SPIN OF VIRTUAL GRAVITONS

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The problem of the spin of an interacting gravitational field is discussed within the framework of linearized gravitational theory. It is shown that virtual gravitons can carry an angular momentum of 2 or 0. Owing to the transversality of the gravitational vertex, only gravitons of spin 2 with a chirality ±2 are emitted. It is shown that gauge invariance plays the role of an auxiliary condition in gravidynamics: only gauge-invariant fields interact with matter. This result is generalized to the case of fields characterized by higher spins and mass zero.

GAUGE invariance in electrodynamics plays the role of a peculiar auxiliary condition restricting the possible values of spin of virtual quanta [1]. The vector potential of the electromagnetic field can be separated into two parts corresponding to spin 1 and 0, and it then turns out that quanta of spin 0 which are not gauge invariant do not interact with the charged particle current. The equations of the general theory of relativity in the approximation linear in the gravitational constant k ($k = 0.59 \times 10^{-38} m_p^{-2}$; m_p is the proton mass, $\hbar = c = 1$) are invariant [2] with respect to the transformation (4) which is analogous to the gauge transformation in electrodynamics. From the formal point of view this similarity is related to the fact that the equations of the free gravitational field coincide in the linear approximation with the equations for particles of spin 2 and mass 0 [3]. But, gauge transformations exist for fields characterized by arbitrary nonzero spin and mass 0 [4], and invariance with respect to these transformations is preserved also when interactions are taken into account both in electrodynamics and in gravidynamics.

In this paper it is shown that within the framework of a linearized theory of gravitation the spin of an interacting gravitational field is equal to 2 or 0 [1)]. Fields corresponding to these spins are gauge-invariant in contradistinction to a field of spin 1 which does not interact with matter. Since a virtual graviton can carry two values of angular momentum, the propagation function for a graviton can be represented in the form of a sum of two Green's functions associated with an exchange of a graviton of definite spin 2 or 0. The correctness of such a decomposition is confirmed by an investigation of the simplest diagrams.

Thus, an interacting gravitational field is characterized by six independent spin states in contradistinction to free gravitons which have only two independent orientations of spin. It is shown that because of the transversality of the gravitational vertex only gravitons of spin 2 and helicity ±2 can be emitted. At the end of this paper we discuss the spin structure of an interacting field of spin s and mass 0. Virtual particles of such a field can carry angular momentum $s - 2k$ ($k = 0, 1, 2, \ldots$).

1. The linearized theory of gravitation has been investigated by a number of authors (cf., for example, [2,5,6]). In this theory the gravitational field is described by the small deviations $h_{\mu\nu}$ of the metric tensor $g_{\mu\nu}$ from the Galilean values:

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}, \quad \delta_{00} = -\delta_{11} = -\delta_{22} = -\delta_{33} = 1; \quad (1)$$

$h_{\mu\nu}$ satisfy the equations

$$h_{\mu\lambda,\,\nu\lambda} + h_{\nu\lambda,\,\mu\lambda} - h_{\lambda\lambda,\,\mu\nu} - h_{\mu\nu,\,\lambda\lambda} + \delta_{\mu\nu}(h_{\lambda\lambda,\,\rho\rho} - h_{\lambda\rho,\,\lambda\rho}) = 16\pi k\theta_{\mu\nu},$$

$$h_{\mu\lambda,\,\lambda\nu} = \delta_{\lambda\rho}\frac{\partial^2 h_{\mu\rho}}{\partial x_\nu \partial x_\lambda}, \qquad h_{\lambda\lambda} = \delta_{\lambda\rho}h_{\rho\lambda}, \quad (2)$$

where $\theta_{\mu\nu}$ is the symmetric energy-momentum tensor for matter.

Under the transformation of coordinates

$$x_\mu' = x_\mu + \xi_\mu \quad (3)$$

(ξ_μ is an arbitrary vector field) $h_{\mu\nu}$ goes over into $h'_{\mu\nu}$:

$$h'_{\mu\nu} = h_{\mu\nu} - \xi_{\mu,\,\nu} - \xi_{\nu,\,\mu}. \quad (4)$$

The requirement of invariance under this trans-

[1)]This result has also been obtained by V. I. Ogievetskii and I. V. Polubarinov (private communication).

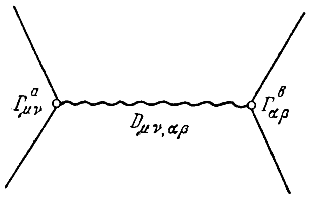

FIG. 1

formation leads to the energy-momentum conservation law, which in the linear approximation has the form

$$\theta_{\mu\nu,\,\mu} = 0. \tag{5}$$

In diagram language equation (5) means that the gravitational vertex is transverse (cf., Fig. 1)

$$q_\mu \Gamma_{\mu\nu} = 0,$$

$$\Gamma_{\mu\nu}(p_1 p_2 q)\,\delta^4(p_1 - p_2 - q) = \frac{1}{(2\pi)^4}\int d^4x\,\langle 2\,|\,\theta_{\mu\nu}(x)\,e^{iqx}\,|\,1\rangle,$$

where $|1\rangle$, $\langle 2|$ are state vectors for particles with momenta p_1, p_2.

The Green's function for a graviton has been obtained in the paper by Gupta[5]:

$$D_{\mu\nu,\,\alpha\beta}(x) = \frac{1}{(2\pi)^4}\int D_{\mu\nu,\,\alpha\beta}(q)\,e^{iqx} d^4q,$$

$$D_{\mu\nu,\,\alpha\beta}(q) = q^{-2}(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \delta_{\mu\nu}\delta_{\alpha\beta}). \tag{6}$$

We note that formula (6) follows from the relationship between the gravitational potentials and $D_{\mu\nu,\alpha\beta}$[6]:

$$h_{\mu\nu} = 8\pi k D_{\mu\nu,\,\alpha\beta}\Gamma_{\alpha\beta}. \tag{7}$$

The general form (the longitudinal terms are of no importance because the vertex is transverse) is

$$q^2 D_{\mu\nu,\,\alpha\beta} = a(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha}) - b\delta_{\mu\nu}\delta_{\alpha\beta}; \quad a,\ b = \text{const.}$$

In the static limit ($q_0 \approx 0$)

$$\Gamma_{00} = m; \quad \Gamma_{0i} = {}^1\!/_2[\mathbf{Mq}]_i \quad (i = 1,\ 2,\ 3)$$

(M is the mechanical angular momentum), and (7) leads to the well known expression[2] for the field produced by a rotating body only for $a = b = 1$.

2. It is clear that the symmetric tensor $h_{\mu\nu}$ represents a superposition of fields with spins 2, 1, 0:

$$h_{\mu\nu} = h_{\mu\nu}^{(2)} + h_{\mu\nu}^{(1)} + h_{\mu\nu}^{(0)}. \tag{8}$$

The expansion (8) can also be obtained explicitly with the aid of an invariant operator for the square of the spin Γ^2 utilized by Ogievetskiĭ and Polubarinov[1,7] (further references are given there):

$$\Gamma^2 = -\Box^{-1}({}^1\!/_2\Sigma_{\mu\nu}\Sigma_{\mu\nu}p_\lambda^2 - \Sigma_{\mu\lambda}\Sigma_{\mu\rho}p_\lambda p_\rho),$$
$$(\Gamma^2)_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta}^{(l)} = l(l+1)\,h_{\mu\nu}^{(l)} \qquad (l = 0,\ 1,\ 2), \tag{9}$$

where $p_\lambda = -i\partial/\partial x_\lambda$, $\Sigma_{\mu\nu}$ are generators of the Lorentz rotations of the components of the tensor $h_{\mu\nu}$:

$$2i(\Sigma_{\mu\nu})_{\alpha\beta}{}^{\gamma\delta} = \delta_{\gamma\alpha}(\delta_{\mu\delta}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\delta}) + \delta_{\delta\beta}(\delta_{\mu\gamma}\delta_{\nu\alpha} - \delta_{\mu\alpha}\delta_{\nu\gamma})$$
$$+ \delta_{\delta\alpha}(\delta_{\mu\gamma}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\gamma}) + \delta_{\gamma\beta}(\delta_{\mu\delta}\delta_{\nu\alpha} - \delta_{\mu\alpha}\delta_{\nu\delta}),$$

$\Box^{-1}$ is the integral operator:

$$\Box^{-1}f(x) = \int D_F(x - x')\,f(x')\,d^4x',$$

where $D_F(x - x')$ is the causal Green's function for the d'Alembertian.

We introduce the projection operator Π^l ($l = 0, 1, 2$):

$$\Pi^{l_1} = \frac{[\Gamma^2 - l_2(l_2+1)][\Gamma^2 - l_3(l_3+1)]}{[l_1(l_1+1) - l_2(l_2+1)][l_1(l_1+1) - l_3(l_3+1)]},$$
$$l_1 \neq l_2 \neq l_3, \qquad \{l_1, l_2, l_3\} \sim \{0, 1, 2\}. \tag{10}$$

It follows from (8) and (10) that

$$(\Pi^l)_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} = h_{\mu\nu}^{(l)}.$$

Calculation leads to the following results:

$$h_{\mu\nu}^{(2)} = h_{\mu\nu} - \Box^{-1}(h_{\alpha\mu,\,\alpha\nu} + h_{\alpha\nu,\,\alpha\mu}) + {}^1\!/_3\Box^{-1}h_{\alpha\alpha,\,\mu\nu}$$
$$+ {}^1\!/_3\delta_{\mu\nu}(\Box^{-1}h_{\alpha\beta,\,\alpha\beta} - h_{\alpha\alpha}) + {}^2\!/_3\Box^{-2}h_{\alpha\beta,\,\alpha\beta\mu\nu},$$
$$h_{\mu\nu}^{(1)} = \Box^{-1}(h_{\alpha\mu,\,\alpha\nu} + h_{\alpha\nu,\,\alpha\mu}) - 2\Box^{-2}h_{\alpha\beta,\,\alpha\beta\mu\nu},$$
$$h_{\mu\nu}^{(0)} = {}^1\!/_3h_{\alpha\alpha}\delta_{\mu\nu} - {}^1\!/_3\delta_{\mu\nu}\Box^{-1}h_{\alpha\beta,\,\alpha\beta} - {}^1\!/_3\Box^{-1}h_{\alpha\alpha,\,\mu\nu}$$
$$+ {}^4\!/_3\Box^{-2}h_{\alpha\beta,\,\alpha\beta\mu\nu}; \tag{11}$$

$h_{\mu\nu}^{(2)}$, $h_{\mu\nu}^{(1)}$ have the right number of independent components—5 and 3—since they satisfy the auxiliary condition $h_{\mu\mu}^{(2)} = h_{\mu\nu,\mu}^{(2)} = 0$, $h_{\mu\nu,\mu\nu}^{(1)} = 0$. There are two fields with spin 0, since a scalar can be formed from a tensor by two methods— $h_{\alpha\alpha}$, $h_{\alpha\beta,\alpha\beta}$. Multiplication of these quantities by $\delta_{\mu\nu}$ or differentiation with respect to x_μ does not, of course, alter the fact that they describe scalar particles, but correspond to different Lagrangians for the interaction of these fields with matter.

A field of spin 1 has one "extra" index which can arise only as a result of differentiation. Therefore, it is from the outset clear that

$$h_{\mu\nu}^{(1)} = B_{\mu,\,\nu} + B_{\nu,\,\mu}, \tag{12}$$

B_μ is a vector field. Substitution of (12) into the left hand side of equation (2) shows that because of its tensor properties alone $h_{\mu\nu}^{(1)}$ satisfies a homogeneous equation independently of $\theta_{\mu\nu}$. It is therefore clear that a field of spin 1 does not interact with matter and is completely determined by the initial conditions. The same can be said also with respect to a part of the field 0. Physical meaning can be ascribed to $h_{\mu\nu}^{(2)}$ and $h_{\mu\nu}^{(0)\prime}$, where

$$h_{\mu\nu}^{(0)\prime} = {}^1\!/_3\delta_{\mu\nu}(h_{\alpha\alpha} - \Box^{-1}h_{\alpha\beta,\,\alpha\beta}). \tag{13}$$

For a given momentum the virtual gravitons are characterized by six independent spin states—five for spin 2 and one for spin 0. Thus, the number of independent states differs by four from the number of components of the tensor $h_{\mu\nu}$, and this is consistent with the existence of four independent transformations (4).

Just as in electrodynamics [1], the physical fields are gauge invariant, while the Lagrangian of the free gravitational field can be expressed only in terms of $h_{\mu\nu}^{(2)}$ and $h_{\mu\nu}^{(0)\prime}$. The invariance of fields of higher spin (1 in electrodynamics and 2 in gravidynamics) already follows from the fact that the gauge tensor (scalar or vector respectively) does not contain fields of higher spin. In contrast to the electromagnetic field the gravitational field carries, generally speaking, two values of angular momentum. Exceptions occur in cases when $\theta_{\alpha\alpha} = 0$, for example, for fields of arbitrary spin s ($\neq 0$) m = 0 [8]. Then $h_{\mu\nu}$ can be subjected to the same requirements as in the case of the free gravitational field $h_{\alpha\alpha} = 0$, $h_{\mu\nu,\mu} = 0$, and the spin of the interacting field is equal to the spin of the free field [7], i.e., 2.

3. We now check the results obtained above directly by means of diagrams. As examples we shall consider the gravitational annihilation of a pair of scalar particles and the vacuum transitions of particles of different spin into gravitons. In order to separate the contributions of the scalar and the quadrupole gravitons we represent the Green's function for a graviton in the form of a sum of Green's functions associated with an exchange of a graviton of definite spin $d^{ll}_{\mu\nu,\alpha\beta}$:

$$D_{\mu\nu,\,\alpha\beta} = d^{22}_{\mu\nu,\,\alpha\beta} + d^{11}_{\mu\nu,\,\alpha\beta} + d^{00}_{\mu\nu,\alpha\beta}.$$

In the case of electrodynamics the analogous equation has the form

$$q^{-2}\delta_{\mu\nu} = d_{\mu\nu}{}^{11} + d_{\mu\nu}{}^{00},$$

where $d^{11}_{\mu\nu}$ is the Green's function for the photon in the Landau gauge:

$$d^{11}_{\mu\nu} = \frac{1}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \tag{14}$$

The commutation relations for the creation operators for gravitons of definite spin follow from (6) and (11):

$$q^{-2}d^{22}_{\mu\nu,\,\alpha\beta} = d^{11}_{\mu\alpha}d^{11}_{\nu\beta} + d^{11}_{\mu\beta}d^{11}_{\nu\alpha} - {}^2\!/_3 d^{11}_{\mu\nu}d^{11}_{\alpha\beta}, \tag{15}$$

$$d^{11}_{\mu\nu,\,\alpha\beta} = q_\mu q_\alpha d^{11}_{\nu\beta} + q_\mu q_\beta d^{11}_{\nu\alpha} + q_\nu q_\alpha d^{11}_{\mu\beta} + q_\nu q_\beta d^{11}_{\mu\alpha},$$

$$q^2 d^{00}_{\mu\nu,\,\alpha\beta} = -\frac{1}{3}\delta_{\mu\nu}\delta_{\alpha\beta} - \frac{2}{3}\delta_{\mu\nu}\frac{q_\alpha q_\beta}{q^2} - \frac{2}{3}\delta_{\alpha\beta}\frac{q_\mu q_\nu}{q^2} + \frac{8}{3}\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4}. \tag{16}$$

$d^{22}_{\mu\nu,\alpha\beta}$ is analogous to the Green's function for particles characterized by s = 2 and m ≠ 0 obtained by Fierz [9]; $d^{11}_{\mu\nu,\alpha\beta}$ represents the propagation function for particles of spin 1 in the case of derivative coupling, which can be easily understood taking (12) into account.

In practical calculations the longitudinal terms can be discarded:

$$D_{\mu\nu,\,\alpha\beta} = D^{22}_{\mu\nu,\,\alpha\beta} + D^{00}_{\mu\nu,\,\alpha\beta}, \qquad q^2 D^{00}_{\mu\nu,\,\alpha\beta} = -{}^1\!/_3\delta_{\mu\nu}\delta_{\alpha\beta},$$

$$q^2 D^{22}_{\mu\nu,\,\alpha\beta} = \delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\nu\alpha}\delta_{\mu\beta} - {}^2\!/_3\,\delta_{\mu\nu}\delta_{\alpha\beta}.$$

The gravitational annihilation of a pair of scalar particles into another pair of scalar particles but of different mass is described by the diagram of Fig. 1 where

$$\Gamma_{\mu\nu}{}^{a,\,b} = \varphi_2{}^*[2p_\mu{}^{a,\,b}p_\nu{}^{a,\,b} - {}^1\!/_2(q_\mu q_\nu - q^2\delta_{\mu\nu})]\varphi_1,$$
$$p_\mu{}^{a,\,b} = {}^1\!/_2(p_1{}^{a,\,b} + p_2{}^{a,\,b})_\mu.$$

In the center of inertia system

$$p_1{}^a = (\omega, \mathbf{p}); \quad -p_2{}^a = (\omega, -\mathbf{p});$$

$$p_1{}^b = (\omega, \mathbf{t}); \quad -p_2{}^b = (\omega, -\mathbf{t});$$

$$\omega^2 - \mathbf{p}^2 = m_a{}^2; \quad \omega^2 - \mathbf{t}^2 = m_b{}^2.$$

The matrix element is

$$M_1 \sim \Gamma^a_{\mu\nu}D^{22}_{\mu\nu,\,\alpha\beta}\Gamma^b_{\alpha\beta} + \Gamma^a_{\mu\nu}D^{00}_{\mu\nu,\,\alpha\beta}\Gamma^b_{\alpha\beta}.$$

Expressing M_1 in terms of the angle θ between the vectors **p**, **t** we obtain

$$\Gamma^a_{\mu\nu}D^{22}_{\mu\nu,\,\alpha\beta}\Gamma^b_{\alpha\beta} \sim P_2(\cos\theta),$$
$$\Gamma^a_{\mu\nu}D^{00}_{\mu\nu,\,\alpha\beta}\Gamma^b_{\alpha\beta} \sim P_0(\cos\theta), \tag{17}$$

where $P_{2,0}$ are Legendre polynomials.

If the initial state of the pair is characterized by a definite relative angular momentum L, then the matrix element differs from 0 only for L = 2 or 0.

From considerations of tensor dimensionality the vacuum transition (Fig. 2) of a scalar particle with four-momentum q into a graviton corresponds to the vertex

$$\Gamma_{\mu\nu} \backsim a(q^2)q_\mu q_\nu + b(q^2)q^2\delta_{\mu\nu}.$$

In virtue of the transversality condition $a(q^2) = -b(q^2)$. Just as in the preceding example, the matrix element is proportional to the sum of two

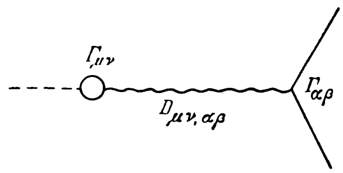

FIG. 2

expressions:

$$M_2 \sim \Gamma_{\mu\nu} D^{22}_{\mu\nu,\,\alpha\beta} \Gamma_{\alpha\beta} + \Gamma_{\mu\nu} D^{00}_{\mu\nu,\,\alpha\beta} \Gamma_{\alpha\beta}.$$

The first term is in fact equal to 0, since

$$\Gamma_{\mu\nu} D^{22}_{\mu\nu,\,\alpha\beta} \Gamma_{\alpha\beta} \sim q_\alpha q_\beta \Gamma_{\alpha\beta} = 0.$$

Thus, we have obtained the natural result that a scalar particle goes over into a graviton which is also scalar.

In considering analogous processes for a vector (or a tensor) field A_μ ($A_{\mu\nu}$) it should be taken into account that virtual particles possess, generally speaking, spins 1 and 0 (2, 1, 0). It can be easily verified that scalar particles of the field A_μ ($A_{\mu\nu}$) go over again into scalar gravitons, while particles of spin 2 of the field $A_{\mu\nu}$ go over into gravitons of spin 2.

4. In contrast to an interacting field the spin of a free gravitational field is equal to 2 [5], and this agrees with the quadrupole character of classical gravitational radiation. Moreover, like all particles of mass 0 [4], the free graviton is characterized by only two independent polarization states [5]. We shall show, following Feynman's discussion [10] of the problem of virtual and real quanta, that of the six types of virtual gravitons only particles of chirality ±2 can be emitted.

The diagram of Fig. 1 corresponds to the matrix element M_3:

$$M_3 = \frac{8\pi k}{\omega^2 - \mathbf{q}^2}[2\Gamma^a_{\mu\nu}\Gamma^b_{\mu\nu} - \Gamma^a_{\mu\mu}\Gamma^b_{\nu\nu}],$$

where ω is the energy of the virtual graviton, while $\mathbf{q}$ is its momentum, $\mathbf{q} \sim \{0, 0, q_3\}$. We transform this expression utilizing the condition of transversality

$$M_3 = \frac{8\pi k}{\omega^2 - q_3^2}[(\Gamma_{11} - \Gamma^a_{22})(\Gamma^b_{11} - \Gamma^b_{22}) + 4\Gamma^a_{12}\Gamma^b_{12}]$$

$$- \frac{1}{q_3^2}(\Gamma^a_{00}\Gamma^b_{00} + \Gamma^a_{11}\Gamma^b_{00} + \Gamma^b_{11}\Gamma^a_{00} + \Gamma^a_{22}\Gamma_{00} + \Gamma^b_{22}\Gamma^a_{00}$$

$$- 4\Gamma^a_{10}\Gamma^b_{10} - 4\Gamma^a_{20}\Gamma^b_{20} - \Gamma^a_{30}\Gamma^b_{30}).$$

A real graviton differs from a virtual one by the fact that for it $\omega = q_3$. If we assume that the graviton is almost real, i.e., that it is absorbed "very far away" from the point of emission, then $\omega \approx q_3$ and in the matrix element we can retain only the pole term:

$$M_3' = \frac{8\pi k}{\omega^2 - q_3^2}[(\Gamma_{11} - \Gamma^a_{22})(\Gamma^b_{11} - \Gamma^b_{22}) + 4\Gamma^a_{12}\Gamma^b_{12}]. \quad (18)$$

On the other hand, the exchange of gravitons of chirality ±2 corresponds to the matrix element M:

$$M_4 = \sum_{i=1,2} \gamma_{\alpha\beta}{}^i \Gamma_{\alpha\beta}{}^a \frac{16\pi k}{\omega^2 - q_3^2} \gamma_{\mu\nu}{}^{i*} \Gamma_{\mu\nu}{}^b,$$

where $\gamma^{1,2}_{\alpha\beta}$ are the polarization tensors (i = 1, 2):

$$\gamma_{\alpha\beta}{}^1 = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \qquad \gamma_{\alpha\beta}{}^2 = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & -1 \end{pmatrix}.$$

Since $M_3' = M_4$, this shows that the only gravitons that can be emitted are those whose component of angular momentum along the linear momentum is given by ±2.

We note that the nonpolar term neglected in formula (18) is responsible for various static effects, and, in particular, for the Newtonian interaction between masses.

5. The results concerning the spin structure of an interacting gravitational field can be easily generalized to the case of particles of arbitrary spin and of mass 0. Free fields of spin s are described by a symmetric tensor of the s-th rank $A_{\mu_1 \ldots \mu}$ ($A_{\mu_1\mu_2\ldots\mu_s} = 0$). Under a gauge transformation [4] $A_{\mu_1\ldots\mu_s}$ go over into $A'_{\mu_1\ldots\mu_s}$:

$$A'_{\mu_1\ldots\mu_s} = A_{\mu_1\ldots\mu_s} + \partial_{\mu_1}\xi_{\mu_2\ldots\mu_s} + \ldots + \partial_{\mu_s}\xi_{\mu_1\ldots\mu_{s-1}}, \quad (19)$$

where $\xi_{\mu_1\ldots\mu_{s-1}}$ is a symmetric tensor of rank s − 1. If the theory is invariant with respect to (19) also when the interaction is taken into account, then there exists a certain conservation law and the vertex is transverse.

A simple generalization of equations (12) and (14) shows that the Green's functions corresponding to an exchange of a virtual particle of spin s − 2k + 1, k = 1, 2, . . . , are proportional to q, and because the vertex is transverse these fields give no contribution to the interaction. Similarly, in expressions for fields of spin s − 2k, k = 1, 2, . . . we have to take into account only terms containing a product of k δ-symbols [cf. (13)].

In Sec. 2 it was shown that the symmetric tensor of the second rank $h_{\mu\nu}$ describes one field of spin 2 and 1 and two fields of spin 0. In the general case the symmetric tensor of the s-th rank $A_{\mu_1\ldots\mu_s}$ describes one field of spin s and s − 1, two fields of spin s − 2 and s − 3, three fields of spin s − 4 and s − 5 etc. Similarly, the gauge tensor $\xi_{\mu_1\ldots\mu_{s-1}}$ represents a superposition of fields of spin s − 1 and s − 2, two fields of spin s − 3 and s − 4, etc. Therefore, there exists one gauge-invariant field corresponding to each value of the spin $s-2k$ ($k = 0, 1, \ldots$) and there are no fields of spin $s-2k+1$ ($k = 1, 2, \ldots$) which are invariant with respect to (19). The proof of the fact that it is just this gauge-invariant combination that appears in front of the product of k δ-symbols is somewhat more complicated and is based on the explicit form of the operator Γ^2 for arbitrary spin and on the generalized formula (10).

Thus, an interacting field $A_{\mu_1 \ldots \mu_s}$ of mass 0 carries, generally speaking, an angular momentum $s-2k$ ($k = 0, 1, 2, \ldots$), and the fields corresponding to these spins are gauge invariant.

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