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ANGULAR MODES IN A GAS LASER

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The difference-frequency spectrum of modes in a helium-neon gas laser with plane-parallel mirrors was investigated. Several beat frequencies were observed. The beat frequencies were identified as difference frequencies between the fundamental mode and those angular modes with smallest indices characteristic of the given cavity.

IT has been shown in several theoretical papers[1,2] that a large number of different types of modes can exist in optical masers. The modes may be classified as either fundamental modes, which exhibit the lowest even order radial symmetry in the electromagnetic-field distribution, or angular modes, which exhibit higher-order even and odd radial symmetry.

The fundamental modes that are set up in real cavities with plane-parallel and confocal circular mirrors were studied experimentally in the papers of Herriott [3], Bennet [4], and others. Besides the fundamental modes in He-Ne lasers with plane-parallel mirrors, there has been experimental detection of only one angular mode exhibiting the lowest odd-order radial symmetry [3]. It is therefore of interest to obtain laser action in angular modes with higher even and odd order radial symmetry for the case of resonators with plane-parallel mirrors; it is also of interest to compare the frequencies of these modes with the theoretical predictions [2].

Since according to theory the frequencies and loss factors of the modes are determined by the resonator parameters (its length and the diameter of its mirrors) we chose in the present work a cavity with a considerably larger Fresnel number than the cavity described by Herriott, and we have examined its frequency spectrum.

EXPERIMENT

The technique of optical mixing (photo heterodyning) [5] was used to study the frequency spectrum of the laser. This method is based on the fact that the photomultiplier which detects the laser emission acts as a quadratic detector whose photocurrent contains a component

$$i \sim \mathbf{E}_1\mathbf{E}_2, \tag{1}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric field vectors of two waves of different frequencies.

A He-Ne gas laser with internal plane-parallel mirrors was used. The mirrors could be adjusted to within 0.2 sec of arc. The length of the cavity was 1 meter, the inside diameter of the tube and the effective diameter of the mirrors was 18 mm. The mirror surfaces were polished flat to 1/400 of the laser wavelength. The mirrors had multilayer dielectric coatings with a reflectivity of

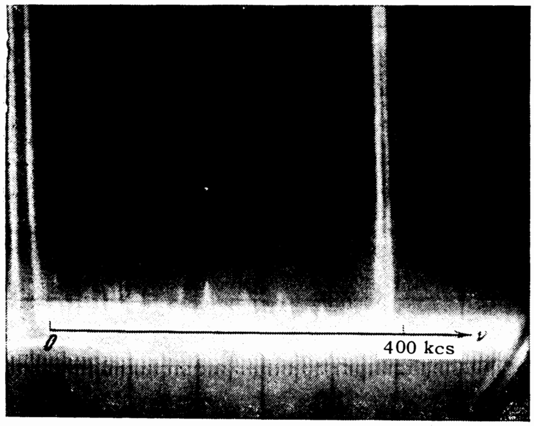

FIG. 1. The spectrum of white noise (frequency markers are shown at 0 and at 400 kc/sec).

98.5 ± 0.5% at $\sim 1.15\ \mu$. Due to the conditions of high-frequency excitation and to the composition of the mixture, we obtained laser action in only one line at 11523 Å.

The spectrum of the modes present in the laser was analyzed as follows.

The output of the laser was put through IFS filters and was incident on the photocathode of a FÉU-28 photomultiplier. The signal from the small (1 kΩ) anode load of the photomultiplier was fed into a wide-band video amplifier with a 10 Mc/sec bandwidth and a gain of 200 and then into an S4-8 panoramic spectrum analyzer, which displayed the spectrum of the oscillations in the photomultiplier current. This arrangement permitted detection of difference frequencies from 20 kc/sec to 10 Mc/sec with an accuracy of ±5 kcs. Figure 1 is a photograph of the spectrum-analyzer display, showing the white-noise spectrum when the photomultiplier was illuminated with incoherent light. When the laser output was incident on the photocathode of the photomultiplier, a number of peaks were observed on the spectrum-analyzer display.

The first of these maxima (cf. Fig. 2a) is the spectrum of the difference frequencies which fall within the linewidth of the oscillator. The increase in the oscillation linewidth due to changes in the angular position of the mirror from 0 to $3'$ is shown in Fig. 2b.

The remaining peaks have frequencies of approximately 300, 600, and 3400 kc/sec (see Fig. 3). We note the following experimental facts: 1. The beat intensity decreases with increasing beat frequency. 2. When the angle between the mirrors and the resonator is changed the position of the beat frequency shifts; this is illustrated in Figs. 4a, b, and c, for the beat frequency at ~ 300 kilocycles.

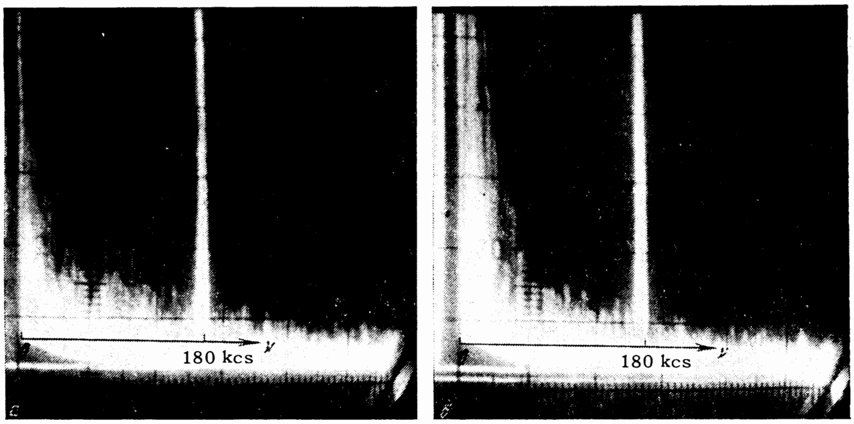

FIG. 2. Beats within the laser linewidth (for a single mode): a – for the minimum misalignment of the mirrors, b – for a mirror misalignment of $3''$ (the frequency markers are at 0 and 180 kc).

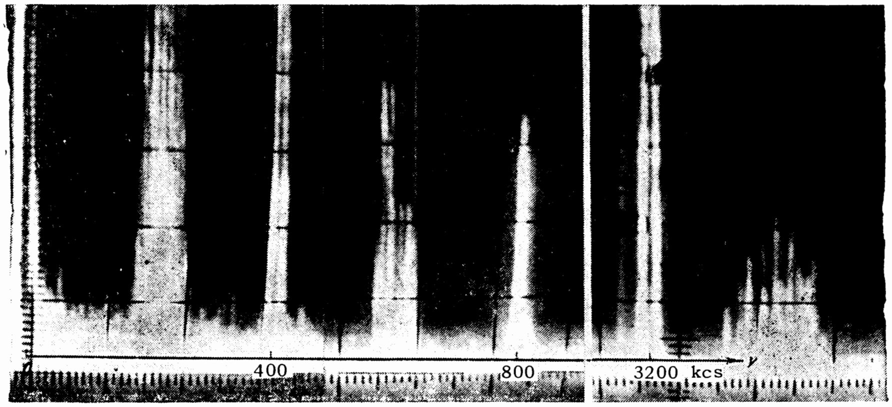

FIG. 3. Beats between the fundamental and angular modes at frequencies near 300, 600 an and 3400 kc/sec (the frequency markers are at 0, 400, 800 and 3200 kc/sec).

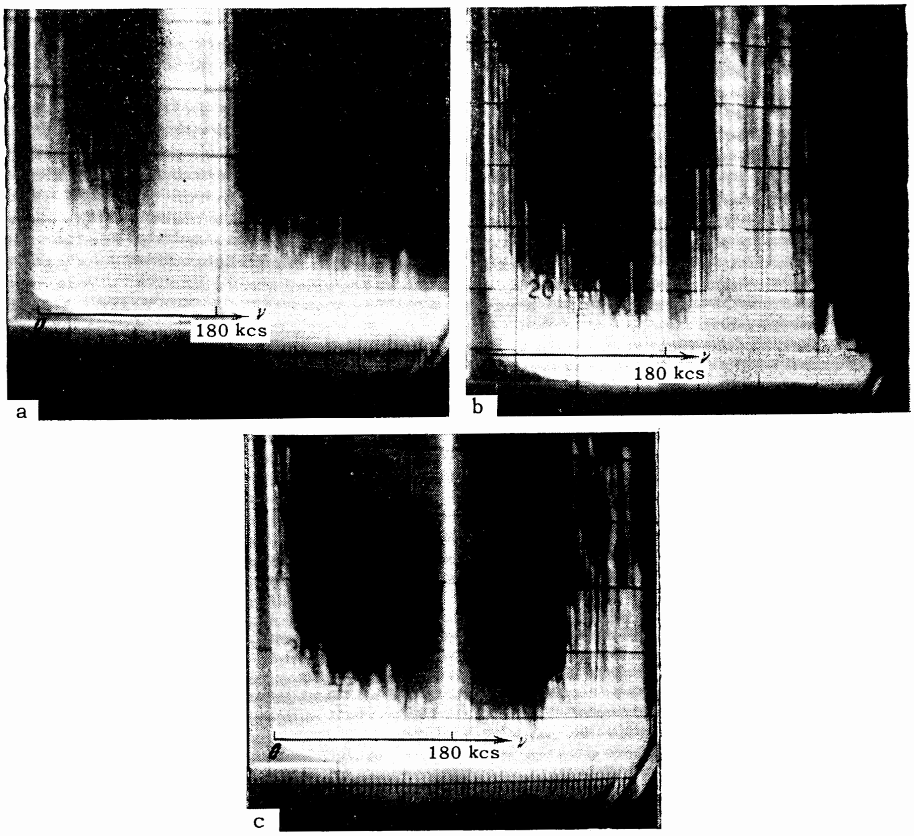

FIG. 4. Displacement of the beat frequency between the fundamental and first odd symmetric mode due to misalignment of the mirrors from zero (a), to 1″ (c) (frequency markers are shown at 0, 180 kc/sec).

3. Beat frequencies of 300 and 600 kilocycles were observed regularly. The beat at 3.4 Mc/sec appeared only after the mirrors had been in use for a long time; at this point a defect was discovered in the coating of one mirror.

DISCUSSION OF EXPERIMENTAL RESULTS

From the condition for oscillation in the fundamental modes it follows that the difference frequency between two fundamental modes ($\Delta\nu = c/4l$, where c is the velocity of light and $2l$ is the cavity length) lies in the vicinity of 150 Mc/sec. Consequently the beat frequencies we observed are not beats between fundamental, even-symmetry modes.

In order to decide which modes can exist in the resonator being investigated, one may use Vaĭnshteĭn's results [2], obtained using rigorous theory for the diffraction at the open end of a waveguide; these results give rather simple expressions for the natural frequencies of the modes and for their damping coefficients in cavities with flat, rectangular or circular mirrors.

According to this theory the frequency difference between two modes lying close to a given fundamental frequency may be expressed as follows

$$\Delta\nu_{m'n'-mn} = \frac{c}{2\pi l}\,\frac{M(M+2\beta)}{[(M+\beta)^2+\beta^2]^2}\,(\nu^2_{m'n'}-\nu^2_{mn}), \qquad (2)$$

where ν_{mn} is the natural frequency of a cavity mode; M is a quantity related to the Fresnel number N which depends on the effective diameter of the cavity mirror (Fig. 2a): $M = \sqrt{8\pi N}$, $N = ka^2/4\pi l$, $k = \omega/c$; ν_{mn} is the n-th root of the Bessel equation $J_n(v) = 0$, and $\beta = 0.824$.

The fractional energy loss for a single pass through the cavity for this mode is

$$\Lambda = 8\nu^2_{mn}\beta\,(M+\beta)/[(M+\beta)^2+\beta^2]^2. \qquad (3)$$

In the table we give estimates for the damping of several modes with the lowest indices which occur in the present cavity (I) and in the cavity described by Herriott [3] (II). We also show in the table estimates for the difference frequencies between those modes which have the lowest diffraction losses (E_{01q}, E_{11q}, E_{12q}, E_{21q}).

Noting that the losses due to transmission and absorption in the mirrors are about 1.5% and that the gain for the tube diameter used does not exceed 2–2.5% over the 500-millimeter length of the discharge, we expect the present cavity to generate modes having diffraction losses less than 0.5%. These modes are E_{01q}, E_{11q}, and E_{21q}. The difference frequencies of the beats between

Laser	Fresnel number	Mode	Damping in %	Beat between modes	Calculated beat freq. (kc/sec)	Observed beat freq. (kc/sec)
I	70	E_{01q}	0.05			
		E_{11q}	0.15	E_{01q} and E_{11q}	470	300
		E_{12q}	0.5	E_{01q} and E_{12q}	2400	—
		E_{13q}	1	E_{01q} and E_{21q}	1000	600
		E_{21q}	0.3			
		E_{22q}	0.7	E_{11q} and E_{21q}	600	
		E_{23q}	1.3			
II	20	E_{01q}	0.3	E_{01q} and E_{11q}	1580	1300
		E_{11q}	0.9	—	—	—
		E_{12q}	3	—	—	—
		E_{21q}	1.8	—	—	—

these modes, calculated according to Eq. (2), are equal to $\Delta\nu_{01q-11q} = 470$ kc/sec, $\Delta\nu_{01q-21q} = 1000$ kc/sec, and $\Delta\nu_{11q-21q} = 600$ kc/sec.

Since the amplitude of each of these angular modes is smaller than that of the fundamental mode [2] and since it is clear from the table that the losses are several times larger, the observation of photomixing [Eq. (1)] between two angular modes is rather improbable. Therefore we expected experimentally to obtain beats in the region of 300 and 600 kc/sec, with difference frequencies $\Delta\nu_{01q-11q}$ and $\Delta\nu_{01q-21q}$, respectively. The observation of a beat in the region of 3.4 Mc/sec may evidently be attributed to the presence of additional diffraction in the damaged part of the coatings, to the occurrence of new angular modes with lower diffraction losses, and to additional beat frequencies between the modes E_{01q} and E^{*}_{11q}.1)

It is striking that Herriott [3] observed only a single beat frequency, corresponding to beats between the fundamental and the first odd symmetric mode. As is clear from the table, this is due to appreciable increase in the diffraction losses for the angular modes with higher indices, which in turn is a consequence of the fact that the tube used by Herriott had a smaller effective diameter than ours.

It is clear from the table that the beat frequencies obtained experimentally in the present laser and in Herriott's laser differ from the theoretical estimates given by formula (2) by 30—50%. This kind of agreement between experiment and theory may be considered satisfactory.

The dependence observed of the beat frequencies on the angular position of the mirrors is qualitatively in agreement with the theoretical calculations of Fox and Li [6]. This dependence also supports the identification of the observed beat frequencies as due to difference frequencies between the fundamental mode and the lowest-order angular modes.

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1)The diameter of the zone with nonuniform coating was about 5 mm, for which N = 7.

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