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SOLUTION OF THE SUPERCONDUCTIVITY EQUATIONS FOR A SYSTEM OF SUPERCONDUCTING AND NORMAL METALS

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The superconductivity equations are solved for a system of superconducting and normal metals. The samples are assumed to be semi-infinite, and the temperature to be close to the superconducting transition temperature. The asymptotic solutions are found at distances from the boundary much greater than the dimensions of the Cooper pair. These asymptotic solutions make it possible to obtain the effective boundary conditions for the Ginzburg-Landau equations.

THE problem of the boundary conditions in modern superconductivity theory has remained an open one to the present time. As was shown by Gor'kov,[1] the superconductivity equations reduce to the Ginzburg-Landau equation at a temperature close to the critical temperature.[2] It follows from the Ginzburg-Landau theory that the wave function of the pair on the boundary between a superconductor and vacuum (or a nonmetal) should satisfy the condition $(\mathbf{n}\cdot\nabla\psi)=0$, where $\mathbf{n}$ is the normal to the surface. This same boundary condition can also be obtained from microscopic theory.[3] The effective boundary conditions at the interface between normal and superconducting metals are obtained in the present work.

We consider the case in which the metals have slightly different chemical potentials and effective electron masses. It can be shown that account of the difference in chemical potentials leads to a change in the boundary conditions by a quantity of the order of $(\Delta\mu/\mu)^2$, which is small ($\lesssim 0.08$) in all experiments carried out to date.[4] As was shown by Falk,[5] account of the difference in effective masses can change the boundary conditions; however, we shall neglect this difference, since the model of weakly bound electrons is used throughout. It can also be assumed that the Debye temperatures $\widetilde{\omega}$ are the same for both metals, since it is shown in what follows that the boundary conditions depend logarithmically on $\widetilde{\omega}$.

Let the superconducting and normal metals fill the half-spaces $z>0$ and $z<0$, respectively. The metals differ in the effective interaction between the electrons. To be precise, inside the superconductor, $\lambda=g_1$ ($g_1<0$), while inside the normal metal, $\lambda=g_2$ ($g_2\geq 0$). We now estimate the dimensions of the region inside which the interaction changes from g_1 to g_2. The electron-phonon interaction is attractive at distances of the order of interatomic distances.[6] The Debye radius of screened Coulomb interaction is $\sim(p_0me^2)^{1/2}\sim 5\times 10^{-8}$ cm. Thus, at distances of the order of 10^{-7}—10^{-8} cm, the interaction between the electrons is attractive in superconductors and repulsive in normal metals (at $T=0$). The change in the value of λ takes place over these same distances. The surface inhomogeneities are of the same order of magnitude. Inasmuch as we shall be interested in quantities which change over much larger distances, it can be assumed that the interaction changes by jumps, while the interface is plane.

A model with four-fermion interaction is used throughout; the interaction is assumed to extend over distances of the order of $v_0/\widetilde{\omega}\sim\xi_0T_c/\widetilde{\omega}\sim 10^{-6}$ cm ($\xi_0=v_0/2\pi T_c$ is the characteristic correlation parameter and is approximately 10^{-4} cm).[6] This is connected with the effective retardation of the electron-phonon interaction. Therefore the behavior of the solutions at distances from the surface less than this value (but larger than the interatomic distance) can depend on the choice of model and indeed has a somewhat different form. For large distances, we always obtain a solution which is independent of the properties of the model.

For our system, the Gor'kov equations[1] have the form

$$[i\omega+\Delta/2m+\mu]G_\omega(\mathbf{r},\mathbf{r}')-\lambda(z)F(\mathbf{r},\mathbf{r},0)F_\omega^+(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),$$

$$[-i\omega+\Delta/2m+\mu]F_\omega^+(\mathbf{r},\mathbf{r}')+\lambda(z)F^+(\mathbf{r},\mathbf{r},0)G_\omega(\mathbf{r},\mathbf{r}')=0,$$

$$F^+(\mathbf{r},\mathbf{r},0)=F^*(\mathbf{r},\mathbf{r},0)=T\sum_\omega F_\omega^*(\mathbf{r},\mathbf{r}),$$

$$\omega=\pi T(2n+1),\qquad \lambda(z)=\begin{cases}g_1 & \text{for}\quad z>0\\ g_2 & \text{for}\quad z<0\end{cases}. \tag{1}$$

It follows from consideration of homogeneity that $F^+(\mathbf{r}, \mathbf{r}, 0) = \psi(z)$. For a temperature close to the transition temperature, one can use perturbation theory and obtain the following integral equation:[1]

$$\psi(z) = -T \sum_{\omega} \int_{-\infty}^{+\infty} G_\omega^{(0)}(\mathbf{r}, \mathbf{l}) \lambda(l_3) \psi(l_3) G_{-\omega}^{(0)}(\mathbf{l}, \mathbf{r}) d\mathbf{l}$$

$$+ T \sum_{\omega} \int_{-\infty}^{+\infty} G_\omega^{(0)}(\mathbf{l}, \mathbf{m}) \lambda(m_3) \psi^*(m_3) G_\omega^{(0)}(\mathbf{m}, \mathbf{r}) \lambda(n_3) \psi(n_3)$$

$$\times G_{-\omega}^{(0)}(n, \mathbf{l}) \lambda(l_3) \psi(l_3) G_{-\omega}^{(0)}(\mathbf{l}, \mathbf{r}) d\mathbf{l}\, d\mathbf{n}\, d\mathbf{m}, \qquad (2)$$

where $G_\omega^{(0)}(\mathbf{r}, \mathbf{r}')$ is the Green's function for free electrons for our system, and satisfies the equation

$$[i\omega + \Delta/2m + \mu] G_\omega^{(0)}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'). \qquad (3)$$

We set $\xi_0 = v_0/2\pi T$. It is not difficult to see that equation (2) can be transformed into the Ginzburg-Landau equation when $z \gg \xi_0$:[1,2,6]

$$\frac{1}{2m}\frac{d^2\psi}{dz^2} + \frac{1}{\eta}\left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8(\pi T_c)^2} g_1^2 |\psi|^2\right]\psi = 0, \qquad (4)$$

where $\eta = 7\zeta(3)\mu/12(\pi T_c)^2$; $\zeta(n)$ is the Riemann zeta function.

The ψ function must decay inside the normal metal over distances of the order of ξ_0 from the boundary.

$$\psi \sim De^{z/\xi_0} \quad \text{for} \quad z < 0, \quad |z| \ll \xi_0. \qquad (5)$$

The ψ function must tend toward a constant inside the superconductor; as follows from Eq. (4),

$$\psi \sim \frac{2\pi T_c}{|g_1|}\left[\frac{2}{7\zeta(3)}\frac{T_c - T}{T_c}\right]^{1/2} = \psi_\infty$$

for

$$z \gg \xi_0, \quad z \to +\infty. \qquad (6)$$

It then follows that close to the boundary the ψ function may turn out to be smaller than its limiting value ψ_∞. Assume that there exists a certain region near the boundary where $|\psi/\psi_\infty| \ll 1$, i.e., $|\psi/\psi_\infty|^3 \ll |\psi/\psi_\infty|$. In this region, the nonlinear part of the integral equation (2) will be much less than its linear part.

We now consider the linear part of Eq. (2):

$$\psi(z) = -T \sum_{\omega} \int_{-\infty}^{+\infty} G_\omega^{(0)}(\mathbf{r}, \mathbf{l}) \lambda(l_3) \psi(l_3) G_{-\omega}^{(0)}(\mathbf{l}, \mathbf{r}) d\mathbf{l}. \qquad (7)$$

The solution of Eq. (7) is given in Appendix A. For the region $\xi \ll z \ll \xi_0/\alpha$, the function ψ can be represented in the form

$$\psi(z) = C(\beta + z);$$

$$\alpha^2 = \frac{12}{7\zeta(3)}\frac{T_c - T}{T_c},$$

$$\beta \approx \xi_0\left[0.7 - \frac{0.5}{2\pi^2/g_2 m p_0 + \ln(2\gamma\tilde{\omega}/\pi T_c)}\right], \qquad (8)$$

C is an arbitrary constant. For $z < 0$ and $|Z| \ll \xi_0$ we have

$$\psi(z) = Ce^{z/\xi_0}/\varphi(z), \qquad (9)$$

where $\varphi(z)$ is a slowly increasing function as $z \to -\infty$. As will be seen from the following, $C \sim \alpha\psi_\infty$; therefore $|\psi/\psi_\infty| \ll 1$ for all $z \ll \xi_0/\alpha$ since $(\beta + z) \ll \xi_0/\alpha$ for these distances. On the other hand, $\xi_0/\alpha \gg \xi_0$, so that in a region where the asymptotic form (8) is applicable, the Ginzburg-Landau equation (4) is also applicable.

The solution of Eq. (4) is given in Appendix B. It has the form

$$\psi(z) = \psi_\infty \,\mathrm{th}\,(\alpha z/\sqrt{2}\xi_0 + C_1), \qquad (10)^*$$

where C_1 is an arbitrary constant. In the region $\xi_0 \ll z \ll \xi_0/\alpha$, the quantity $\alpha z/\xi \ll 1$, so that we can write

$$\psi = \psi_\infty \,\mathrm{th}\, C_1 + \psi_\infty \alpha z/\sqrt{2}\xi_0 \,\mathrm{ch}^2 C_1. \qquad (11)^\dagger$$

As is seen from what follows, $C_1 \sim \alpha \ll 1$, so that Eq. (11) can be simplified:

$$\psi \cong \psi_\infty C_1 + \psi_\infty \alpha z/\sqrt{2}\xi_0. \qquad (12)$$

Equating the expansions (8) and (12), we find the arbitrary constants:

$$C_1 = \alpha\beta/\sqrt{2}\xi_0, \qquad C = \alpha\psi_\infty/\sqrt{2}\xi_0. \qquad (13)$$

Since $C_1 \sim \alpha$ and $C \sim \alpha\psi_\infty$, the assumptions that have been made are confirmed.

Thus, the solution of Eq. (2) with $z \gg \xi_0$ has the form

$$\psi(z) = \psi_\infty \,\mathrm{th}\,(\alpha z/\sqrt{2}\xi_0 + \alpha\beta/\sqrt{2}\xi_0), \qquad (14)$$

while for $z < 0$ and $|z| \gg \xi$ we have

$$\psi(z) = \alpha\psi_\infty e^{z/\xi_0}/\sqrt{2}\xi_0\varphi(z). \qquad (15)$$

The value of β is determined in (8), while the function $\varphi(z)$ is found in (A.17). The wave function $\psi(z)$ can be obtained from (10) if we apply the following effective boundary condition to it for $z = 0$:

$$\beta d\psi(0)/dz = \psi(0). \qquad (16)$$

Equation (16) permits the natural generalization

$$-\beta(\mathbf{n}\nabla\psi) = \psi, \qquad (17)$$

where **n** is the normal vector outward drawn rela-

*th = tanh
†ch = cosh

tive to the superconductor, while the values of the functions ψ and $\nabla\psi$ are taken on the interface between the superconductor and the normal metal.

It is necessary to remark that this boundary condition is "effective" in the sense that it gives the correct solution for $z \gg \xi_0$. When $z = 0$, the function ψ does in reality not satisfy any such simple conditions.

In conclusion, I express my gratitude to Prof. B. T. Geĭlikman for suggesting the theme and for his constant interest in the research, and also to A. I. Larkin who provided essential help in setting up the problem.

APPENDIX A

Let us solve the integral equation (7):

$$\psi(z) = -g_1T\sum_{\omega}\int_1 G_\omega^{(0)}(\mathbf{r},\mathbf{l})\psi(l_3)G_{-\omega}^{(0)}(\mathbf{l},\mathbf{r})\,d\mathbf{l} - g_2T\sum_{\omega}\int_2 G_\omega^{(0)}(\mathbf{r},\mathbf{l})\psi(l_3)G_{-\omega}^{(0)}(\mathbf{l},\mathbf{r})\,d\mathbf{l}, \quad (A.1)$$

where $G_\omega^{(0)}(\mathbf{r},\mathbf{r}')$ is the Green's function for free electrons in the metal; integration in the first term of the right hand side is carried out over the volume of the superconductor, and in the second term, over the volume of the normal metal. The kernel of the integral equation (A.1) depends on the difference in coordinates and, because of this difference, is damped over distances of the order of ξ_0.[1,6] Therefore, for $z < 0$ and $|z| \gg \xi_0$, we have

$$\psi \sim De^{z/\xi_0}. \quad (A.2)$$

On the other hand, for $z \gg \xi_0$, the solution should slowly approach the constant value it takes on for the bulk superconductor:

$$|\psi| \sim C \text{ for } z \gg \xi_0. \quad (A.3)$$

We shall seek $\psi(z)$ in the form

$$\psi(z) = \frac{1}{2\pi}\int_{-\infty+i\delta}^{+\infty+i\delta} e^{-ikz}f(k)\,dk; \qquad f(k) = f^+(k) + f^-(k),$$

$$f^+(k) = \int_0^\infty e^{ikz}\psi(z)\,dz, \qquad f^-(k) = \int_{-\infty}^0 e^{ikz}\psi(z)\,dz. \quad (A.4)$$

It follows from (A.3) that the function $f^+(k)$ is analytic for all $\operatorname{Im} k > 0$, while it follows from (A.2) that the function $f^-(k)$ is analytic for all $\operatorname{Im} k < 1/\xi_0$. Transforming to Fourier components in (A.1), we get the equation

$$f^+(k) + f^-(k) = |g_1|K(k)f^+(k) - g_2K(k)f^-(k), \quad (A.5)$$

$$K(k) = \frac{T}{(2\pi)^3}\sum_\omega\int_{-\infty}^{+\infty} G_\omega^{(0)}(\mathbf{p})G_{-\omega}^{(0)}(\mathbf{k}-\mathbf{p})\,d\mathbf{p}. \quad (A.6)$$

In the calculation of the integral (A.6), it is necessary to take into consideration the "smearing out" of the interaction by an amount of the order of $\xi_0T/\tilde{\omega} \sim 10^{-6}$ cm; therefore, the Green's function $G_\omega^{(0)}(p)$ should be multiplied by

$$\theta_p = \begin{cases} 1 \text{ for } |p^2/2m - \mu| < \tilde{\omega} \\ 0 \text{ for } |p^2/2m - \mu| > \tilde{\omega} \end{cases}. \quad (A.7)$$

We are interested in the behavior of $\psi(z)$ at distances $z \gg \xi_0T/\tilde{\omega} \sim 10^{-6}$ cm, so that it suffices to compute $K(k)$ for $k \ll \tilde{\omega}/T\xi_0$. As $k \to \infty$, the kernel $K(k) \to 0$, but the manner of its vanishing can depend on the choice of model.

After summation over all frequencies and integration with account of the cut (A.7) for $k \ll \tilde{\omega}/T\xi_0 = 2\pi m\tilde{\omega}/p_0$[7], we get

$$K(k) = \frac{mp_0}{2\pi^2}\left\{\ln\frac{\tilde{\omega}}{2\pi T} + \frac{2\pi i m}{p_0k}\ln\left[\frac{\Gamma(1/2 + ip_0k/4\pi mT)}{\Gamma(1/2 - ip_0k/4\pi mT)}\right]\right\}, \quad (A.8)$$

whence it is seen that $K(k)$ is an analytic function at the pole

$$|\operatorname{Im} k| < 2\pi mT/p_0 = 1/\xi_0;$$

for $k = i(2n+1)/\xi_0$, $n = 0, \pm1, \pm2, \ldots$, it has branch points of the logarithmic type.

We introduce the notation: $1 - |g_1|K(k) = R_1(k)$, $1 + g_2K(k) = R_2(k)$. Expanding $K(k)$ in powers of k, it is easy to ascertain that the function $R_1(k)$ has two symmetrically placed zeros on the real axis for $k = \pm\alpha/\xi_0$, where

$$\alpha^2 = \frac{12}{7\zeta(3)}\frac{T_c - T}{T_c}, \quad (A.9)$$

T_c is the transition temperature.

The function $R_2(k)$ does not have zeros in the region of its analyticity $|\operatorname{Im} k| < 1/\xi_0$. We represent $R_1(k)$ in the form

$$R_1(k) = \frac{k^2\xi_0^2 - \alpha^2}{k^2\xi_0^2 + 1}\frac{N_1^+(k)}{N_1^-(k)},$$

where $N_1^+(k)$ is analytic, not having zeros of the function for any $\operatorname{Im} k > 0$; $N_1^-(k)$ is analytic and does not have zeros of the function for all $\operatorname{Im} k < 1/\xi_0$.

The function $R_2(k)$ can be represented in similar fashion:

$$R_2(k) = \frac{N_2^-(k)}{N_2^+(k)},$$

where the functions $N_2^\mp(k)$ are analytic and do not have zeros in the corresponding half planes.

We rewrite Eq. (A.5) in the form

$$\frac{(k^2\xi_0^2 - \alpha^2)N_1^+(k)N_2^+(k)f^+(k)}{(k\xi_0 + i)} = -(k\xi_0 - i)N_1^-(k)N_2^-(k)f^-(k). \quad (A.10)$$

As $k \to \infty$, both sides of (A.10) increase no more rapidly than a polynomial of first degree, since $f^{\pm}(k)$, $N^{\pm}_{1,2}(k) \to 0$ as $k \to \infty$. On the other hand, the left side of (A.10) is analytic and does not have zeros for Im $k > 0$; the right side is analytic and does not have zeros for Im $k < 1/\xi_0$. It therefore follows that both sides of (A.10) are equal to constants. For convenience, we write the constant in the form $iCN_1^+(0)N_2^+(0)\xi_0^2$; then

$$f^+(k) = iC\xi_0^2 \frac{N_1^+(0)N_2^+(0)(k\xi_0 + i)}{N_1^+(k)N_2^+(k)(k^2\xi_0^2 - \alpha^2)}. \tag{A.11}$$

For $z > 0$, we have

$$\psi(z) = \frac{1}{2\pi}\int_{-\infty+i\delta}^{+\infty+i\delta} e^{-ikz} f^+(k)\,dk.$$

We carry out a cut in the k plane from the point $k = -i/\xi_0$ to $k = -i\infty$ along the imaginary axis. Deforming the contour of integration so that it passes along the left and right edges of the cut and around the points $k = \pm\alpha/\xi_0$ in the counter-clockwise direction, we get

$$\psi(z) = \frac{CN_1^+(0)N_2^+(0)\xi_0}{2\alpha} \times \left\{\frac{(\alpha + i)e^{-i\alpha z/\xi_0}}{N_1^+(\alpha/\xi_0)N_2^+(\alpha/\xi_0)} + \text{c.c.}\right\} + \int_\gamma. \tag{A.12}$$

The remaining integral is exponentially small for $z \gg \xi_0$, so that it can be neglected in this region.

As is well known,[6] Eq. (2) gives the correct solution for ψ_∞ with accuracy to terms of order α^2. Therefore, it is sufficient to calculate $N_{1,2}(\alpha/\xi_0)$ with accuracy to terms of order α, since account of terms of higher order determines the ψ function with accuracy to α^3. The computations are entirely analogous to the corresponding contributions to the problem of radiative equilibrium.[8] We finally obtain

$$N_1^+\left(\frac{\alpha}{\xi_0}\right) = \left[\frac{7\zeta(3)}{12\ln(2\gamma\tilde{\omega}/\pi T_c)}\right]^{1/2}(1 - 0.3i\alpha),$$

$$N_2^+\left(\frac{\alpha}{\xi_0}\right) = \left\{1 - i\alpha\frac{0.5}{2\pi^2/g_2 mp_0 + \ln(2\gamma\tilde{\omega}/\pi T_c)}\right\} \times\left[1 + g_2\frac{mp_0}{2\pi^2}\ln\frac{2\gamma\tilde{\omega}}{\pi T_c}\right]^{-1/2}. \tag{A.13}$$

For $\xi_0 \ll z \ll \xi_0/\alpha$, one can write

$$\psi(z) = C(\beta + z) \tag{A.14}$$

(β is defined in (8)).

For $z < 0$, we have

$$\psi(z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ikz} f^-(k)\,dk. \tag{A.15}$$

Since $f^-(k)$ has no singularities for Im $k < 1/\xi_0$, the nearest singularity of the integrand of (A.15) is the branch point for $k = i/\xi_0$. We make a cut in the k plane from $k = i/\xi_0$ to $k = i\infty$, so that

$$\psi(z) = e^{z/\xi_0}\frac{i}{2\pi}\int_0^\infty e^{kz}[f_2^-(k) - f_1^-(k)]\,dk. \tag{A.16}$$

where $f_1^-(k)$ is the value of $f^-(k)$ on the left and $f_2^-(k)$ the value on the right of the cut. Therefore, one can write

$$\psi(z) = Ce^{z/\xi_0}/\varphi(z);$$

$$\varphi(z) = C\left\{-\frac{i}{2\pi}\int_0^\infty e^{kz}[f_1^-(k) - f_2^-(k)]\,dk\right\}^{-1}. \tag{A.17}$$

APPENDIX B

For solution of Eq. (4), we make the usual substitution

$$z \to \frac{p_0}{2\pi m T_c} z, \qquad \psi \to \frac{2\pi T_c}{|g_1|}\left[\frac{2(T_c - T)}{7\zeta(3)T_c}\right]^{1/2}\psi = \psi_\infty\psi;$$

then Eq. (4) is transformed to

$$\frac{d^2\psi}{dz^2} + \alpha^2(1 - |\psi|^2)\psi = 0 \tag{B.1}$$

(the value of α is defined in (A.9)).

The first integral of this equation is found by using the paper of Ginzburg and Landau,[2] and we find the integration constant from the conditions

$$\psi \to 1, \qquad \psi' \to 0 \qquad \text{as} \qquad z \to +\infty. \tag{B.2}$$

As a result, we get the equation $(\psi')^2 = \alpha^2(1 - \psi^2)^2/2$ which is easily integrated:

$$\psi = \text{th}(\alpha z/\sqrt{2}\xi_0 + C_1), \tag{B.3}$$

$$\psi = \text{cth}(\alpha z/\sqrt{2}\xi_0 + C_2), \tag{B.4*}$$

$$\psi = 1. \tag{B.5}$$

It is easy to see that the solutions (B.4) and (B.5) can in no manner satisfy the condition $|\psi| \ll 1$. On the other hand, the nonlinear integral equation (2) for $T < T_C$ always admits a unique solution (if one neglects a phase factor), which satisfies this condition and the conditions at infinity (5), (6). Therefore, (B.4) and (B.5) cannot be asymptotic solutions of Eq. (2), so that they can be discarded.

Thus the solution of Eq. (4) which satisfies the conditions (B.2) and which is the asymptotic solutions of (2) has the form (in the usual variables)

$$\psi = \psi_\infty\,\text{th}(\alpha z/\sqrt{2}\xi_0 + C_1). \tag{B.6}$$

*cth = coth

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