FIELD THEORY WITH NONLOCAL INTERACTIONS. IV. QUESTIONS OF CONVERGENCE, CAUSALITY, AND GAUGE INVARIANCE

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Submitted to JETP editor July 31, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 622-631 (February, 1965)

A simple method is indicated for the construction of a nonlocal field theory which is relativistically invariant, unitary, and free from divergences, and which is causal so long as the values of the kinematic invariants of the problem do not exceed a certain limiting value. A nonlocal electrodynamics is constructed which in addition satisfies the requirement of gauge invariance. A number of problems are discussed which relate to the description of macroscopic bodies and the introduction of additional vectors into the interaction.

1. Preceding papers by one of the writers [1-3] have been based on the hypothesis that the well-known difficulties of nonlocal field theory (NFT) are not organically inherent in it, but are a result of a too direct way of generalizing the apparatus of ordinary field theory. In particular this relates to the invalid identification of a number of concepts and quantities—the Lagrangian and the Hamiltonian (with sign reversed), the criteria for causality and compatibility, the Green's functions in the Heisenberg and in representations, and so on—which are identical only in local theory.

It was shown in [1-3] that the problems of mathematical compatibility and of causality, and also a number of questions of dynamical description, can be solved in a positive way in NFT. Among the problems remaining unsolved are those of convergence and macroscopic causality (and also of gauge invariance in electrodynamics). As has already been shown by Bloch, [4] in a NFT with a "hard" form-factor in the vertex part of the interaction Lagrangian specific divergences will arise with respect to the angles of the pseudo-euclidean space, owing to violation of the Feynman rules of procedure because of the acausality of the theory. In other words, the divergences are associated with large values of the space and time components of the virtual momenta while their four-dimensional squares are small. An analysis based on the diagram technique formulated in [3] shows that the form-factor eliminates only logarithmic divergences of the local theory (in particular, fermion proper-energy divergences, see also [5]). Quadratically divergent matrix elements, on the other hand, remain divergent in the NFT also; furthermore it is not merely a matter of the appearance of an infinite constant, and divergences arise only for definite (spacelike) momenta of the diagram. Thus this type of NFT is in any case not applicable to the extremely important case of non-renormalizable theories.

There have been several papers devoted to the removal of this difficulty. A type of form-factor proposed by Bloch [4] does not lead to divergences, but turned out to be in contradiction with the correspondence principle and the condition of macroscopic causality. In a method developed by Heitler and his co-workers [6] convergence was achieved at the cost of renouncing relativistic invariance. A paper by Levy [7] introduced an averaging over an external timelike vector; there are, however, serious difficulties with unitarity (see below, Sec. 7). Finally, very recently I. E. Tamm (private communication) has proposed a method for removing the divergences with respect to angles by using a curved momentum space.

The situation in the problem of causality is still more complicated. Leaving a detailed analysis of this problem to a future occasion, we point out at once that there is still no sufficiently clear criterion of macroscopic causality which can go beyond the framework of mathematical analogies and correspond directly to the conditions of a physical experiment.[1)] In this connection it seems to us that the results of the well known papers by Stueckel-

[1)]We emphasize the essential difference between the criterion of microcausality in the axiomatic method, which is designed to select as narrow a type of theory as possible, and the criterion of macrocausality in NFT, which is necessary to verify the absence of contradictions between the predictions of the theory and experiment.

berg and Wanders [8] and by Slavnov and Sukhanov [9] show only that the mathematical type of macrocausality criterion (in all probability too severe) is not satisfied. We point out, on the other hand, that it has been shown by one of the writers (A.L.) that even this criterion can be satisfied in the corresponding order of perturbation theory by choosing the real part of the matrix element different from that used in [8].

Meanwhile, the analysis made by one of the writers [3] shows that in ordinary NFT with "hard" form-factors there is a marked breakdown of the analytic properties of the matrix elements owing to the appearance of close-in singularities (singularities which do not go out to infinity with increase of the "cut-off" momentum Λ).[2)] These singularities, which have their source in the high virtual momenta, will evidently necessarily lead to violation of a reasonable macrocausality condition. From the mere fact that the dispersion relations are violated it is evident that such singularities cannot be allowed to appear.

Questions of gauge invariance in electrodynamics are discussed in Sec. 5.

2. The difficulties which have been indicated in NFT with a hard form-factor in the vertex part of the interaction Lagrangian have common roots—the facts that there is no Feynman avoidance of the poles and that it is impossible to go over to Euclidean momentum space. At the same time, keeping the Feynman way of going around all the singularities clearly is in contradiction with the unitarity condition.

It is an essential fact that the solution of the difficulties with convergence and close-in singularities by no means requires that the Feynman rules be kept for all singularities. A matrix element can always be divided into two parts (for definiteness, 1 and 2) such that in one of them (2) the integration over virtual momenta is accomplished in the finite region. For example, we can identify part 1 with the real, and part 2 with the imaginary, part of the matrix element. According to what has been said, it suffices to require that the Feynman rules for going around singularities be kept only for part 1. An arbitrary avoidance of singularities can be used in part 2. This fact enables us simultaneously to preserve the unitarity of the matrix element and escape from the difficulties in question.

Since it is known that part 1 with the Feynman procedure is microcausal, it is necessary to impose definite causality conditions on part 2. Without proposing to give here a general formulation of the causality criterion, we shall formulate a condition which at any rate assures complete absence of acausal effects for not too large energies. This condition reduces to the requirement that the matrix element in the momentum representation satisfy the usual microcausality condition as long as no one of the external kinematic invariants that characterize it exceeds a prescribed quantity Λ^2. In other words, it is necessary that part 2 of the matrix element, which contains the anti-Feynman passages around singularities, show a threshold behavior.

We note that the degree to which causality in its space-time aspect is violated at momenta larger than Λ has so far not been investigated, owing to the lack of the appropriate criterion. In any case, at momenta smaller than Λ the analytic properties of the matrix element remain exactly the same as in local theory. Accordingly the only difference between the dispersion relations and the ordinary dispersion relations is that their absorptive part at momenta larger than Λ is not directly connected with the total cross section. It is well known that this region makes an extremely small contribution to the dispersion relations.

3. We now proceed to the realization of the indicated program. We fix for ourselves the goal of constructing an expression for the S matrix which satisfies the following requirements: a) relativistic invariance, b) unitarity, c) formal existence of the correct local limit, d) convergence, e) macrocausality in the sense indicated in Sec. 2.

According to [1,2] the most general expression for the S matrix that satisfies conditions a)—c) is of the form

$$S = \tilde{T}_\lambda \exp\left(i \int_0^1 d\lambda \sigma(\lambda) \right), \tag{1}$$

where

$$\sigma(\lambda) = \int d^4x L_{int}(x)$$

is the action corresponding to local theory with the Lagrangian λL_{int}, in which a "smearing" is introduced in one way or another, and $\tilde{T}_\lambda$ is the symbol for antichronological ordering with respect to λ.

There are quite a number of ways of introducing the "smearing." Since the fundamental structural elements of a diagram are the vertex parts and the propagation functions, the simplest methods involve modifications of these elements. More rea-

[2)] Generally speaking, these singularities appear in all matrix elements. Some of them can be removed by a suitable "smearing out" of the counterterms (cf. [2])

listic methods can be based on changes in the metric of momentum space.[11]

As has already been indicated, the introduction of a form-factor into the vertex part leads to insuperable difficulties. A "smearing" of the propagation function leads to different results. It is simplest to take as a basis the Pauli-Villars regularization method in the form given to it in papers by Bogolyubov and his co-workers.[12,9] One introduces the nonphysical operators

$$\tilde{\varphi}(x) = \varphi(x) + \int_{\Lambda^2}^{\infty} d\varkappa^2 \sqrt{\rho(\varkappa^2)}\,\varphi_\varkappa(x)$$

with the commutator

$$[\tilde{\varphi}(x),\ \tilde{\varphi}(y)] = -i\tilde{D}(x-y),$$

$$\tilde{D}(x-y) = D(x-y) - \int_{\Lambda^2}^{\infty} d\varkappa^2 \rho(\varkappa^2) D_\varkappa(x-y).$$

Here

$$\int_{\Lambda^2}^{\infty} d\varkappa^2\, \varkappa^{2n}\, \rho(\varkappa^2) = \mu^{2n}$$

and D is the ordinary commutator function. In order to eliminate the nonphysical states one introduces a projection operator P onto the space of physical amplitudes

$$P\tilde{\varphi}^{(+)} = \varphi^{(+)}P, \qquad \tilde{\varphi}^{(-)}P = P\varphi^{(-)},$$

and from the definition of this operator it follows that

$$P : \tilde{\varphi}(x) \ldots \tilde{\varphi}(y) : P = P : \varphi(x) \ldots \varphi(y) : P.$$

We define the "smeared" action by the relation

$$\sigma(\lambda) = P\tilde{\sigma}(\lambda)P, \tag{2}$$

where the tilde indicates replacement of the Heisenberg field operators in the local Lagrangian by operators $\tilde{\varphi}_\Gamma$ which are defined by equations of the type

$$\tilde{\varphi}_\Gamma(x) = \tilde{\varphi}(x) + \int d^4y\, \tilde{D}^R(x-y)\tilde{f}_\Gamma(y).$$

The presence of the operators P guarantees that there are no nonphysical states in the initial and final wave functions.

We shall prove that the theory constructed in this way satisfies the requirements d) and e). First we convince ourselves that there are no divergences. As is well known, the cause of their appearance in local field theory is the presence of products of singularities of functions D^R and $D^\pm$, which contain at least one D^R function. In the type of theory now to be considered this function appears only in regularized form (it arises only on account of the last equation), and therefore it contains no singularities. This means that the matrix element converges.

We shall prove, furthermore, that the matrix element constructed in this way differs from the local matrix element as to analytic properties only by distant cuts (cuts which begin at momenta of the order of Λ). For this purpose we note that the S matrix regularized by the Pauli-Villars method (which is obviously nonunitary), being of the form

$$S' = P\tilde{T}_\lambda \exp\left(i\int_0^1 d\lambda\tilde{\sigma}\right)P, \tag{3}$$

has the analytic properties of local theory, containing only Feynman functions.

Let us consider the difference between the terms of S and S′ in the n-th order of perturbation theory:

$$\int d^4x_1 \ldots d^4x_n \{P\tilde{L}^{(i_1)}_{int}(x_1)P \ldots P\tilde{L}^{(i_n)}_{int}(x_n)P - P\tilde{L}^{(i_1)}_{int}(x_1)\ldots\tilde{L}^{(i_n)}_{int}(x_n)P\}, \tag{4}$$

where $\tilde{L}^{(i)}_{int}$ is the appropriate component of the Lagrangian. Replacing the intermediate factors P by $1-(1-P)$, we find that (4) reduces to a sum of terms, each of which contains at least one intermediate factor $1-P$. Taking the matrix element between states p_i and p_j and using the translational invariance, we can write a typical term of (4) in the form

$$\sum_n \langle p_i|\tilde{L}^{(i_1)}(0)\ldots|p_n\rangle\langle p_n|\tilde{L}^{(i_k)}(0)\ldots|p_f\rangle \times \delta^4(p_i-p_n)\delta^4(p_n-p_f),$$

where the intermediate state corresponds to the operator $1-P$ and has $p_n^2 > \Lambda^2$.

It is obvious from the conservation laws that (4) vanishes if the external kinematic invariants of the problem, constructed from p_i and p_f, are smaller than some threshold value determined by the quantity Λ. This result is physically obvious, since breakdown of the analytic properties can occur only at the threshold for production of nonphysical particles.

4. The type of theory considered in the preceding section is distinguished by its closed operator form, but the corresponding diagram technique is hardly a simple one. Therefore we shall consider below another type of NFT which satisfies the conditions a)—e) and corresponds to a simple diagram technique.

We shall construct the matrix element of the S matrix in the following way: its antihermitian part is equal to the real part of the Pauli-Villars-regularized matrix element (3), and the Hermitian part is chosen in accordance with the unitarity condition. This condition, written in the form

$$2S_n^h = -\sum_{i=1}^{n-1} (S_{n-i}^h + S_{n-i}^a)(S_i^h - S_i^a), \qquad (5)$$

shows that the Hermitian part of a matrix element can be expressed in terms of the known antihermitian parts of lower orders in perturbation theory.

Furthermore the conditions a)—d) are satisfied in an obvious way. As for condition e), by substituting in the right member of (5) the expression $S^a = S'^a$ where S'^a is the antihermitian part of the S matrix (3) and comparing the resulting expression with S'^h, we can repeat the proof carried out in the preceding section.

The meaning of these results is obvious: the Hermitian part of the unitary S matrix must contain only physical intermediate states. According to this, we can give for the way of constructing the S matrix in this type of NFT the following simple formulation: the regularized matrix element is constructed by means of the usual diagram technique, and the nonphysical terms containing distant poles are then struck out from its Hermitian part.[3)]

As an illustration we give the expression for the proper energy of a particle in the Hearst-Thirring model. Discarding the distant cuts in the Hermitian part of the matrix S′:

$$S' \sim i^{-1} \int d^4p \tilde{D}_F(p) \tilde{D}_F(p+k),$$

we get $S = S' + M$, where

$$M \sim i \int_{\Lambda^2}^{\infty} d\varkappa^2 \rho(\varkappa^2) \Big\{ 2\theta(k^2 - (\varkappa+\mu)^2) \times \Big[\Big(1 - \frac{(\varkappa+\mu)^2}{k^2}\Big)\Big(1 - \frac{(\varkappa-\mu)^2}{k^2}\Big) \Big]^{1/2} - \theta(k^2 - 4\varkappa^2) \Big(1 - \frac{4\varkappa^2}{k^2}\Big)^{1/2} \Big\}.$$

In accordance with what was said above, the quantity S contains no divergences and differs from S′ only by distant cuts which start at the points $\Lambda + \mu$ and 2Λ.

[3)]Recently a method of formulating the theory directly in Euclidean momentum space and subsequently continuing it analytically into the physical region has gained currency. According to what has been said above, this method could eliminate the difficulties considered in this paper. Obviously, however, the condition of unitarity is not satisfied for the expression obtained in this way [use of the expression (1) in this case is impossible, because there are no free-field operators in the Euclidean space]. The method considered in this section corresponds essentially to constructing only the antihermitian part of the matrix element in the Euclidean space.

In the types of NFT considered in the last sections the use of the Pauli-Villars regularization has been dictated only by considerations of simplicity. In principle the indicated regularization can be carried out by using complex masses, i.e., choosing the propagation function in the form

$$\tilde{D}_F(k) = \frac{1}{k^2 - M^2 + i\varepsilon} - \frac{1}{k^2 - M^2 + i\Lambda^2}$$

This kind of construction is analogous to the introduction of a hard form-factor of the McManus type, but differs from it by converging in arbitrary order in perturbation theory. The question is still open, however, as to whether this type of theory is causal.

In concluding this section we point out that the results obtained above, which use the Pauli-Villars regularization, are in complete agreement with the fact that the breakdown of unitarity of the S matrix also has a threshold behavior.[13] We emphasize that the unitarity of (3) at momenta smaller than Λ of course gives no basis for asserting complete equivalence of the Pauli-Villars regularization and the corresponding nonlocal theory. The point is the deep difference between breakdowns of causality and unitarity. Whereas the latter would mean a deep physical contradiction with the probability treatment of quantum mechanics, breakdown of the causality condition "in the small" means only violation of a formal condition obtained by extrapolation of the classical condition of causality (for details see [14], and also the additional arguments in [15]).

5. We shall go further into the application of the method in question to quantum electrodynamics, i.e., to a case which may be of practical interest in connection with anticipations of the results of experiments with colliding beams. A serious difficulty encountered by a number of authors (cf. e.g., [16]) lies in the formal violation of gauge invariance in nonlocal electrodynamics—the appearance of a nonzero photon mass. We shall indicate a simple way to overcome this difficulty (concerning other possibilities see [7]).

As is well known, formal gauge invariance of the theory is assured by the fact that the momentum operators and the vector potential always come in in the form of the combination $\hat{p} - e\hat{A}$. Then the transformation $A \to A + \nabla\Phi$, $\psi \to \psi e^{-ie\Phi}$ keeps all observable quantities unchanged. Therefore it is necessary in any event that in introducing a "smearing" one preserve the combination in question unchanged. That this condition is nontrivial in NFT is due to the fact that the form-factor is equivalent to a set of differentiation op-

erators, i.e., can be put in the form of a function of the momentum operators.

On the other hand, the fulfilling of this condition still by no means guarantees actual gauge invariance. As one of the writers once showed[17] (cf. also [18]), the origin of the photon mass is from the factor $\exp[ie\{\Phi(x') - \Phi(x)\}]$, which appears on gauge transformation of the current $[\bar{\psi}(x), \gamma_\mu\psi(x')]$, if in this operator we temporarily refer $\bar{\psi}$ to the point x' and ψ to the point x (sic), in order to make the operator unambiguous. There then appear nonzero terms (actually infinite in the limit $x' \to x$), which arise from the multiplication of terms of the expansion of the exponential which vanish in this limit and infinite terms of the expansion of the rest of the expression. It is from the development of this indeterminate form that a nonzero photon mass arises.

It is clear from this that in NFT, where there are no singularities of the current for $x' \to x$, gauge invariance should hold, provided only that the formal condition we have stated is not violated in the "smearing." We note at once that a literal extension to electrodynamics of the method expounded in the preceding sections leads to violation of this condition. In fact, the regularization of the field ψ gives

$$\tilde{\psi}_\Gamma(x) = \bar{\psi}(x) + e\int dy \tilde{D}^R(x-y)\,\tilde{\hat{A}}(y)\,\tilde{\psi}_\Gamma(y).$$

Considering for simplicity the case of two masses M_1 and M_2, we get the equation

$$[(\hat{p} - M_1)(\hat{p} - M_2) - e(M_2 - M_1)\,\tilde{\hat{A}}(x)]\,\tilde{\psi}_\Gamma(x) = 0,$$

in which, besides the combination $\hat{p} - e\hat{A}$, the operator $\hat{p}$ occurs by itself. This corresponds directly to the well known fact that Pauli-Villars regularization, when applied to each of the lines of a diagram, leads to a nonzero photon mass. There will obviously be an analogous situation if an ordinary form-factor is introduced.

It is well known[19] that gauge invariance in the Pauli-Villars method can be achieved by the use of a single auxiliary mass for all of the propagation functions that enter the make-up of a closed fermion cycle. This suggests a method for constructing a gauge-invariant NFT. It is necessary to "smear" not each separate field operator, but the entire current as a whole:

$$\tilde{j}_{\mu\Gamma}(x) = -(ie/2)\Big\{[\bar{\psi}_\Gamma(x), \gamma_\mu\psi_\Gamma(x)] + \int_{\Lambda^2}^{\infty} d\varkappa^2 \sqrt{\rho(\varkappa^2)}\,[\bar{\psi}_{\Gamma\varkappa}(x), \gamma_\mu\psi_{\Gamma\varkappa}(x)]\Big\};$$

$$\psi_{\Gamma\varkappa}(x) = \psi_\varkappa(x) + e\int dy\, D_\varkappa^R(x-y)\,\bar{A}(y)\,\psi_{\Gamma\varkappa}(x). \quad (6)$$

It can be seen from the corresponding Dirac equations that the current operator (6), being finite, is conserved. Therefore, using the fact that

$$\tilde{\sigma} \sim \int d^4x \tilde{j}_\Gamma(x)\bar{A}_\Gamma(x),$$

we can verify that the S matrix is gauge invariant.

It is easy to show that this choice corresponds exactly to the rule about cycles indicated above. In fact, the auxiliary masses come in only in the internal lines of the diagram. On the other hand, according to (6), at each vertex the masses of the two fermion lines are the same. It is obvious that this will be true for all of the fermion lines that make up a cycle.

6. The solution of the problems of causality and convergence would be decidedly simplified if it were possible to introduce into the theory some timelike four-vector N. By means of it one could localize the deviation from the ordinary theory "in the small" (in the Euclidean sense), for example, by introducing in the form-factor the argument $(x-x')^2 - (x-x', N)^2/2N^2$. Attempts of this sort have been made at various times.

We shall show that, unfortunately, this path must be abandoned. The vector N can either be external, or can relate to the system of particles itself. In the former case an averaging over the direction of this vector is necessary, since otherwise the relativity principle is violated by distinguishing a certain reference system (the one in which the vector N reduces to its time component). The averaging process, however, leads to deep difficulties owing to the pseudoeuclidean character of the metric (the integral over directions of the vector N diverges). Devious ways of overcoming this difficulty, involving a transition to a complex Lorentz group, can lead to violation of the unitarity of the S matrix.[7]

The latter possibility for N leads to still more obvious difficulties. It is natural to choose for the vector N the (conserved) energy-momentum vector of the system of interacting particles. Then, however, the result (for example, the scattering cross section) will depend essentially on just what particles are included in the composition of the system. In particular, inclusion in it of distant particles which are in no way connected with it leads to changes of the characteristics of the scattering. Thus there is set up a nonphysical connection between arbitrarily remote objects, which leads to a glaring violation of the physical causality principle.

7. The violation of the causality principle at sufficiently large values of the kinematic invari-

ants poses quite a number of problems relating to macroscopic bodies. This is the case not only with the specific schemes discussed above, but with any nonlocal theory with an additional dimensional parameter.

The point is that even in the region of small energies the interaction of macroscopic bodies is characterized by large (comparable with Λ) values of the kinematic invariants (we may note, for example, that an elementary length of $\sim 10^{-17}$ cm corresponds to a momentum Λ of only $\sim 10^{-10}$ in c.g.s. units). Therefore the question as to the correctness of the description of macroscopic bodies in the framework of NFT becomes a very crucial one, in particular in regard to the absence of acausal effects in the interaction of such bodies.

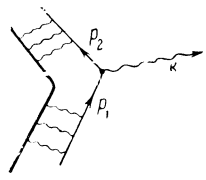
First of all, it is desirable (though perhaps not strictly essential) that the equations of the theory that are applicable not to elementary particles but directly to a macroscopic body should lead to correct classical results—that is, that the usual correspondence principle should hold. If in the initial formulation we regard Λ as a fixed number, then when the mass M of the body exceeds Λ acausal effects will arise in the classical region. A simple way out of this difficulty (at least in the framework of a theory with a nonlocal interaction) is to take from the beginning as the basis of the theory not a fixed value of Λ, but a dimensionless parameter $\alpha = \Lambda/M \gg 1$. This gives, in particular,

$$D_F = \frac{1}{k^2 - \mu^2 + i\varepsilon} - \int_{\alpha^2}^{\infty} \frac{dx^2 \rho(x^2)}{k^2 - M^2 x^2 + i\varepsilon}.$$

With this formulation all acausal cuts are shifted into the unobservable region.

There is, however, a deeper and more important requirement, which is that a macroscopic body constructed from elementary particles whose interaction is described by NFT must obey the usual causal equations of motion.

Here we must consider two cases. First we assume that there are interacting in a direct way [4] only a comparatively small number of particles, all of whose kinematical invariants are smaller than Λ. At the same time these particles are assumed to be connected with heavy bodies whose



kinematic invariants exceed Λ. An example is the scattering of charged pith balls, or scattering of light by a pith ball.

Although the propagation function of the body as a whole will depend on its total mass, this mass does not occur in the regularized expressions for the vertex part and the propagation function. In fact, let us consider the simplest vertex (see the figure, where the thick line corresponds to the heavy body, and the thin line to the particle considered). The momenta p_1 and p_2 which determine the acausal cuts do not lie on the mass shell. Their degrees of virtualness ($v_{1,2} = p_{1,2}^2/m^2 - 1$), however, are determined not by the mass of the heavy body, but by the energy E_{bd} of binding of the particle to this body. It follows from the uncertainty principle that $v_{1,2} \sim E_{bd}/m \ll \Lambda^2/m^2$.

As for the momentum k, in the Compton effect it lies exactly on the mass shell, and in the scattering of pith balls, whose impact parameter d is large under the conditions of a classical experiment, the regularized Green's function actually involves not k, but the quantity [5] $\hbar/d$ ($kd \sim e^2M/p_\infty \gg \hbar$), which is many orders of magnitude smaller. We can see this by writing out the matrix element in coordinate form (for simplicity in the lowest order of perturbation theory):

$$M \sim \int dxdy\, j_{AB}(x) j_{CD}(y) D_F(x-y),$$

where j is the current of the transition, and under the conditions of the experiment $j_{AB}(x) j_{CD}(y)$ is different from zero only for $x - y \sim d$. Going back to the momentum representation, we verify that the assertion that was made is correct.

The second case corresponds to the direct interaction of such a large number of elementary particles that their total kinematic invariants exceed

[4] By a direct interaction we mean the presence of a direct coupling between each pair of particles of the system. As for "relay" diagrams, which correspond to direct interaction of only a small number of neighboring atoms, the corresponding matrix element will not involve the total kinematic invariants of the system. This case therefore represents no danger from the present point of view.

[5] It is actually this parameter which occurs also in the quantum-mechanical expression corresponding to the scattering of elementary particles with wave functions in the form of plane waves. In this case $\hbar/d_{eff} \sim k$. But in the case considered the conditions of the experiment are such (there are no small impact parameters because the spheres cannot come close together) that the quantity $\hbar/d$ is much smaller than k, and also consequently than Λ.

Λ. This case requires for its realization extraordinarily high values of the density because of the finite range of the forces (for Coulomb interactions one must take the Debye screening into account). The corresponding energy per particle in such a state is extremely large.

The occurrence of such a situation requires extraordinarily great energies, and therefore it has no direct bearing on the problem of macroscopic bodies in the form in which we are considering it here.

The writers are grateful to I. E. Tamm and the members of the seminar he conducts for a discussion of this work, and also to M. A. Lifshitz for fruitful discussions.

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Translated by W. H. Furry
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