel}^2 + \frac{\sigma_0(T_e)E_{\perp}^2}{1 + \Omega^2\tau_i^2(T_e)} = \sigma_0(T_0)E_{\parallel}^2\left(\frac{T_e}{T_0}\right)^{3/2}$$
$$+\, \sigma_0(T_0)E_{\perp}^2\left(\frac{T_e}{T_0}\right)^{3/2}\left\{1 + \Omega^2\tau_i^2(T_0)\left(\frac{T_e}{T_0}\right)^3\right\}^{-1}, \qquad (3.2)$$

where σ_0 is the electric conductivity in the absence of a magnetic field, and $E_{\parallel}$ and $E_{\perp}$ are the electric field components parallel and perpendicular to the magnetic field. In the immediately following sections we shall be interested in the case of heating that is practially independent of the magnetic field. This can occur either for $\Omega\tau_i \ll 1$ or for $E_{\perp}/E_{\parallel} \ll 1 + \Omega^2\tau_i^2$.

On the basis of all preceding considerations the set of heat balance equations for a system of electrons, ions, and photons will be

$$\frac{3}{2}\dot{T}_e = \frac{\sigma E^2}{n} - \frac{1}{\tau_{ef}}(T_e - T_f) - \frac{1}{\tau_{ei}}(T_e - T_i),$$
$$\frac{3}{2}\dot{T}_i = \frac{1}{\tau_{ei}}(T_e - T_i),$$
$$3\dot{T}_f = \frac{n}{N}\frac{1}{\tau_{ef}}(T_e - T_f) - \frac{T_f - T_0}{\tau_0}. \qquad (3.3)$$

These equations must now be interpreted.

1. The coefficient 3 in the left-hand side of the third equation pertains to the case of large photon heating $T_f \gg T_0$ with a near-Maxwellian distribution. (The coefficient would be $\sim$2.8 in the case

of small heating, with a near-Planckian distribution.)

2. We assume that the region of the high electric field has dimensions considerably greater than the mean free path. The temperatures T_e, T_i, and T_f are averaged over the volume of this region. We assume, finally, that the system is cooled slowly by photonic heat transfer; this cooling is represented by the second term in the right-hand side of the third equation containing the cooling time τ_0.

3. We shall investigate the behavior of the plasma in a high electric field when the radiation pressure is so large that the ratio n/N is small. It was shown in [3] that in this case the momentum relaxation of photons proceeds mainly through Compton, rather than bremsstrahlung, processes. The small ratio can also occur even when Coulomb scattering predominates over Compton scattering in the electron equation.

4. The quantities T_f, τ_{ef}, and τ_0 have only qualitative significance. However, we shall require these only for order of magnitude estimates, so that their presence in (3.3) does not prevent us from using these equations for exact quantitative results.

For a preliminary estimate of the different terms in the system of equations we shall obtain their first solution by making the electric conductivity σ and the time constants independent of the particle temperatures (as in the case of small heating). For simplicity we shall consider the two limiting cases when the ratio τ_{ei}/τ_{ef} is much larger and much smaller than unity. The first case, $\tau_{ef} < \tau_{ei}$, exists when $\gamma \lesssim M$, where $\gamma = \tau_f/\tau_i$ is the ratio of the momentum relaxation times due to scattering by photons and ions [see Eq. (1.10) of [1]]. Then (3.3) has the solutions

$$T_e \cong T_0 + \frac{\sigma E^2}{n}\left[\tau_{ef}\left(1 - \exp\left(-\frac{2t}{3\tau_{ef}}\right)\right) + \frac{n}{N}\tau_0\left(1 - \exp\left(-\frac{t}{3\tau_0}\right)\right)\right],$$

$$T_f \cong T_0 + \frac{\sigma E^2}{N}\left[\tau_0\left(1 - \exp\left(-\frac{t}{3\tau_0}\right)\right) + \tau_{ef}\left(\exp\left(-\frac{2t}{3\tau_{ef}}\right) - \exp\left(-\frac{t}{3\tau_0}\right)\right)\right],$$

$$T_i \cong T_0 + \frac{\sigma E^2}{n}\left[\tau_{ef}\left(1 + \frac{\tau_{ef}}{\tau_{ei} - \tau_{ef}}\exp\left(-\frac{2t}{3\tau_{ef}}\right) - \frac{\tau_{ei}}{\tau_{ci} - \tau_{ef}}\exp\left(-\frac{2t}{3\tau_{ei}}\right)\right) + \frac{n}{N}\tau_0\left(1 + \frac{\tau_{ei}}{\tau_{ei} - 2\tau_0} \times \exp\left(-\frac{2t}{3\tau_{ei}}\right) - \frac{2\tau_0}{2\tau_0 - \tau}\exp\left(-\frac{t}{3\tau_0}\right)\right)\right]. \quad (3.4)$$

These solutions lead to the following conclusions:

1. During a time of the order τ_f only the electron subsystem is heated, but not the photon subsystem. At the expiration of this time electron heating is slower in the ratio n/N and photon heating begins. This second stage continues for a time of the order τ_0, with the temperature difference $T_e - T_f$ becoming constant. A steady state is reached after the time τ_0.

2. The ion temperature follows the electron temperature almost exactly, differing from the latter only by an amount depending on the small parameters τ_{ef}/τ_{ei} and τ_{ei}/τ_0.

In the second limiting case, $\tau_{ei} \ll \tau_{ef}$, during a time of the order τ_{ei} only electrons are heated; when $\tau_{ef} > t > \tau_{ei}$ the heating is extended to the ions and the difference $T_e - T_i$ remains constant. After a time τ_{ef} energy begins to be acquired by photons, so that electron heating is sharply reduced; the difference $T_e - T_i$ becomes very small, but the difference $T_e - T_f$ remains constant. The analytic form of the solution is too complicated for presentation here.

From an examination of the two limiting cases we arrive at the conclusions that, (a) after some time of greater order than τ_{ei} or τ_{ef} the temperatures T_i and T_e become practically identical, and (b) the behavior of the system during a time of the order τ_{ef} in the first case and τ_{ei} in the second case differs greatly from its behavior during a time τ_0. These conclusions are so general that they should continue to hold true for order of magnitude estimates even when the temperature dependence of the systems of parameters is taken into account in the case of large heating.

Adding the first two equations of (3.3) and assuming $T_e = T_i$, we obtain equations for T_e and T_f:

$$3\dot{T}_e = \frac{\sigma E^2}{n} - \frac{T_e - T_f}{\tau_{ef}},$$

$$3\dot{T}_f = \frac{n}{N}\frac{T_e - T_f}{\tau_{ef}} - \frac{T_f - T_0}{\tau_0}. \quad (3.5)$$

Eliminating T_f and using (2.2) for $\sigma(T_e)$, we arrive at an integro-differential equation for $z = T_e/T_0$:

$$3\tau_{ef}\dot{z} + z = \alpha z^{3/3} + 1 + \frac{n}{3N} \times \int_0^t \exp\left\{\left(\frac{\tau_{ef}}{\tau_0} + \frac{n}{N}\right)\frac{t' - t}{3\tau_{ef}}\right\}\frac{z(t') - 1}{\tau_{ef}}dt', \quad (3.6)$$

where $\alpha = \sigma(T_0)E^2\tau_{ef}/nT_0$. The integral in the right-hand side equals $T_f/T_0 - 1$ and thus takes photon heating into account.

We shall solve this equation in the limiting cases of small and large photon heating, i.e., small and large values (compared with unity) of the integral in (3.6). For small heating we can neglect the integral by comparison with unity, obtaining

$$3\tau_{ef}\dot{z}+z=\alpha z^{3/2}+1, \tag{3.7}$$

which possesses a solution that is independent of time when the parameter α is smaller than some critical value α_0. In this case we have $\alpha = z^{-1/2} - z^{-3/2}$ with the maximum $\alpha_0 = 2/3^{3/2}$ for $z = z_0 = 3$.

In this case, the maximum electron heating consistent with a steady state of the system is given by $T_e/T_0 = 3$; in the absence of radiation pressure the value is 1.5. For α close to α_0 we have

$$z=z_0-2\cdot 3^{5/4}(\alpha_0-\alpha)^{1/2}, \tag{3.8}$$

and for α close to zero we have

$$z=1+\alpha. \tag{3.9}$$

When $\alpha > \alpha_0$ electrons cannot transfer to photons all the energy that they derive from the electric field. Electron heating is therefore unbounded and thermal instability arises. A similar case in the absence of radiation pressure, when the electrons can transfer energy only to ions, was considered in[5]. The conditions permitting the dropping of the integral in (3.6) are simplest when a steady state is possible. The integral then equals

$$\frac{n}{N}\left(\frac{n}{N}+\frac{\tau_{ef}}{\tau_0}\right)^{-1}\left\{1-\exp\left[-\left(\frac{n}{N}+\frac{\tau_{ef}}{\tau_0}\right)\frac{t}{3\tau_{ef}}\right]\right\}$$

and is small in two cases. The first case is given by

$$n/N \ll \tau_{ef}/\tau_0. \tag{3.10}$$

Substituting the values of τ_{ef} and $\tau_0 \approx L^2 n\Sigma_f c^{-1}$, we then obtain

$$L< l_f(mc^2/T)^{1/2}. \tag{3.10a}$$

This inequality denotes such great outward heat transfer by photons that they are not appreciably heated after any period of time.

As the second condition, if (3.10) does not occur, the given integral becomes small after a time

$$t\ll\tau_{ef}\left(\frac{n}{N}+\frac{\tau_{ef}}{\tau_0}\right)^{-1}\sim\tau_{ef}\frac{N}{n},$$

i.e., the photons are not still not heated up. Since photon heating was not considered in all the preceding discussions, τ_{ef} = const was assumed in (3.6)—(3.9).

We shall now consider the opposite limiting case of large photon heating, i.e., $z, y = T_f/T_0 \gg 1$, which occurs, according to (3.10), when $\beta = \tau_{ef} n/\tau_0 N \ll 1$. After a time $t > \tau_{ef}N/n$ steady-state temperatures are established for the electrons, $z = T_e/T_0$, and for the photons, $y = T_f/T_0$; these satisfy the equations

$$\alpha z^{3/2}=y(z-y),\qquad y(z-y)=\beta(y-1), \tag{3.11}$$

where $\tau_{ef} = \tau_{ef}(T_0)$ is assumed in α.

Solving the second equation for y, substituting this solution into the first equation, and neglecting β compared with z^2, we obtain an equation that is identical with (3.8):

$$\alpha_1 z_1^{3/2}=z_1-1, \tag{3.12}$$

where

$$z_1=z(1+\beta)^{-1},\qquad \alpha_1=\alpha(1+\beta)^{1/2}\beta^{-1}.$$

The investigation of (3.8) leads to the conclusion that the maximum electron heating is given by $z_0 = 3(1+\beta)$, i.e., there is negligible increase; the electric field for thermal stability is determined from

$$\alpha_0=\frac{2}{3^{3/2}}\frac{\beta}{(1+\beta)^{1/2}},$$

which is smaller by approximately the factor $\beta^{1/2}$. Near this critical value, by analogy with (3.8), we have

$$z_1=z_{10}-2\cdot 3^{5/4}(\alpha_{10}-\alpha_1)^{1/2},$$

whence

$$z=z_0-2[3(1+\beta)]^{5/4}\beta^{-1/2}(\alpha_0-\alpha)^{1/2}.$$

We can assume that our approximations—the introduction of the photon temperature T_f and the time constants τ_{ef} and τ_0—affect only the parameter β but do not affect our physical results. We note, particularly, that in the steady state maximum electron heating is practically identical in the limiting cases of fast and slow photon cooling.

We have investigated the conditions leading to thermal instability in the limiting case of small and large photon heating in a plasma with large radiation pressure. It is also possible to state certain general criteria for the possibility that instability will occur. Let us consider, for example, that electrons are heated by an electric field while the product of their electric conductivity and the energy transfer time τ_{en} is proportional to the degree n of electron heating, and that they transfer energy to a reservoir either directly or through the medium of another subsystem (as in the two cases already discussed). The heat balance equations analogous to (3.11) then become $\alpha z^n = z - 1$. The maximized solution of this equation for α gives $z_0 = n/(n-1)$.

It follows that for $n = 1$ thermal instability arises only in the case of infinite heating z_0, while for $n < 1$ instability does not occur. Therefore thermal instability is possible if

$$n = \partial \ln \sigma\tau_\varepsilon / \partial \ln T. \qquad (3.13)$$

4. ACOUSTIC INSTABILITY

In a plasma, as in a solid, an electric current can excite oscillations having a phase velocity smaller than the electron drift velocity.[6] With small radiation pressure, when the electrons are scattered mainly by ions rather than by photons, the excitation of sound oscillations, i.e., acoustic instability, is impossible, because thermal instability arises in even a weaker electric field; we shall now show this for two cases.

1. The ions are not appreciably heated and act as a heat reservoir. The energy balance equation analogous to (3.7) is $\alpha z^3 = z - 1$. Then, in complete analogy with Sec. 3, we find that steady state solutions are possible for

$$\alpha < \alpha_0 = 4/27, \qquad z < z_0 = 1.5.$$

For the maximum value we have

$$\alpha = \alpha_0 = \sigma E^2 \tau_{ei} / nT_0 = 4/27 = 1.15 (eE\tau_i)^2 M m^{-2} T_0^{-1},$$

since, according to Landshoff,[7] $\sigma = 2.62\, ne^2 \tau_i/m$ and the time for energy transfer from electrons to ions is $\tau_{ei} = \tau_i \sqrt{\pi M/4m}$, where τ_i is the momentum transfer time [Eq. (1.5) of [1]]. Utilizing the electron drift velocity $v_d = 2.62\, eE\tau_i/m$ and the sound velocity $s = (10T/3M)^{1/2}$, we obtain $(v_d/s)_{max} \approx 0.7 < 1$. Thermal instability thus actually arises when the drift velocity is below the velocity of sound.

2. The ions are heated considerably and transfer energy to neutral atoms in a time τ_0 longer than the time τ_{ei} for energy exchange between electrons and ions; we have $\beta = \tau_{ei}/\tau_0 \ll 1$. The neutral atoms are now not heated and act as a heat reservoir. The system of equations for $z = T_e/T_0$ and $x = T_i/T_0$ is

$$\alpha z^3 = x - z, \qquad z - x = \beta(x - 1).$$

In complete analogy with (3.11) of the preceding section, a steady state is possible for $\alpha_0 < 4\sqrt{\beta}/27$. Since the maximum field is reduced, acoustic instability is also not reached. Thermal instability does not arise only if the electron and ion concentration in the plasma is so low that electrons are scattered mainly by neutral atoms; it is easily seen that in (3.13) the exponent is $n = -\frac{1}{2} < 1$, and acoustic instability is trivially possible in sufficiently high fields.

Unlike the foregoing cases, in a plasma with large radiation pressure acoustic instability is possible under certain conditions. According to (3.6) we have $\alpha_0 = \sigma E^2 \tau_{ef}/nT_0 = 2/3^{3/2}$; therefore the drift velocity in the critical field producing thermal instability is

$$v_d^0 \cong 1.6 e\tau_i m^{-1} \sqrt{nT_0/\sigma\tau_{ef}}.$$

On the other hand, the velocity of sound in a plasma with large radiation pressure is given by $s^2 = 2TN/3Mn$ [8] if the sound frequency ω is smaller than the reciprocal of the time required to establish equilibrium between electrons and photons (τ_{ef}^{-1}).

At higher frequencies $\omega \gg 1/\tau_{ef}$ the velocity of sound is derived from the previous formula. In the first case we therefore have

$$\frac{v_d^0}{s} \approx 1.9 \left(\frac{M}{m}\right)^{1/2} \left(\frac{T}{mc^2}\right)^{5/4},$$

and in the second case

$$\frac{v_d^0}{s} \approx 0.86 \left[\frac{N}{n}\frac{M}{m}\left(\frac{T}{mc^2}\right)^{5/2}\right]^{1/2}.$$

It is thus seen that in a sufficiently rarefied plasma at a sufficiently high temperature the condition $v_d > s$ is fulfilled in the entire frequency interval before thermal instability arises; at a lower temperature it is fulfilled for high frequencies, at least. In the second case of Sec. 3 (slow cooling of the system) the critical electric field and maximum drift velocity are reduced by a factor of $\beta^{1/4}$, so that this factor should be inserted in the right-hand side of the equation for v_d/s.

5. BEHAVIOR OF PLASMA WITH LARGE RADIATION PRESSURE IN A HIGH MAGNETIC FIELD

In Secs. 3 and 4 we considered the case when plasma heating in an electric field is practically independent of the magnetic field. We shall now consider the opposite case of a high field, when heating results mainly from the second term in (3.2):

$$\frac{\sigma_0(T_0)E^2}{\Omega^2\tau_i^2(T_0)}\left(\frac{T_0}{T_e}\right)^{3/2}$$

Under these conditions the heat balance equation in the first case of Sec. 3 (the absence of photon heating) becomes

$$\alpha = z^{3/2}(z - 1). \qquad (5.1)$$

This equation can be solved for any degree of heating and accordingly for arbitrarily high electric fields, i.e., in this case thermal instability is absent ($n = -3/2 < 1$) and acoustic instability is possible. In this case, moreover, a third type of instability appears, associated with a descending current-voltage characteristic. Indeed, for large heating $z \gg 1$ it follows from (5.1) that $z = \alpha^{2/5} \sim E^{4/5}$; consequently the current component parallel to the electric field, $j = \sigma E$, is represented by the proportion $j \sim E^{-1/5}$.

[1] L. É. Gurevich and V. I. Vladimirov, JETP 44, 166 (1963), Soviet Phys. JETP 17, 116 (1963).

[2] L. É. Gurevich and V. I. Vladimirov, JETP 47, 300 (1964), Soviet Phys. JETP 20, 201 (1965).

[3] V. I. Vladimirov, ZhTF 35, 267 (1965), Soviet Phys. Tech. Phys. 10,

[4] S. I. Braginskiĭ, JETP 33, 459 (1957), Soviet Phys. JETP 6, 358 (1958).

[5] A. V. Gurevich, JETP 39, 1296 (1960), Soviet Phys. JETP 12, 904 (1961).

[6] Vedenov, Velikhov, and Sagdeev, UFN 73, 701 (1961), Soviet Phys. Uspekhi 4, 332 (1961).

[7] R. Landshoff, Phys. Rev. 76, 904 (1949).

[8] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (The Mechanics of Continuous Media), Gostekhizdat, 1954.

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