ANOMALOUS RESISTANCE AND MICROWAVE RADIATION FROM A PLASMA IN A STRONG ELECTRIC FIELD

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An experimental and theoretical study was made of the mechanism of the observed anomalous resistance of a collision-free plasma [1] in a toroidal set-up with a strong electric field (100 V/cm) parallel to a quasistationary magnetic field. It is shown that the anomalous resistance is the result of excitation of intense Langmuir oscillations that carry momentum away from the plasma. The theoretical estimate of the resistance is in good agreement with experiment. Intense microwave radiation from the plasma is observed. It is shown that the observed microwave signals can be explained as being due to a superposition of two effects: radiation of electromagnetic waves at double the plasma frequency from the volume of the plasma, and excitation of surface waves.

1. MECHANISM OF ANOMALOUS RESISTANCE

WE have observed earlier [1] anomalous resistance of a plasma in a strong electric field parallel to the confining magnetic field. Inasmuch as the observed active resistance could be attributed neither to collisions between electrons and ions nor to collisions with neutral atoms or with the walls of the discharge chamber, a hypothesis was advanced that this resistance might be due to two-stream instability arising when the current velocity u of the electrons exceeds their thermal velocity v_t. Let us investigate this mechanism in greater detail and let us attempt to estimate numerically the anomalous active resistance caused by it.

In the experiments in question, the plasma density is 10^{11}—10^{12} cm^{-3} and the amplitude of the high-frequency electric field is $E \sim 10$—100 V/cm, so that the condition for the "runaway" of all the electrons [2] is satisfied with ample margin. After turning on the high-frequency electric field E_θ, all the electrons become accelerated relative to the ions, and attain within a time $\sim 10^{-9}$ sec a translational velocity higher than the initial thermal velocity $v_{t0} \sim 10^8$ cm/sec. This gives rise to two-stream instability with an increment $(m/M)^{1/3}\omega_{pe} \sim 3 \times 10^9$ sec^{-1}, and electrostatic waves of large amplitude are excited in the plasma within several nanoseconds. The electrons interact with these waves and transfer to them their energy and their translational momentum. The waves, in turn, can be absorbed by the electrons, ions, or the discharge-chamber walls, if the plasma pinch is not detached from the walls [1)].

Under the conditions of the experiment in question, the thermal velocity of the ions is small compared with the phase velocities of the waves, and there is little absorption of the waves by the ions. The fast ions could not be retained in a discharge chamber with a small diameter 3 cm at a longitudinal magnetic field ~ 3 kG.

Therefore the subsequent fate of the energy that is anomalously dissipated in the plasma depends on the ratio of the wave absorption by the electrons (turbulent heating of the electrons [3]) to the drift of the waves to the chamber walls. Let us estimate the role of the latter mechanism of wave absorption. The time τ that the waves go off to the walls is approximately equal to r/v_{gr}, where v_{gr} is the radial component of the group velocity of the waves. Nonlinear interaction of the waves produced in the plasma in the presence of two-stream instability leads to excitation of an entire spectrum of all possible waves. It is clear that the fastest to move off to the walls will be those waves which have the largest radial group velocity v_{gr}. It is precisely these waves which determine the momentum balance in the system.

The largest group velocity in the radial direction is possessed by short Langmuir waves with wave vector $k \sim k_D = \omega_{pe}/v_{Te}$. If the electron

[1)]The importance of taking into account the last effect was pointed out to us by L. I. Rudakov.

plasma frequency ω_{pe} exceeds the electron cyclotron frequency ω_{He}, then the dispersion equation for the Langmuir waves is of the form

$$\omega = k_{\parallel}u \pm (\omega_{pe}^2 + k^2 v_t^2)^{1/2}, \tag{1}$$

where $k_{\parallel}$ is the component of the wave vector in the direction of the longitudinal electric field E_θ. It follows therefore that when $k \sim k_D$ the radial component of the group velocity is approximately equal to v_t, that is,

$$\tau \approx r / v_t. \tag{2}$$

Waves with $k \sim k_D$ are rapidly absorbed by electrons. However, the time of damping of the waves by the electrons increases exponentially with decreasing k [3], and even when $k \sim k_D/3$ the time τ for the waves to move off to the walls becomes smaller than the attenuation time. The absorption of short waves with $k \gtrsim k_D/3$ leads to heating of the electrons.

We shall now show that in plasma installations with strong longitudinal electric field, where absorption of the waves due to two-stream instability by the walls of the discharge chamber greatly exceeds the absorption of these waves by the plasma, a linear relation should exist between the current velocity of the electrons and the electric field, that is, an anomalous active resistance of a collision-free plasma should actually be observed. To this end, we estimate the energy and momentum of the plasma waves.

So long as the energy of the waves is small compared with the energy of the translational motion of the electrons, effective "pumping over" of the latter into the former takes place, and the energy density of the plasma waves increases exponentially: $W \sim \exp[(m/M)^{1/3}\omega_{pe}t]$. But when the wave energy reaches the translational-motion energy, the corresponding energy "pumping" ceases [5], and a stationary mode is established [2)], in which

$$W \approx mnu^2 / 2. \tag{3}$$

As to the wave momentum, assuming axial symmetry with respect to the direction of the field E_θ, we can consider the resultant momentum to be directed along the field E_θ. Then the energy density is connected with the momentum density P by the relation

$$W = (\omega / k_{\parallel})P.$$

Inasmuch as in the case of two-stream instability $u > v_t$, it follows from (1) that $\omega/k_{\parallel} \approx u$ when $k_{\parallel} \sim k \sim k_D$, that is,

$$W \approx uP. \tag{4}$$

It is obvious that if the wave energy density of the waves, given by equation (3), has a stationary value, then the momentum density P is also stationary, and the drift of momentum to the wall, averaged over the time τ, is offset by the transfer of momentum from the electrons to the waves. Inasmuch as the electron obtains from the wave and (in the stationary mode) gives up to the plasma waves a momentum eE_θ per unit time, it is obvious that

$$neE_\theta \approx \frac{P}{\tau}. \tag{5}$$

Solving (2)—(5) simultaneously, we obtain

$$u \approx \frac{er}{2mv_T} E_\theta, \tag{6}$$

q.e.d. Going over to the current density $j = neu$, we can rewrite (6) in the form

$$j = \frac{1}{\rho_{eff}} E_\theta, \tag{7}$$

where

$$\rho_{eff} \sim 2 \cdot 10^{11} \frac{\sqrt{T_e}}{nr} \Omega \cdot \text{cm}, \tag{8}$$

the electron temperature T_e being expressed in eV.

Getting somewhat ahead of ourselves, we note that the experimental $I(E_\theta)$ curve shown in Fig. 5 is in splendid agreement with formulas (7) and (8) over a wide range of variation of E_θ, both in the sense of the general character of the functional relation and in the order of magnitude of the observed anomalous active resistance.

2. EXPERIMENTAL SETUP

In the present work we used an installation which was essentially analogous to that described in [1]. The discharge chamber 1 (see Fig. 1) constituted a glass toroid with large diameter 30 cm and small diameter 3 cm. The confining quasi-stationary magnetic field with intensity 3 kOe was produced by a system of 28 coils 2. A corrugated field mode, which is optimal from the point of view of all the observed results, was set up by choosing the currents in the coils. The electric vortical field of the loop was produced by a high-frequency shock circuit 3, constituting a turn 150 cm wide fed from a broad ribbon line placed inside the turn. The frequency of the circuit was 5 Mc, the instan-

[2)] This stationary mode is reached as indicated above, within $\sim 10^{-9}$ sec, which is much shorter than the period of the high-frequency field E_θ.

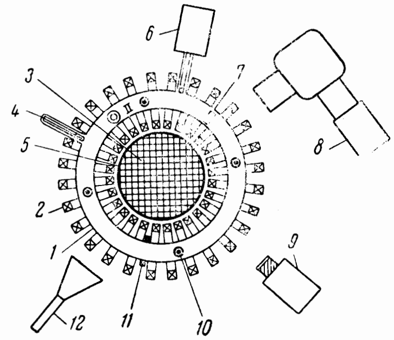

FIG. 1. Diagram of setup: 1 – glass chamber, 2 – coils of quasistationary magnetic field, 3 – high-frequency shock circuit, 4 – probe for measuring transverse velocities of the electrons and ions, 5 – carbon cylinder, 6 – x-ray pickup, 7 – microwave sounding of the plasma at 3 cm wavelength, 8 – monochromator, 9 – hard gamma radiation pickup, 10 – titanium injectors, 11 – Rogowski loop, 12 – horn for the investigation of microwave radiation of the plasma.

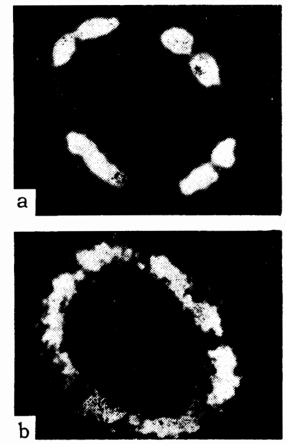

FIG. 2. Filling of the torus with plasma: a – 1 μsec after operation of the injector; b – 20 μsec after operation of the injectors. The dark spots on the image of the glowing torus are due to the presence of parts that blocked the light, and the apparent ellipticity of the image is due to faults in the optical system.

taneous power reached 10^9 W, and the oscillation damping time was approximately 1 microsecond.

To ensure homogeneity of the electric field E_θ along the perimeter of the torus, a special enclosing circuit was used—a homogeneous carbon cylinder 5, the current through which was several times the current in the plasma. Under these conditions, any field inhomogeneity should have been smoothed out within a time $\sim 10^{-8}$ sec. The discharge chamber was first pumped out to 10^{-5} mm Hg, and the hydrogen plasma injected transversely to the magnetic field through four pairs of titanium injectors 10 spaced $\pi/2$ apart. The homogeneity of the plasma filling of the torus was monitored by high-speed photography in white light and in the light of the H_β line with the aid of a multistage electron-optical converter. The exposure of each photograph frame was approximately 1 μsec. The delay of the instant of photography could be smoothly varied within the range from 0 to 100 microseconds relative to the instant of operation of the plasma injectors. As shown in Fig. 2a, during the first ~5 μsec after operation of the injectors one could observe the propagation of the glow of the plasma from the injectors along the perimeter of the torus. Some 15—20 μsec later, the illumination encompassed the entire perimeter of the torus, and by the time 30 μsec was reached, when the experiments with the turning on of the high-frequency circuit were started, the illumination became completely uniform (Fig. 2b).

The plasma density was estimated from the blocking of a rf sounding beam of 3 cm wavelength (Fig. 3). Under the operating conditions, the density of the pre-conditioned plasma was

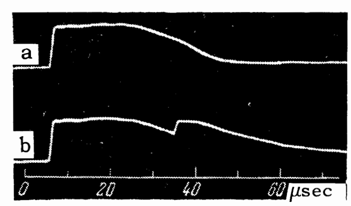

FIG. 3. Blocking of a radio beam of 3 cm wavelength by the plasma: a – following operation of the titanium injectors (shock circuit disconnected), b – following operation of the titanium injectors and the shock circuit.

5×10^{11}—10^{12} cm^{-3}. The upper limit of the total number of neutral atoms in the chamber volume was estimated by determining the pressure jump in the installation after the operation of the injectors with the vacuum pump turned off. It was shown that the number of neutral atoms did not exceed 10^{13} cm^{-3}.

Figure 4 shows an oscillogram of the electric field E_θ, obtained with the aid of a special carbon loop placed along the perimeter of the torus, and an oscillogram of the current in the plasma, meas-

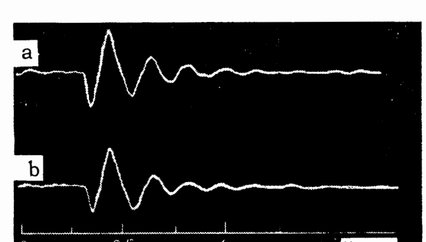

FIG. 4. Oscillograms of electric field E_θ (a) and of the current (b).

ured with a broadband Rogowski loop. Comparison of the oscillograms shows that the resistance of the plasma turn, as in[1], is purely active. The magnitude of the resistance changes somewhat over the time of current flow, amounting in a typical case up to approximately 30 ohms at the first maximum of the current, approximately 20 ohms at the second maximum, and again 30 ohms and more in the succeeding maxima of the current. From electrotechnical considerations it follows that inasmuch as the inductive reactance of the equivalent copper torus at the same frequency amounts to 7 ohms, a barely distinguishable phase shift, $\sim 20°$, should be noticed in the second half-cycle. Such a small phase shift was actually observed for the second half-cycle of the current following a detailed comparison of the oscillograms of Fig. 4 with each other.

Figure 5 shows the dependence of the discharge current on the electric field E_θ (curve 1—for the first current maximum, curve 2—for the second current maximum). We see that within the limits of variation of E_θ, from 10 to 70 V/cm, the $I(E_\theta)$ plot is linear, in accordance with the theoretical formula (7), and the value of the anomalous resistance is well described by formula (8) with $n \sim 10^{12}$ cm^{-3} and $T_e \sim 10^3$ eV. However, in view of the fact that the anomalous active resistance depends only on the square root of the temperature (8), an estimate of the electron temperature from the value of the resistance cannot be regarded as sufficiently convincing.

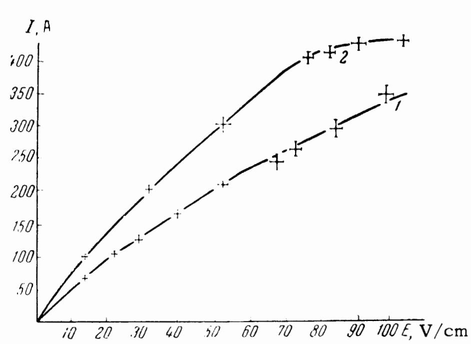

FIG. 5. Dependence of the discharge current on the loop voltage.

Measurements of the x-ray emission, started already in[1], also indicate that the electrons acquire energies of 10^3 eV and more, but it is difficult to judge in this case whether the high-energy x-rays are due to the motion of electrons along the axis or transverse to the accelerating electric field E_θ. We have therefore carried out a direct measurement of the transverse component of the electron velocity with the aid of a probe of the Faraday-cup type, with magnetic analysis of the particle velocities (4 in Fig. 1).

The probe, introduced into the plasma transversely to the magnetic field, was a cylinder closed on its end, inside of which was placed a moving collector of charged particles. The gap a between the end wall of the outer cylinder and the particle collector could be varied over a wide range. In the end wall there were several dozen openings of small diameter. The total area of all the openings was chosen such as to make the density of the electrons and ions in the gap a small and make the Debye radius of the plasma in the gap larger than a. Under these conditions, it is possible to separate the charges with the aid of the magnetic field, so that the particles whose Larmor radius is smaller than half the distance to the collector should not strike the collector, that is,

$$2\frac{mcv_\perp}{eH} < a. \tag{9}$$

To check the operation of the probe, experiments were carried out in which the product aH was maintained constant but both a and H were varied. Within certain limits of variation of a and H, it was possible to observe constancy of the probe readings at a constant product aH. For small values of aH, the current to the collector was negative and corresponded to the gathering of electrons, while at large values of aH the current was positive and corresponded to gathering of ions. Measurements of the collector current as functions of aH have shown that the transverse velocity of the electrons corresponded under optimal conditions to a value $T_e \sim 10^3$ eV, while the transverse ion velocity did not exceed values corresponding to energy $\sim 10^2$ eV. Heating of the electrons to $T_e \sim 10^3$ eV, which followed from different types of measurements, can be attributed to the mechanism indicated above, absorption of the shortest Langmuir waves by the electrons, while the low ion temperature agrees with the impossibility of confining faster ions at the small dimensions and low magnetic field intensities of the present installation.

A qualitative confirmation of the heating of the plasma as a result of operation of the high-frequency circuit is the jump in plasma density, noticed from the repeated blocking of the probing rf beam of 3 cm wavelength (Fig. 3b). An investigation of the H_β emission and x-rays from the plasma has shown that their durations are of the order of several microseconds and of 1 μsec, respectively, while the corresponding signals vary smoothly and are hardly modulated by the variation of the field E_θ.

3. REGISTRATION OF PLASMA MICROWAVE OSCILLATIONS

In view of the explanation of the anomalous active resistance of the plasma as being due to the buildup of intense plasma waves, a search for microwave radiation from the plasma was undertaken and was successful. The microwave signals from the plasma were registered with an external waveguide equipped with a horn antenna and loaded with a broadband detector head (12 on Fig. 1). A typical oscillogram of the envelope of the detected microwave signal received by the horn antenna is shown in Fig. 6. The power of the received signal was 10 mW. The oscillograms of Fig. 6 show that the microwave signal is quite deeply modulated at double the plasma current frequency, and that the maxima of the intensity of the microwave radiation coincide in time with the current maxima. It must also be noted that the most intense microwave signal is observed during the second current half-cycle.

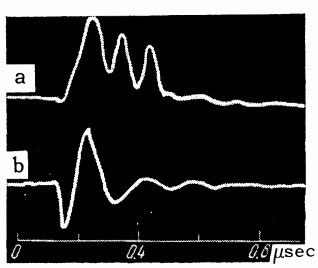

FIG. 6. Oscillograms of microwave signal from a plasma (a) and oscillograms of the current (b).

For a rough estimate of the spectrum of the microwave signals from the plasma, we used waveguides beyond cutoff, namely waveguides with λ_{max} = 3.5, 4.4, and 7 cm. It turned out that the wavelengths of the recorded microwave signals lie in the range from 3.5 to 7 cm. In this measurement method we can obviously draw no conclusions concerning the presence or absence of radiation with wavelength more than 7 cm. The electric-field amplitude $\mathscr{E}$ of the radiation with $\lambda < 3.5$ is at least one order of magnitude smaller than in the radiation in the 3.5–7 cm range, and the power is two orders of magnitude smaller. The experimentally obtained wavelength range of the microwave plasma signal was in good agreement in order of magnitude with ω_{pe}.

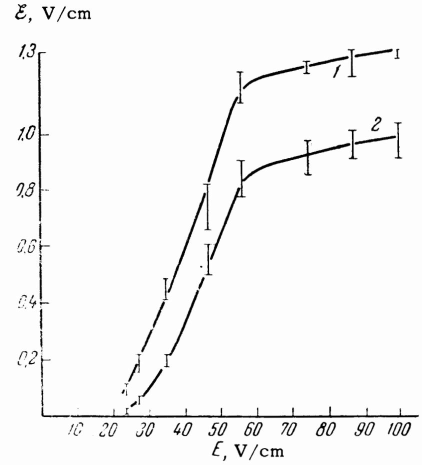

FIG. 7. Dependence of the amplitude of the microwave field on the electric field E_θ.

We determined next the dependence of the field amplitude $\mathscr{E}$ of the observed electromagnetic radiation on the intensity of the accelerating electric field E_θ (see Fig. 7). To this end we used a waveguide open on one end with λ_{max} = 7 cm, loaded on the other end with a broadband detecting head. Curve 1 of Fig. 7 was obtained at a distance R = 2.5 cm from the center of the small cross section of the toroidal chamber to the end of the waveguide, while curve 2 was obtained for a distance R = 7.5 cm. A distinguishing feature of both curves is the saturation of $\mathscr{E}$ when $E_\theta \sim 60$ V/cm is reached. An analogous saturation under like values of E_θ was noted also on the curves showing the dependence of the current on E_θ (see Fig. 5). We can therefore conclude that the microwave signal amplitude is determined not directly by the magnitude of the accelerating electric field E_θ, but by a quantity nu which depends on it (n is the plasma density and u is the current electron velocity).

We can notice also on Fig. 7 a certain difference between curves 1 and 2 in the region of small E_θ far from the saturation point. Curve 2 is similar to a parabola, $\mathscr{E} \sim E_\theta^2$, whereas curve 1 has a more complicated character.

Finally, measurements were made of the law governing the fall-off of the microwave signal from the plasma with increasing distance R from the center of the small section of the torus to the end of the waveguide with λ_{max} = 4.4 cm (waveguide cross section 22 × 10 mm). As indicated above, the wavelengths of the microwave radiation lie in the range $\lambda > 3.5$ cm, that is, when a waveguide with λ_{max} = 4.4 cm is used $\mathscr{E}(R)$ is plotted for a narrow section of the microwave spectrum: 3.5 cm $< \lambda <$ 4.4 cm. The measurements were

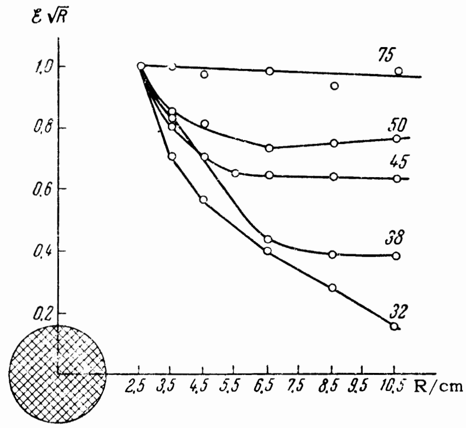

FIG. 8. Dependence of the amplitude of the microwave field ℰ on the distance R to the center of the plasma. The numbers on the curves denote the value of the field E_θ in V/cm.

made for different values of E_θ—from the smallest to those at which saturation of the current and of the microwave signal was observed. In Fig. 8, the initial points of all the curves are normalized to unity. The abscissas represent the distances R, and the ordinates the products $\mathscr{E} R^{1/2}$. When R is small compared with the large radius of the torus, the radiation can be regarded as cylindrically symmetrical. In this case it would be natural to expect the amplitude of the microwave signal to have a fall-off that is characteristic of cylindrical geometry

$$\mathscr{E} \sim R^{-1/2}, \tag{10}$$

corresponding in the coordinates of Fig. 8 to straight lines parallel to the abscissa axis. Such a fall-off was observed earlier by Suprunenko et al[6] in investigations of microwave radiation from a direct discharge with runaway electrons that cause a two-stream instability. In our setup, Eq. (10) holds true at large values of E_θ (curve 1 of Fig. 8). However, the smaller R, the larger the deviations from this variation of signal fall-off (curves 2—5 on Fig. 8). At small values of E_θ, the fall-off of $\mathscr{E}$ with distance is described more readily by an exponential law rather than by (10).

The experimental results obtained with respect to the microwave signals from the plasma can be interpreted theoretically in the following fashion. Aamodt and Drummond[7] have shown that in systems with a high level of plasma oscillations the nonlinear interaction between the oscillations leads to emission of electromagnetic waves from the plasma, with frequency $2\omega_{pe}$, and estimated the energy Q of the electromagnetic waves radiated from a unit volume of plasma in a unit time:

$$Q \sim 10\left(\frac{T_e}{mc^2}\right)^{5/2}\left(\frac{W}{nT_e}\right)^2 nT\omega_{pe}. \tag{11}$$

At values R small compared with the large radius of the torus, the geometry of the plasma pinch differs little from cylindrical, and we can write for the amplitude $\mathscr{E}$ of the electric field of the electromagnetic wave at a distance R from the plasma axis the following relation:

$$\frac{Qr^2}{2R} = \frac{c}{8\pi}\mathscr{E}^2, \tag{12}$$

where r—small radius of the torus and c—velocity of light. Solving (11) and (12) simultaneously with (4), we find that when R = 5 cm and T $\sim 10^3$ eV the amplitude of the electric field of the wave is ~ 3 V/cm, in good agreement with the data of Fig. 7.

From (11) and (12) follows also the relation

$$\mathscr{E} \sim mnu^2, \tag{13}$$

which apparently explains the character of curve 2 of Fig. 7. Finally, from (12) we obtain the well-known law for the fall-off of a cylindrical wave (10), which is experimentally verified [see Fig. (8)] for all R at large values of E_θ and for large distances R in the case of small E_θ.

After determining experimentally that the wavelength λ of the plasma microwave radiation exceeds 3.5 cm, and recognizing that in accordance with[7] the frequency of the microwave radiation is equal to $2\omega_{pe}$, we find that the density of the radiating plasma is $\sim 3 \times 10^{11}$ cm^{-3}. Apparently the radiation comes from the plasma periphery.

The experimentally observed deviations from (10) can apparently be attributed to the surface waves which occur in this case, and which have already been considered theoretically (see, for example,[8]). Inasmuch as the nonlinear interactions of the waves generated in the case of a two-stream instability excite other types of waves, surface waves can also be excited with characteristic frequency $\omega_{pe}/\sqrt{2}$. The electric field of the surface wave should decrease outside the plasma like

$$\mathscr{E} \sim R^{-1/2}\exp(-kR), \tag{14}$$

where k—wave vector of the surface wave in the direction of the plasma axis. Obviously, the surface waves make a noticeable contribution to the microwave field intensity only at a distance R comparable with the lengths of the surface waves, and at larger distances the microwave field is determined by the volume radiation and decreases like (10) with increasing R.

The deviations from (10) observed for small R cannot be attributed to the effect of the quasi-stationary zone of volume microwave radiation. If this deviation were to be due to this effect, then its magnitude would be the same on all the curves of Fig. 8, regardless of the value of E_θ (in other words, regardless of the radiation power).

To estimate the energy of the surface waves we can start from the fact[9] that the nonlinear interaction of the waves leads to a uniform distribution of the energy over the degrees of freedom of the collective oscillations of the plasma. It follows therefore that the energy of the surface waves is

$$W_{\text{sur}} \approx \mathscr{E}^2_{\text{sur}}/4\pi$$

and is proportional to the energy of the plasma wave, equal to $mnu^2/2$. In such a case

$$\mathscr{E}_{\text{sur}} \sim u, \qquad (15)$$

whereas, according to (13), the field of the electromagnetic waves radiated by the volume of the plasma is

$$\mathscr{E}_{\text{vol}} \sim u^2.$$

From a comparison of (13) and (15) it follows that at small values of translational electron velocity u, and consequently at small values of E_θ the electric field of the surface waves, with a characteristic fall-off given by (14), should predominate near the plasma surface, whereas for large u and E_θ there should predominate electromagnetic waves radiated from the volume of the plasma, with fall-off (10). These considerations explain quite well the data of Fig. 8.

In conclusion the authors thank E. K. Zavoĭskiĭ for continuous interest in this work, L. I. Rudakov for valuable discussions of the theoretical problems, P. I. Blinov and V. A. Skoryupin for supplying individual measuring instruments, and A. E. Bazhenov and M. K. Volodin for help in preparing the setup.

[1] Fanchenko, Demidov, Elagin, and Ryutov, JETP **46**, 497 (1964), Soviet Phys. JETP **19**, 337 (1964).

[2] B. A. Trubnikov, Voprosy teorii plazmy (Problems of Plasma Theory), Gosatomizdat, No. 1, 163, 1963.

[3] Babykin, Gavrin, Zavoĭskiĭ, Rudakov, and Skoryupin, JETP **43**, 411 and 1547 (1962), Soviet Phys. JETP **16**, 295 and 1092 (1963).

[4] V. D. Shafranov, op. cit.[2], No. 3, 44, 1963.

[5] A. A. Vedenov, ibid. No. 3, 233, 1963.

[6] Suprunenko, Faĭnberg, Tolok, Sukhomlin, Reva, Burchenko, Rudnev, and Volkov, Atomnaya ĕnergiya **14**, 349 (1963).

[7] R. E. Aamodt and W. E. Drummond, J. Nucl. Energy **C6**, 147 (1963/4).

[8] Ya. B. Faĭnberg and M. F. Gorbatenko, ZhTF **29**, 549 (1959), Soviet Phys. Tech. Phys. **4**, 487 (1959).

[9] Dikasov, Rudakov, and Ryutov, JETP **48**, 913 (1965), translation in press.

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