NONUNIFORM RESONANCE IN AN ANTIFERROMAGNETIC PLATE

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The frequencies of nonuniform resonance in a plate are calculated for antiferromagnets of various types (uniaxial with positive and negative anisotropy constants; structures permitting weak ferromagnetism). It is shown that dipole-dipole interactions are important in the extreme long-wave part of the spectrum (magnetostatic modes of oscillation), for those values of the field **H** at which reversal of the magnetic moments of the sublattices of the antiferromagnet occurs.

THE study of ferro- and antiferromagnetic resonance is connected with the excitation of high-frequency magnetic oscillations in dielectric specimens whose dimensions are appreciably smaller than the wavelength. As is well known, these oscillations can be divided into uniform and nonuniform. The nonuniform oscillations (Walker modes) depend significantly on the shape of the specimen. A qualitative idea of the structure of the spectrum of the Walker oscillations can be obtained by studying magnetic oscillations in a plate.[1]

The present note gives the results of a calculation of Walker[2] oscillations in an antiferromagnetic plate in the magnetostatic case. For the solution of this problem, it is necessary to make use of an expression for the magnetic susceptibility. The high-frequency magnetic susceptibility tensor of an antiferromagnet, at various constant magnetic fields, was calculated by Kaganov and Tsukernik;[3] these authors, however, did not take account of dipole-dipole interaction. Allowance for dipole-dipole interaction leads to a dependence of the frequency spectrum on the direction of the wave vector **k**. But for small wave vectors, $\sin\theta_k$ (θ_k is the angle between the vector **k** and the z axis) becomes indeterminate, and for resolution of this indeterminacy it is necessary to solve Maxwell's equations with the appropriate boundary conditions.

We will consider a uniaxial antiferromagnet, composed of two mirror sublattices, and an antiferromagnet with weak ferromagnetism. In the first case, the crystal symmetry axis **n**, the constant external field $\mathbf{H}_0$, and the coordinate axis z are perpendicular to the plane of the plate (Fig. 1). In the absence of an external magnetic field, the magnetic-moment vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ of the sublattices are parallel to the chosen axis; in a magnetic field, the orientation of the magnetic-moment vectors is determined by the size of the field.

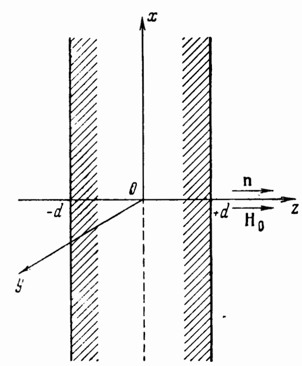

FIG. 1.

If we introduce the scalar potential of the magnetic field by means of the relation $\mathbf{h} = -\nabla\varphi$, and if we suppose that in the plane of the plate it has the form $\varphi(x, y, z) = \psi(z)e^{i\kappa\cdot\rho}$ (ρ is the radius vector in the plane of the plate, κ is the wave vector), then $\psi(z)$ is determined by the equations

$$d^2\psi / dz^2 + \varkappa_i^2\psi = 0 \quad \text{for} \quad |z| < d,$$

$$d^2\psi / dz^2 - \varkappa^2\psi = 0 \quad \text{for} \quad |z| > d, \qquad (1)$$

$$\varkappa_i^2 = -\varkappa^2[1 + 4\pi(\chi_{xx}\cos^2\varphi + \chi_{yy}\sin^2\varphi)] / (1 + 4\pi\chi_{zz}),$$

where 2d is the thickness of the plate, χ_{ik} are the components of the magnetic susceptibility tensor, and φ is the angle between the x axis and the vector κ.

In the treatment of specific cases, the magnetic susceptibility tensor will be written without allowance for spatial dispersion; this is justified

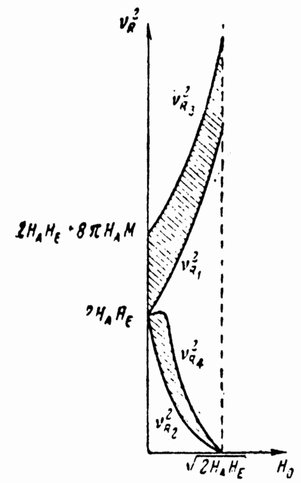

FIG. 2.

for sufficiently long waves, when the condition $c/\omega \gg a\sqrt{\Theta_C/\lambda(\mu M)}$ is satisfied. Here Θ_C is the Curie-Néel temperature, a is the lattice constant, and λ is the anisotropy constant. Furthermore, it is supposed that the dimensions of the plate are much smaller than the wavelength of the magnetostatic oscillations.

1. If $0 < H_0 < \sqrt{2H_AH_E}$, then the components of the tensor have the form[3]

$$\chi_{xx} = \chi_{yy} = H_AM \times\left\{\frac{1}{2H_AH_E-(H_0-\nu)^2}+\frac{1}{2H_AH_E-(H_0+\nu)^2}\right\};$$
$$\chi_{xy} = -\chi_{yx}; \quad \chi_{zz} = 0, \tag{2}$$

where $H_E = \alpha M$; $H_A = (\lambda + \eta)M$; α is the constant of exchange interaction of the sublattices; λ and μ are the anisotropy constants; $\nu = \omega/\gamma$; and γ is the gyromagnetic ratio. In this case, the frequencies of nonuniform resonance are determined by the expression

$$\nu_R^2 = H_0^2 + 2H_AH_E + 4\pi H_AM\frac{u^2}{u^2+v^2} \pm\left[4H_0^2\left(2H_AH_E+4\pi H_AM\frac{u^2}{u^2+v^2}\right) + 16\pi^2H_A^2M^2\left(\frac{u^2}{u^2+v^2}\right)^2\right]^{1/2}. \tag{3}$$

Here u and v are connected by the relation $\cot v = v/u$ for a symmetrical solution of the system (1), and by the relation $\tan v = -v/u$ for an antisymmetric solution ($u = \kappa d$).

For the limiting case $v \gg u$, we have two limit points:

$$\nu_{R_1}^2 \approx (H_0+\sqrt{2H_AH_E})^2; \quad \nu_{R_2}^2 \approx (H_0-\sqrt{2H_AH_E})^2. \tag{4}$$

If, on the other hand, $v \ll u$, then the limit points of the resonance frequencies lie at the frequencies

$$\nu_{R_3}^2 \approx H_0^2+2H_AH_E+4\pi H_AM +[4H_0^2(2H_AH_E+4\pi H_AM)+16\pi^2H_A^2M^2]^{1/2},$$
$$\nu_{R_4}^2 \approx H_0^2+2H_AH_E+4\pi H_AM -[4H_0^2(2H_AH_E+4\pi H_AM)+16\pi^2H_A^2M^2]^{1/2}, \tag{5}$$

All the resonance frequencies are contained in the intervals

$$\nu_{R_2}^2 < \nu_R^2 < \nu_{R_4}^2; \quad \nu_{R_1}^2 < \nu_R^2 < \nu_{R_3}^2$$

(cf. Fig. 2).

2. If $\sqrt{2H_AH_E} < H_0 < 2H_E$, then a symmetric arrangement of the magnetic moments of the sublattices with respect to the antiferromagnetic axis **n** is energetically more advantageous. If we denote the angle between the magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$ by 2θ ($\theta \approx \pi/2$), the components of the magnetic susceptibility tensor take the form[3]

$$\chi_{xx} = 4H_EM\cos^2\theta/(4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta - \nu^2),$$
$$\chi_{yy} = \frac{M}{H_E}\frac{4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta}{4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta - \nu^2};$$
$$\chi_{xy} = -\chi_{yx}, \quad \chi_{zz} = 0, \quad \cos\theta = H_0/M(2\alpha - \lambda + \eta). \tag{6}$$

The x axis lies in the plane of the magnetic moments, the y axis perpendicular to it.

The resonance frequencies in this case have the form

$$\nu_R^2 = (4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta)\times\left(1+4\pi\frac{M}{H_E}\sin^2\varphi\frac{u^2}{u^2+v^2}\right) +16\pi H_EM\cos^2\theta\cos^2\varphi\frac{u^2}{u^2+v^2}. \tag{7}$$

If $\varphi = \pi/2$, then

$$\nu_R^2 = (4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta)\times\left(1+4\pi\frac{M}{H_E}\frac{u^2}{u^2+v^2}\right). \tag{8}$$

If $\varphi = 0$, then

$$\nu_R^2 = 4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta + 16\pi H_EM\cos^2\theta\frac{u^2}{u^2+v^2}. \tag{9}$$

When $v/u \gg 1$,

$$\nu_{R_1}^2 = 4H_E^2\cos^2\theta - 2H_AH_E\sin^2\theta. \tag{10}$$

When $v/u \ll 1$,

$$\nu_{R_2}^2 \approx (H_E + 4\pi M\sin^2\varphi)(4H_E\cos^2\theta - 2H_A\sin^2\theta) + 16\pi H_EM\cos^2\theta\cos^2\varphi. \tag{11}$$

All the resonance frequencies are contained in the interval $\nu_{R_1}^2 < \nu_R^2 < \nu_{R_2}^2$ (Fig. 3).

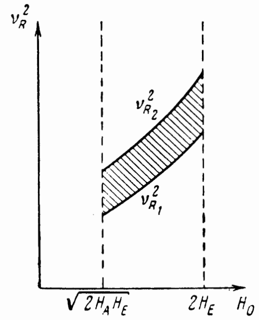

FIG. 3.

3. If the external field H_0 exceeds $2H_E$, then the antiferromagnet goes over to a ferromagnetic state. The components of the tensor χ_{ik} have the form[3]

$$\chi_{xx}=\chi_{yy}=\frac{2M(H_0+H_a)}{(H_0+H_a)^2-\nu^2};\qquad \chi_{xy}=-\chi_{yx};\quad \chi_{zz}=0,$$

where $H_a \equiv (\lambda-\eta)M$. The resonance frequencies are

$$\nu_R^2=(H_0+H_a)^2+8\pi M(H_0+H_a)\frac{u^2}{u^2+v^2}. \tag{12}$$

When $v/u \gg 1$,

$$\nu_{R_1}^2\approx (H_0+H_a)^2, \tag{13}$$

when $v/u \ll 1$,

$$\nu_{R_2}^2\approx (H_0+H_a)^2+8\pi M(H_0+H_a). \tag{14}$$

The resonance frequencies lie in the interval $\nu_{R_1}^2<\nu_R^2<\nu_{R_2}^2$ (Fig. 4).

From the formulas presented for the frequencies of nonuniform resonance, (7) to (9) and (12), it is clear that dipole-dipole interaction is important in the extreme long-wave part of the spectrum (the magnetostatic type of oscillation) at those values of the magnetic field H at which a reversal of the magnetic moments of the sublattices occurs in the antiferromagnetic crystal.

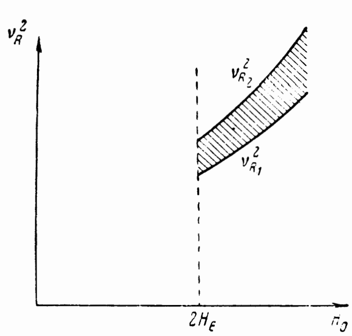

FIG. 4.

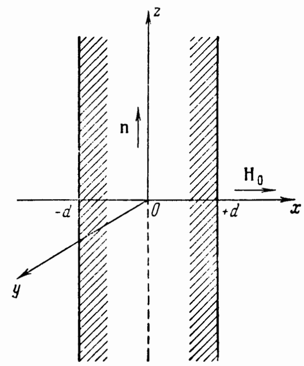

FIG. 5.

4. We consider a uniaxial antiferromagnet with weak ferromagnetism, for which, in the absence of an external field, the antiferromagnetism vector $L = M_1 - M_2$ is perpendicular to the axis of symmetry n of the crystal. The plane of the plate is parallel to the axis n, and the external field is applied perpendicular to the plane of the plate. We choose a system of coordinates such that the x axis is directed along the external field H_0 and the z axis along the axis of symmetry of the crystal (Fig. 5). The field $H_0 \ll H_E$. In this case[4]

$$\chi_{xx}=\chi_0\frac{\nu_2^2}{\nu_2^2-\nu^2};\qquad \chi_{yy}=\chi_0\frac{(H_0+H_D)^2}{\nu_1^2-\nu^2};$$
$$\chi_{zz}=\chi_0\frac{\nu_1^2}{\nu_1^2-\nu^2};\qquad \chi_{yz}=-\chi_{zy}, \tag{15}$$

where $\chi_0\equiv\chi_\perp$ is the transverse static susceptibility;

$$\nu_1=(H_0H_D+H_0^2)^{1/2},\quad \nu_2=(2H_AH_E+H_D^2+H_0H_D)^{1/2} \tag{16}$$

for structures of type $n_z^+2_d^-$ (cf. [4]), and

$$\nu_1=(4H_D^2+5H_0H_D+H_0^2)^{1/2};$$
$$\nu_2=(2H_AH_E+H_D^2+H_0H_D)^{1/2} \tag{16a}$$

for structures of type $4_z^-2_d^+$; $H_D = d_1/2M$, where d_1 is a parameter that is responsible for the noncollinearity of the mechanical moments of the sublattices; g enters in the expression $\gamma = ge/2mc$.

The frequencies of nonuniform resonance are determined by the expression

$$\nu_R^2=\frac{1}{2(1+v^2/u^2)}\Big\{\frac{v^2}{u^2}[\nu_1^2+\nu_2^2(1+4\pi\chi_0)]+\nu_1^2+\nu_2^2+l^2(\varphi)\pm\Big(\Big\{\frac{v^2}{u^2}[\nu_1^2+\nu_2^2(1+4\pi\chi_0)]+\nu_1^2+\nu_2^2+l^2(\varphi)\Big\}^2-4\Big(1+\frac{v^2}{u^2}\Big)\times\Big[\frac{v^2}{u^2}\nu_1^2\nu_2^2(1+4\pi\chi_0)+\nu_2^2(\nu_1^2+l^2(\varphi))\Big]\Big)^{1/2}\Big\}, \tag{17}$$

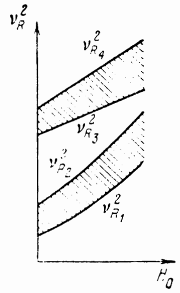

FIG. 6.

where v and u are found from the equation

$$\cot v = (1 + 4\pi\chi_{xx}) v / u$$

for the "symmetric" solution and

$$\tan v = -(1 + 4\pi\chi_{xx}) v / u$$

for the "antisymmetric" solution of the system (1);

$$l^2(\varphi) = 4\pi\chi_0 (H_0 + H_D)^2 \cos^2\varphi + \nu_1^2 \sin^2\varphi,$$

where φ is the angle between the z axis and the vector κ.

When $v/u \ll 1$,

$$\nu_{R_3}^2 \approx \nu_2^2; \quad \nu_{R_2}^2 \approx \nu_1^2 + l^2(\varphi);$$

when $v/u \gg 1$,

$$\nu_{R_1}^2 \approx \nu_1^2; \quad \nu_{R_4}^2 \approx \nu_2^2(1 + 4\pi\chi_0).$$

The resonance frequencies are contained in the intervals

$$\nu_{R_1}^2 < \nu_R^2 < \nu_{R_2}^2, \qquad \nu_{R_3}^2 < \nu_R^2 < \nu_{R_4}^2 \qquad (18)$$

(cf. Fig. 6).

If in formulas (15) to (17) we set $d_1 = 0$, we go over to the case of a uniaxial antiferromagnet with a negative anisotropy constant.

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Translated by W. F. Brown, Jr.
124