DESTRUCTION OF THE SUPERFLUIDITY OF He^3 *IN A MAGNETIC FIELD*

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It is shown that the superfluidity of liquid He^3 must be destroyed by a sufficiently strong magnetic field (of the order of 4×10^4 Oe), since the formation of Cooper pairs ceases to be energetically advantageous. In the model of Gor'kov and Galitskiĭ the destruction of superfluidity involves a first-order phase transition; in this case supercooling and superheating is possible and at any given temperature there exist, besides the thermodynamic critical field H_c, two other critical fields H_{c1}, H_{c2} ($H_{c1} < H_c < H_{c2}$) which define the limits of metastability. In the Anderson-Morel model the surface tension at a superfluid-normal interface is negative; therefore the destruction of superfluidity in a magnetic field must proceed in much the same way as in a type-II superconductor. In this case $H_{c2} < H_{c1} < H_c$, with $H_{c2} = 0$, and for $H > 0$ ($= H_{c2}$) layers of normal phase appear. At higher fields liquid He^3 goes over completely into the normal phase.

THE superfluidity of He^3 was predicted theoretically [1-4] and has recently been observed experimentally by Peshkov[5]. According to the theory, Landau's criterion for superfluidity is fulfilled in liquid He^3 at sufficiently low temperatures because the excitations of the liquid form bound Cooper pairs. Apparently these Cooper pairs have even orbital angular momentum l and therefore zero spin s.

The theory of formation of Cooper pairs with nonzero orbital angular momentum has been investigated by a number of authors [6-10]. However, the question of the symmetry of the superfluid phase of He^3 remains unresolved at present. Gor'kov and Galitskiĭ [8] assume an isotropic excitation spectrum in the superfluid phase, while according to Anderson and Morel [6,7] the energy gap must be anisotropic.

There is, of course, no Meissner effect in superfluid He^3. Nevertheless, in a strong magnetic field $H \sim \Delta/\mu$ (where Δ is the gap in the elementary excitation spectrum and μ, the magnetic moment of the He^3 atom, is 2.127 nuclear magnetons) the formation of Cooper pairs in a singlet state becomes energetically unfavorable, so that superfluidity must be destroyed.

In the Gor'kov-Galitskiĭ model destruction of superfluidity must involve a first-order phase transition. In this case superheating and supercooling is possible and at any given temperature there exist, besides the thermodynamic critical field H_c, two other critical fields H_{c1} and H_{c2} ($H_{c1} < H_c < H_{c2}$) which define the limits of metastability; in the region $H_{c1} < H < H_{c2}$ the normal phase is metastable and, conversely, in the region $H_c < H < H_{c2}$ the superfluid phase is metastable. In a field less than H_{c1} the normal phase is completely unstable against the formation of Cooper pairs; in a field greater than H_{c2} the Cooper pairs must break up, i.e., the superfluid phase is unstable.

In the Anderson-Morel model the surface tension at a superfluid-normal phase boundary is negative and $H_c > H_{c1} > H_{c2}$ with $H_{c2} = 0$. In this case the destruction of superfluidity must proceed in much the same way as in a type-II superconductor [11]: in a field $H > 0$ ($= H_{c2}$) layers of normal phase are formed, and as the field is increased further liquid He^3 goes over completely into the normal phase.

Below we shall first use the method of Gor'kov and Galitskiĭ to calculate H_c and H_{c2} (Sec. 1). The critical field H_{c1} is calculated in Sec. 2 without using the results of [6-10]. We believe our method to be free of some possible drawbacks of the methods of these references. The question of the destruction of superfluidity in the Anderson-Morel model is discussed in Sec. 3.

1. THERMODYNAMIC RELATIONS FOR THE GOR'KOV-GALITSKIĬ MODEL. DETERMINATION OF THE SUPERHEATING CRITICAL FIELD H_{c2}

To determine the thermodynamic critical field we use the fact that at the transition point the free energies of the superfluid and normal phases are equal (we neglect the compressibility of He^3); they are given by:

$$F_s = F_{s0} - \int_0^H M_s(H)\,dH, \quad F_{nh} = F_{n0} - \int_0^H M_n(H)\,dH, \tag{1}$$

where M_s and M_n are the magnetization in the superfluid and normal state respectively. Therefore the critical magnetic field H_c satisfies the equation:

$$\int_0^{H_c} [M_n(H) - M_s(H)]\,dH = F_{n0} - F_{s0}. \tag{2}$$

In the two most interesting cases a) $\mu H \ll T$ and b) $\mu H \gg T$, the magnetizations M_s and M_n are proportional to the magnetic field[1)]: $M_s = \chi_s H$, $M_n = \chi_n H$. Under these conditions the critical field is given by:

$$H_c(T) = \{2(F_{n0} - F_{s0}) / (\chi_n - \chi_s)\}^{1/2}. \tag{3}$$

The susceptibility of the normal phase is $\chi_n = \mu^2 m p_0/\pi^2$ where m is the effective mass of an excitation in He^3 and p_0 is the Fermi momentum. The susceptibility of the superfluid phase χ_s was calculated in [12,13]. For pairing in a singlet state

$$\chi_n - \chi_s = N_s(T) N^{-1} \chi_n, \tag{4}$$

where $N_s(T)/N$ is the concentration of the superfluid component. The quantities $F_{n0} - F_{s0}$ and $N_s(T)/N$ are given by the same formulae as in superconductivity theory [14].

At zero temperature we have:

$$H_c(0) = \Delta(0) / \mu\sqrt{2} = 44000\text{e}. \tag{5}$$

where we have used the fact that $\Delta(0) = 1.75\,T_c$ and substituted the value $T_c = 0.0055$°K found by Peshkov [5].

Near T_c the critical magnetic field changes with temperature according to the formula

$$H_c(T) = \frac{\Delta(T)}{\mu} = \frac{1.53 T_c}{\mu}\left(1 - \frac{T}{T_c}\right)^{1/2}. \tag{6}$$

The heat of the transition has the form

$$Q = -T[M_n(H_c) - M_s(H_c)]\,dH_c/dT. \tag{7}$$

At low temperatures

$$Q = C_{n0}(T) T, \tag{8}$$

where $C_{n0}(T) = m p_0 T/3$ is the specific heat of the normal phase; while for $T \rightarrow T_c$

$$Q = \chi_n \Delta^2(T) / 4\mu^2 = 0.82 C_{n0}(T_c) T_c (1 - T/T_c). \tag{9}$$

The specific heat of the superfluid phase is increased in the presence of the magnetic field. At low temperatures the change of specific heat with field can be pronounced:

$$C_{sh} = C_{s0}\left\{1 + \frac{1}{2}(\mu H/T)^2\right\}. \tag{10}$$

The critical fields H_{c1} and H_{c2} cannot be determined from thermodynamic considerations alone and to calculate them we must return to the microscopic theory. In this section we calculate the superheating critical field H_{c2}.

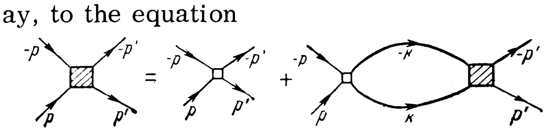
In a field $H > H_{c2}$ the superfluid phase is completely unstable against the break-up of Cooper pairs, since the energy gain due to the ordering of spins in the magnetic field, $2\mu H$, exceeds the binding energy of pairs at the temperature in question, $2\Delta(T)$, so that there appear quasiparticles with negative energy $\Delta(T) - \mu H$[2)]. Therefore the superheating critical field bears a simple relation to the energy gap $\Delta(T)$, namely

$$H_{c2}(T) = \Delta(T) / \mu. \tag{11}$$

2. SINGULARITIES OF THE VERTEX PART FOR ZERO TOTAL MOMENTUM OF THE COLLIDING PARTICLES. CALCULATION OF THE CRITICAL FIELD H_{c1}

In a field less than H_{c1} the normal phase is completely unstable against the formation of pairs. It can be shown [15] that this instability is signalled by the fact that in the thermodynamic diagram technique the vertex part $\mathcal{T}_{\alpha\beta;\gamma\delta}(p, q-p; p', q-p')$, when considered as a function of the fourth component of total 4-momentum q_4 and analytically continued from a discrete set of points on the imaginary axis into the upper half of the complex q_4 plane, has a pure imaginary pole. As we reduce the field, this instability sets in at the value of the field $H = H_{c1}$. Obviously the pole in the vertex part appears first for $q = 0$, i.e., for zero total 4-momentum. Thus, at the point $H = H_{c1}$ the thermodynamic vertex part $\mathcal{T}_{\alpha\beta;\gamma\delta}(p - p; p', -p') \equiv \mathcal{T}_{\alpha\beta;\gamma\delta}(p, p')$ tends to infinity. This property can be used to find the value of H_{c1}.

The relevant function $\mathcal{T}_{\alpha\beta;\gamma\delta}(p, p')$ can be calculated by summing the series of "ladder" graphs [15]. This summation leads, in the usual way, to the equation



(12)

1)This can be shown by a microscopic calculation of magnetic moment.

2)The energy of an elementary excitation with momentum p and spin projection σ in the magnetic field is equal to $\epsilon_p + 2\mu H\sigma$, where $\epsilon_p = \sqrt{\Delta^2 + \zeta_p^2}$ is the energy of the quasi-particle in absence of the field. This result may be obtained by Green's-function methods.

where the unshaded block denotes the bare vertex $\mathcal{T}^{(0)}_{\alpha\beta;\gamma\delta}$

$$\mathcal{T}^{(0)}_{\alpha\beta;\gamma\delta}(p,p') = V'(\mathbf{p},\mathbf{p}')(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma}) + V''(\mathbf{p},\mathbf{p}')(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}). \tag{13}$$

Here $V'(p, p')$ and $V''(p, p')$ are respectively the even and odd parts of the interaction potential $V(p, p')$, which in our model is nonzero only in a shell of width $2\tilde{\omega}$ around the Fermi surface, and within this shell depends only on the angle between the vectors p and p′; accordingly we may expand it in Legendre polynomials:

$$V(\mathbf{p},\mathbf{p}') = \sum_l (2l+1)V_l P_l(\hat{\mathbf{p}}\hat{\mathbf{p}}'), \quad \hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|. \tag{14}$$

The explicit form of the equation for the vertex part $\mathcal{T}_{\alpha\beta;\gamma\delta}$ is:

$$\mathcal{T}_{\alpha\beta;\gamma\delta}(p,p') = \mathcal{T}^{(0)}_{\alpha\beta;\gamma\delta}(p,p') - \frac{T}{2(2\pi)^3}\sum_{\omega_k}\int d\mathbf{k}\,\mathcal{T}^{(0)}_{\alpha\beta;\mu\nu}(p,k)\,\mathfrak{G}_{\mu\eta}^{(0)}(k) \times \mathfrak{G}_{\rho\nu}^{(0)}(-k)\,\mathcal{T}_{\eta\rho;\gamma\delta}(k,p'), \tag{15}$$

where

$$\mathfrak{G}^{(0)}_{\alpha\beta}(\mathbf{p},\omega_n) = \frac{1}{\omega_n-\zeta_p\mp\mu H}\delta_{\alpha\beta}, \quad \omega_n = i(2n+1)\pi T. \tag{16}$$

It should be noted that in the approximation we are using the vertex part $\mathcal{T}_{\alpha\beta;\gamma\delta}(p, p')$ does not depend on the fourth components of the momenta p and p′; this is obvious from (15) and (13).

Expanding the function $\mathcal{T}_{\alpha\beta;\gamma\delta}(p, p')$ in Legendre polynomials

$$\mathcal{T}_{\alpha\beta;\gamma\delta}(p,p') = \sum_l (2l+1)\mathcal{T}^{(l)}_{\alpha\beta;\gamma\delta}P_l(\hat{\mathbf{p}}\hat{\mathbf{p}}') \tag{17}$$

and substituting (13), (14), and (17) in (15), we obtain the following relation for the coefficients $\mathcal{T}^{(l)}_{\alpha\beta;\gamma\delta}$:

$$\mathcal{T}^{(l)}_{\alpha\beta;\gamma\delta} = V_l(\delta_{\alpha\gamma}\delta_{\beta\delta}\pm\delta_{\alpha\delta}\delta_{\beta\gamma}) + \frac{T\rho_l}{2}\sum_{\omega_k}\int d\zeta_k \times\{\mathfrak{G}^{(0)}_{\alpha\eta}(k)\mathfrak{G}^{(0)}_{\beta\rho}(-k)\pm\mathfrak{G}^{(0)}_{\beta\eta}(k)\mathfrak{G}^{(0)}_{\alpha\rho}(-k)\}\mathcal{T}^{(l)}_{\eta\rho;\gamma\delta}. \tag{18}$$

We have introduced the notation

$$\rho_l = -V_l m p_0/2\pi^2. \tag{19}$$

In Eq. (18) the minus sign must be taken for even l and the plus sign for odd l.

For even l the quantities $\mathcal{T}^{(l)}_{\alpha\beta;\gamma\delta}$ have the form

$$\mathcal{T}^{(l)}_{\alpha\beta;\gamma\delta} = \mathcal{T}^{(l)}(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma}), \tag{20}$$

where

$$\mathcal{T}^{(l)} = V_l\left\{1+\frac{T\rho_l}{2}\sum_{\omega_k}\int d\zeta_k\mathfrak{G}^{(0)}_{\alpha\alpha}(k)\,\mathfrak{G}^{(0)}_{-\alpha,-\alpha}(-k)\right\}^{-1}. \tag{21}$$

After some simple but tedious calculations which we shall not reproduce here, we get the following final expression for $\mathcal{T}^{(l)}$ for even l:

$$\mathcal{T}^{(l)} = V_l\left\{1-\frac{\rho_l}{2}\int_0^{\tilde{\omega}}\frac{d\zeta}{\zeta}\left(\operatorname{th}\frac{\zeta+\mu H}{2T}+\operatorname{th}\frac{\zeta-\mu H}{2T}\right)\right\}^{-1}. \tag{22}*$$

Thus the critical field H_{c1} can be determined from the equation

$$1 = \frac{\rho_l}{2}\int_0^{\omega}\frac{d\zeta}{\zeta}\left[\operatorname{th}\frac{\zeta+\mu H_{c1}}{2T}+\operatorname{th}\frac{\zeta-\mu H_{c1}}{2T}\right]. \tag{23}$$

For T = 0 we must make the substitution in the integrand

$$\operatorname{th}\frac{\zeta\pm\mu H_{c1}}{2T}\to\operatorname{sign}(\zeta\pm\mu H_{\text{к}1}),$$

after which the integral is easily calculated. As a result the relation (23) takes the simple form

$$1 = \rho_l\ln\frac{\tilde{\omega}}{\mu H_{c1}} \tag{24}$$

and the critical field H_{c1} at T = 0 is given by

$$H_{c1}(0) = \Delta(0)/2\mu = 31000\,\text{Oe}. \tag{25}$$

where $\Delta(0)$ is the energy gap calculated by the method of Gor'kov and Galitskii [9]: $\Delta(0) = 2\tilde{\omega}\exp(-1/\rho_l)$.

Comparing formulae (5), (11) and (25), we see that in the Gor'kov-Galitskii model $H_{c1}(0) = H_c(0)/\sqrt{2}$ and $H_{c2}(0) = \sqrt{2}\,H_c(0)$, i.e., $H_{c1}(0) < H_c(0) < H_{c2}(0)$. These inequalities actually hold for all temperature regions. Thus in the Gor'kov-Galitskiĭ model H_{c1} is the super-cooling critical field.

To determine the function $H_{c1}(T)$ near the critical temperature T_c, we must expand the right-hand side of Eq. (22) in powers of μH_{c1} and $T_c - T$. After some simple algebra, which we omit, we obtain the relation

$$(\mu H_{c1})^2 = 8T_c^2\left(1-\frac{T}{T_c}\right)\Bigg/\int_0^{\infty}\frac{\operatorname{th}^2 x}{x^2}dx. \tag{26}$$

Using the relation found in superconductivity theory [14] for the temperature dependence of the energy gap $\Delta(T)$, we can prove that for $T \to T_c$

$$H_{c1}(T)\to\Delta(T)/\sqrt{2}\mu\to\frac{1}{\sqrt{2}}H_c(T). \tag{27}$$

In the intermediate temperature region $H_{c1}(T)$ cannot be expressed so simply in terms of $\Delta(T)$.

*th = tanh.

It should be noted that in calculating H_{c1} we have not made use of the method of Gor'kov and Galitskiĭ [8]. Moreover, the above results do not depend on the use of the widely used but very artificial model involving the "reduced" BCS Hamiltonian, for which the Anderson-Morel solutions are asymptotically exact [9].

3. DESTRUCTION OF SUPERFLUIDITY IN THE ANDERSON-MOREL MODEL

In the Anderson-Morel model the energy gap is anisotropic and satisfies the equation

$$\Delta(\hat{\mathbf{p}}) = -\frac{\pi}{(2\pi)^4}\int d\mathbf{p}' V(\mathbf{p}, \hat{\mathbf{p}}')\frac{\Delta(\hat{\mathbf{p}}_1)}{\varepsilon(\hat{\mathbf{p}}')}\operatorname{th}\frac{\varepsilon(\hat{\mathbf{p}}')}{2T}. \quad (28)$$

In the general case the solution of this equation has the form

$$\Delta(\mathbf{p}) = \sum_{l,m}\Delta_{lm}Y_{lm}(\hat{\mathbf{p}}), \quad (29)$$

where the coefficients Δ_{lm} are connected by the relations

$$\Delta_{lm} = \rho_l \sum_{l',m'}\Delta_{l'm'}\int_0^{\tilde{\omega}} d\zeta \int d\hat{\mathbf{p}} \times \frac{Y_{l'm'}(\hat{\mathbf{p}})Y_{lm}(\hat{\mathbf{p}})}{(\zeta^2+|\Delta(\hat{\mathbf{p}})|^2)^{1/2}}\operatorname{th}\frac{(\zeta^2+|\Delta(\hat{\mathbf{p}})|^2)^{1/2}}{2T}. \quad (30)$$

At $T = 0$

$$F_{n0} - F_{s0} = \frac{mp_0}{16\pi^3}\sum_{l,m}|\Delta_{lm}|^2. \quad (31)$$

Anderson and Morel have shown [7] that of the solutions of the form

$$\Delta(\hat{\mathbf{p}}) = \sum_m \Delta_m Y_{2m}(\hat{\mathbf{p}}), \quad (32)$$

the one with the lowest value of F_{s0} at $T = 0$ is:

$$\Delta(\hat{\mathbf{p}}) = \Gamma\Delta(0)\left[2^{-1/2}Y_{20} + \frac{1}{2}(Y_{22} - Y_{2,-2})\right]; \quad (33)$$

$$\ln\Gamma = 1.154$$

(where $\Delta(0)$ is the gap calculated by the Gor'kov-Galitskiĭ method). In (29) the coefficients Δ_{lm} with $l \neq 2$ are small compared to Δ_{2m} in view of the fact that the coupling constant $\rho_2 \ll 1$ [16]. This fact allows us to calculate the value of the thermodynamic critical field for the Anderson-Morel model from (3):

$$H_c^{AM}(0) = 0.89H_c(0) = 39000\,\text{Oe}, \quad (34)$$

where $H_c(0)$ is the critical field for the Gor'kov-Galitskiĭ model [cf. formula (5)]. Thus, $H_c^{AM}(0) > H_{c1}(0)$ [cf. formula (25)]. On the other hand, the critical field H_{c2} in the Anderson-Morel model is given by

$$H_{c2}^{AM}(T) = |\Delta(T)|_{min}/\mu, \quad (35)$$

where $|\Delta|_{min}$ is the minimum value of the energy gap on the Fermi surface. For the solution (33) the minimum value $|\Delta|_{min} = 0$.

This value is attained at the points of intersection of the lines $\cos\theta = \pm 1/\sqrt{3}$, , $\sin 2\varphi = 0$, on which the real and imaginary parts respectively of $\Delta(\hat{\mathbf{p}})$ vanish. Obviously, taking harmonics with $l \neq 2$ into account will only lead to an unimportant shift in these lines, while the minimum value of $|\Delta|$ remains zero. This result remains valid at finite temperature. Thus,

$$H_{c2}^{AM} = 0. \quad (36)$$

Accordingly we have for the Anderson-Morel model the inequalities $H_{c2}^{AM} < H_{c1} < H_c^{AM}$. In this case the pattern of destruction of superfluidity is quite different from that considered in the previous sections. In fact, in the interval $0 < H < H_{c1}$ the pure normal and pure superfluid states are both unstable and for any finite field $H > 0$ ($= H_{c2}^{AM}$) layers of normal phase appear, i.e , He^3 goes over into a mixed state. Accordingly, the surface tension at a superfluid-normal boundary must be negative. A similar situation occurs in superconducting alloys (type-II superconductors)[11]. The final destruction of superfluidity obviously involves a first-order phase transition. The calculation of the corresponding critical field H_{c3}^{AM} is extremely complicated, since it requires a knowledge of the thermodynamic functions of the mixed state. We can only state that $H_{c3}^{AM} \geq H_c^{AM}$ since for $H < H_c^{AM}$ the inequality $F_{nh} > F_{sh}$ is satisfied. On the other hand, obviously H_{c3}^{AM} has order of magnitude Δ/μ. In the interval $H_{c1} < H < H_{c3}$ the normal phase can exist in a metastable (supercooled) state.

Thus we may expect that the destruction of superfluidity of He^3 at $T = 0$ will require a field of order 4×10^4 Oe. Observation of this phenomenon should constitute convincing confirmation of current ideas about the nature of superfluid Fermi systems.

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