

*ELECTROMAGNETIC SPECTRUM OF FERROMAGNETIC METALS IN A STRONG ELECTRIC FIELD AND ITS EXCITATION*

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We consider the spectrum of electromagnetic oscillations of a ferromagnetic metal in the presence of a stationary external electric field. We show that a new oscillation branch, whose frequency depends appreciably on the electric field at small values of the wave vector, appears along with the ordinary spin oscillations. Instability of the new oscillations arises if the electric field strength exceeds a certain critical value. This is manifest in a change from damping to build-up of oscillations with a wave vector almost parallel to the magnetization.

A new branch of the electromagnetic spectrum, different from the spectrum of the equilibrium crystal, is produced in a ferromagnetic metal in the presence of an electric field. The real part of the frequency in this branch differs from the ordinary spin-wave frequency only when  $ck \ll \sqrt{4\pi\sigma\omega_0}$ , where  $\sigma$  is the electric conductivity,  $k$  the wave vector,  $c$  the speed of light,  $\omega_0 = 4\pi gM_0$ ,  $g$  the gyromagnetic ratio, and  $M_0$  the magnetization. When the electric field exceeds a certain critical value, these oscillations with wave vector  $k$  almost parallel to  $M_0$  become unstable and start to grow.

The system of equations leading to these phenomena consists of the equation of the current in a weak magnetic field ( $\eta \equiv (\sigma'_B B_0 + \sigma'_M M_0) / \sigma \ll 1$ ):

$$\mathbf{J} = \sigma\mathbf{E} + \sigma'_B[\mathbf{E}\mathbf{B}] + \sigma'_M[\mathbf{E}\mathbf{M}], \tag{1}^*$$

the equations of motion of the magnetic moment

$$\frac{d\mathbf{M}}{dt} = g[\mathbf{M}\mathbf{H}^{(e)}] - \frac{\lambda}{M^2}[\mathbf{M}[\mathbf{M}\mathbf{H}^{(e)}]], \tag{2}$$

Maxwell's equations †

$$\begin{aligned} \text{rot } \mathbf{H} &= \frac{4\pi}{c}\mathbf{J}, & \text{rot } \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ \text{div } \mathbf{B} &= 0, & \text{div } \mathbf{E} &= 4\pi en \end{aligned} \tag{3}$$

and the continuity equations

$$\frac{\partial n}{\partial t} + \text{div } \mathbf{J} = 0. \tag{4}$$

Here  $\mathbf{H}^{(e)}$  is the effective magnetic field,<sup>[1]</sup>  $\mathbf{B}$  the magnetic induction,  $\mathbf{E}$  the electric field,  $n$  the electron concentration, and  $\sigma'_B$  and  $\sigma'_M$  the normal and

anomalous Hall conductivities, respectively. The subscript zero denotes the constant parts of the corresponding quantities.

Linearizing (1)–(4), using the smallness of the parameter  $\omega/\sigma$ , and assuming that the vectors  $\mathbf{M}_0$ ,  $\mathbf{B}_0$ , and  $\mathbf{k}$  are parallel, we obtain dispersion equations for the waves with right-hand and left-hand polarizations:

$$\begin{aligned} \xi(\omega - \mathbf{v}_B\mathbf{k})[\omega \mp (\Omega_h + \omega_0)(1 \mp i4\pi\lambda/\omega_0)] \\ \pm \xi\omega_0(1 \mp i4\pi\lambda/\omega_0)[\mathbf{v}_M\mathbf{k} \pm i\eta\omega] \\ + i[\omega \mp \Omega_h(1 \mp i4\pi\lambda/\omega_0)](\omega_0 - \xi\eta\omega) = 0. \end{aligned} \tag{5}$$

Here

$$\begin{aligned} \xi &= 4\pi\sigma\omega_0/c^2k^2, & \mathbf{v}_B &= \sigma'_B c\mathbf{E}_0/\sigma, \\ \mathbf{v}_M &= \sigma'_M c\mathbf{E}_0/4\pi\sigma, & \Omega_h &= Ia^2k^2/\hbar + gH_0, \end{aligned} \tag{6}$$

$I$  is a quantity of the order of the Curie temperature,  $a$  the lattice constant,  $H_0$  the magnetic field, including the external magnetic field, the anisotropy field, and the demagnetizing field. These equations lead to two branches of the spectrum, which, under the conditions

$$(\mathbf{v}_B\mathbf{k})/\omega_0, (\mathbf{v}_M\mathbf{k})/\omega_0, \lambda/\omega_0, \Omega_h/\omega_0 \ll 1$$

take the form

$$\omega_1 = \pm\omega_0(1 + \eta/\xi) - i(4\pi\lambda + \omega_0/\xi), \tag{7}$$

$$\omega_2 = \frac{[\xi(\mathbf{v}\mathbf{k}) - i\Omega_h](1 \mp i4\pi\lambda/\omega_0)}{\xi(1 \mp i4\pi\lambda/\omega_0)(1 \mp i\eta) \mp i}; \tag{8}$$

$$\mathbf{v} = \mathbf{v}_B + \mathbf{v}_M.$$

When  $\xi \ll 1$ , that is, when  $k$  is large, practically no oscillations are excited in the first branch, which is strongly attenuated. When  $\xi \gg 1$ , the attenuation is weak.<sup>[1]</sup>

\* $[\mathbf{E}\mathbf{B}] = \mathbf{E} \times \mathbf{B}$   
†rot = curl

The oscillations of the second branch are weakly attenuated and are therefore excited at both large and small values of the parameter  $\xi$ .

The expression for  $\omega_2$  simplifies in the limiting cases  $\xi \ll 1$  and  $\xi \gg 1$ .

In the former case

$$\omega_2 = \pm \Omega_h - i\xi[\Omega_h \mp (\mathbf{vk})] - i4\pi\lambda\Omega_h/\omega_0. \quad (9)$$

Here, as in (11), we have discarded terms containing products of small parameters, for example,  $\lambda\eta/\omega_0$ .

If

$$|\mathbf{vk}| > \Omega_h(1 + 4\pi\lambda/\xi\omega_0), \quad (10)$$

then the damping gives way to growth, but this condition can be realized only in electric fields that are difficult to attain.

In the latter case

$$\omega_2 = (\mathbf{vk}) - \frac{i}{\xi}[\Omega_h \mp (\mathbf{vk})] \pm i\eta(\mathbf{vk}). \quad (11)$$

Oscillations of this type attenuate weakly when  $v \gg \Omega_k/\xi k |\cos \vartheta|$ , and start to grow if

$$v > v_{cr} = \frac{k\Omega_h}{(k^2 + 4\pi\sigma\omega_0\eta/c^2)|\cos \vartheta|}, \quad (12)$$

where  $\vartheta$  is the angle between  $\mathbf{k}$  and  $\mathbf{E}_0$ .

The velocity  $v_{cr}$  is minimal when  $\vartheta = 0$  and vanishes when  $k = 0$ .

If  $gH_0 < 36\pi\sigma\omega_0 I a^2 \eta / \hbar c^2$ , then  $v_{cr}$  increases monotonically with  $k$ , and for the inverse inequality the velocity is a nonmonotonic function of  $k$  and has in the case when  $gH_0 \gg 36\pi\sigma\omega_0 I a^2 \eta / \hbar c^2$  a minimum at

$$k \approx \sqrt{\hbar g H_0 / I a^2}. \quad (13)$$

In this case

$$E_{cr} = \frac{8\pi\sigma}{(4\pi\sigma_B' + \sigma_M')c} \sqrt{\frac{I a^2 g H_0}{\hbar}}. \quad (14)$$

To decrease the field  $E_{cr}$  at which instability sets in, it is necessary to choose a material in which the parameter  $H_0 = H_a + H_d + H$  is minimal ( $H_a$ ,  $H_d$  and  $H$  are respectively the anisotropic field, the demagnetizing field, and the external field).

The parameter  $H_0$  can be decreased by various means. First, the anisotropy field fluctuates in different materials over a wide range and may even reverse sign at some temperature and composition of the material<sup>[2]</sup>. Second, the parameter  $H_0$  can be reduced by choosing the sign of the magnetic field opposite to the sign of the anisotropy field<sup>[3]</sup>. From the expression (14) for the critical field it follows that the material must have a maximum ratio  $\sigma'/\sigma$ . There are several alloys in which the anisotropy field is close to zero and at the same

time the anomalous Hall conductivity is relatively large. Thus, for example, for sandust (83–85% Fe, 12–9% Si, and 3–8% Al), the anisotropy field is close to zero. The anisotropy constant is close to zero<sup>[4]</sup>, while the anomalous Hall constant  $R_M = \sigma'_M/\sigma^2 \approx 6 \times 10^{-9}$  V-cm/A-G. Putting  $\sigma \approx 10^4 \Omega^{-1} \text{ cm}^{-1}$  and  $gH_0 \approx 10^6$  sec, we obtain  $E_{cr} \approx 1$  V/cm. Near the point where the anisotropy constant reverses sign,  $gH_0$  can be assumed to be even smaller. A relatively large anomalous Hall constant and a near-zero anisotropy is possessed also by the alloy of the permalloy class  $\sim 80\%$  Ni, 17% Fe, 3% Mo<sup>[5,6]</sup>.

We have considered waves with wave vector  $\mathbf{k}$  parallel to the magnetization  $\mathbf{M}_0$ . In the case when  $\mathbf{k} \parallel \mathbf{E}_0 \perp \mathbf{M}_0$ ,  $\mathbf{B}_0$  the dispersion equation takes the form

$$\begin{aligned} & [i\xi(\omega - v_B k) - \omega_0] \{i\xi[(\omega - v_B k)(\omega^2 - \Omega_1^2) + v_M k \Omega_1 \omega_0] \\ & - \omega_0(\omega^2 - \Omega_1^2) + i(4\pi\lambda\omega/\omega_0)[2i\xi\Omega_1(\omega - v_B k) \\ & - i\xi v_M k \omega_0 - \omega_0(2\Omega_h + \omega_0)]\} = 0. \end{aligned} \quad (15)$$

Here  $\Omega_1 = \Omega_k + \omega_0$ .

When  $\xi \ll 1$ , this equation leads to the well-known spin-wave spectrum<sup>[1]</sup>, which is hardly affected by the electric field, and to two strongly attenuated branches of oscillations.

When  $\xi \gg 1$  we obtain high-frequency oscillations  $\pm \Omega_1$ <sup>[1]</sup> and two low-frequency oscillations: strongly attenuating

$$\omega_1 = v_B k - i\omega_0/\xi \quad (16)$$

and weakly attenuating

$$\omega_2 = v k - i\Omega_h/\xi. \quad (17)$$

In the last expression we have discarded terms of the order of  $\xi v k/\omega_0$ .

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<sup>1</sup>Akhiezer, Bar'yakhtar, and Peletminskii, JETP **35**, 228 (1958), Soviet Phys. JETP **8**, 157 (1959).

<sup>2</sup>R. M. Bozorth, Ferromagnetism, Van Nostrand, 1951, Ch. 12.

<sup>3</sup>G. I. Rado and J. R. Weertman, J. Phys. Chem. Sol. **11**, 315 (1959).

<sup>4</sup>J. L. Snoek, New Developments in Ferromagnetic Materials, Elsevier, N. Y., 1946.

<sup>5</sup>J. M. Lavine, Phys. Rev. **123**, 1273 (1961).

<sup>6</sup>I. M. Puzeĭ, Izv. AN SSSR ser. fiz. **16**, 549 (1962).