ANNIHILATION OF HIGH-ENERGY ANTIBARYONS

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Annihilation of high-energy baryon-antibaryon pairs with formation of mesons and γ-quanta is considered. Expressions for the helicity amplitudes are obtained in two asymptotic regions; the amplitudes are used to calculate the differential cross section and various polarization coefficients. Isotopic relations between the cross sections are derived for various values of Reggeon isospin.

1. INTRODUCTION

A major step forward in the dynamic theory of strong interactions was the clarification of the connection between the asymptotic value of the amplitude $A(t, s)$ at high energies s and the singularities of the partial amplitudes $f_j(t)$ as functions of the angular momentum [1]. The asymptotic amplitudes of different processes were investigated under the assumption that the main contribution is made by the pole of the partial amplitude, viz: for elastic scattering through small angles by the vacuum pole, and for large-angle scattering by the pole whose trajectory describes some fermion family. An investigation of the many-particle terms of the unitarity condition [2,3] has made it possible to conclude that moving branch points can appear in the relativistic theory.

However, many features of the purely pole-type asymptotic values, namely the spin structure of the amplitudes, the factorization and isotopic relations, and the oscillatory behavior of the scattering amplitudes at large angles, are all valid also in the case when the analytic properties in the j-plane are more complicated than in the pole situation.

We consider in this paper the antibaryon annihilations $\overline{N} + N \to \pi + \gamma$ and $\overline{Y} + N \to K + \gamma$ at large energies and small final-particle production angles. From the point of view of the Regge-pole hypothesis, the amplitudes of these processes are determined by the contribution of the fermion Regge poles, the specific features of which were explained by Gribov [4]. For the processes in question, there are two asymptotic regions, which depend on the angles of production of the γ quantum and of the meson. One region ($t \to \infty$, $u = \text{const}$, $s \to -\infty$) corresponds to the production of a γ quantum in the direction of the antibaryon momentum, in which case the meson is emitted in the direction of the nucleon momentum. The second region ($t \to \infty$, $u \to -\infty$, $s = \text{const}$) corresponds to the production of a γ quantum in the direction of the nucleon momentum.

2. STRUCTURE OF THE AMPLITUDE IN THE s-CHANNEL

In order to obtain the asymptotic value of the amplitudes, for example in the second region, we start from partial expansions in the s-channel, use the Sommerfeld-Watson transformation, and separate the contribution of the outermost singularity.

In the s-channel the invariant amplitudes are determined in the following fashion [5]:

$$T_s = \bar{u}(p_2)\Gamma[\hat{\varepsilon}\hat{k}A_1(s,t,u) + ((\varepsilon p_1)(kp_2) - (\varepsilon p_2)(kp_1))A_2(s,t,u) - i(\hat{\varepsilon}(kp_1) - \hat{k}(\varepsilon p_1))A_3(s,t,u) - i(\hat{\varepsilon}(kp_2) - k(\varepsilon p_2))A_4(s,t,u)]u(p_1), \quad (1)$$

where $\Gamma = i\gamma_5$ if the internal parity of the (KNY) system is negative; $\Gamma = 1$ if this parity is positive; p_1, p_2—four-momenta of the nucleon and hyperon, ϵ—γ-quantum polarization vector, k—its four-momentum, $s = -(k + p_1)^2$, $u = -(k - p_2)^2$, $t = -(p_1 - p_2)^2$.

Inasmuch as the parity of the (KNY) system has not yet been established reliably, we investigate here both possibilities.

We consider first the case of negative (KNY) parity. We introduce the amplitude F_s in the center of mass system (c.m.s.) of the s-channel, normalized in such a way that the differential cross section is

$$d\sigma / d\Omega = |(\chi_Y{}^* F_s \chi_N)|^2 q_s / k_s.$$

The amplitude F_s has the following structure:

$$F_s = i(\boldsymbol{\sigma\varepsilon})F_{1s}(w_s, z_s) + i(\boldsymbol{\sigma}\hat{\mathbf{q}})(\boldsymbol{\sigma\varepsilon})(\boldsymbol{\sigma}\hat{\mathbf{k}})F_{2s}(w_s, z_s) + i(\boldsymbol{\sigma}\hat{\mathbf{k}})(\boldsymbol{\varepsilon}\hat{\mathbf{q}})F_{3s}(w_s, z_s) + i(\boldsymbol{\sigma}\hat{\mathbf{q}})(\boldsymbol{\varepsilon}\hat{\mathbf{q}})F_{4s}(w_s, z_s), \tag{2}$$

where $w_s = s^{1/2}$, $z \equiv \cos\theta_s$ (θ_s—meson production angle), $\hat{\mathbf{k}}$—unit vector along the photon momentum, $\hat{\mathbf{q}}$—unit vector along the meson momentum.

The scalar amplitudes F_{is} obey the symmetry conditions

$$F_{1s}(w_s, z_s) = F_{2s}(-w_s, z_s), \quad F_{3s}(w_s, z_s) = F_{4s}(-w_s, z_s). \tag{3}$$

We introduce the helicity amplitudes in the s-channel

$$f_{1s} \equiv \left(\tfrac{1}{2} 0 \,|F_s|\, \tfrac{1}{2} 1\right) = 2\sin\frac{\theta_s}{2}\sum_j f_1{}^j(\sqrt{s})[P'_{j+1/2}(z_s) + P'_{j-1/2}(z_s)],$$

$$f_{2s} \equiv \left(\tfrac{1}{2} 0 \,|F_s|\, -\tfrac{1}{2} -1\right) = 2\cos\frac{\theta_s}{2}\sum_j f_2{}^j(\sqrt{s})[P'_{j+1/2}(z_s) - P'_{j-1/2}(z_s)], \tag{4}$$

$$f_{3s} \equiv \left(\tfrac{1}{2} 0 \,|F_s|\, -\tfrac{1}{2} 1\right) = -2\cos\frac{\theta_s}{2}\sum_j f_3{}^j(\sqrt{s}) \times\left[\left(\frac{2j-1}{2j+3}\right)^{1/2} P'_{j+1/2}(z_s) - \left(\frac{2j+3}{2j-1}\right)^{1/2} P'_{j-1/2}(z_s)\right],$$

$$f_{4s} \equiv \left(\tfrac{1}{2} 0 \,|F_s|\, \tfrac{1}{2} -1\right) = -2\sin\frac{\theta_s}{2}\sum_j f_4{}^j(\sqrt{s}) \times\left[\left(\frac{2j-1}{2j+3}\right)^{1/2} P'_{j+1/2}(z_s) + \left(\frac{2j+3}{2j-1}\right)^{1/2} P'_{j-1/2}(z_s)\right].$$

Then, if we introduce the partial amplitudes corresponding to transitions with definite total angular momentum j and definite parity

$$f_1{}^j(\sqrt{s}) = \frac{1}{2}[h_1{}^j(\sqrt{s}) + h_2{}^j(\sqrt{s})],$$

$$f_3{}^j(\sqrt{s}) = \frac{1}{2}[h_3{}^j(\sqrt{s}) - h_4{}^j(\sqrt{s})],$$

$$f_2{}^j(\sqrt{s}) = \frac{1}{2}[h_1{}^j(\sqrt{s}) - h_2{}^j(\sqrt{s})],$$

$$f_4{}^j(\sqrt{s}) = \frac{1}{2}[h_3{}^j(\sqrt{s}) + h_4{}^j(\sqrt{s})],$$

then, in accordance with (3), we obtain for them the following symmetry properties:

$$h_1{}^j(\sqrt{s}) = h_2{}^j(-\sqrt{s}), \quad h_3{}^j(\sqrt{s}) = h_4{}^j(-\sqrt{s}). \tag{5}$$

3. ASYMPTOTIC BEHAVIOR FOR $t \to \infty$ AND $s = \text{const} < 0$

Relations (5) play the central role in the theory of fermion Regge poles. They lead to a connection between the singularities of the partial amplitudes with opposite parity, and in particular the connection between the residues of the amplitudes at the pole.

In order to obtain the asymptotic amplitudes at $t \to \infty$ and $s = \text{const} < 0$, we transform in the well known fashion the sums over j in (4) into integrals; we deform the integration contour to extend it to the complex domain. We assume further that when $t \to \infty$ and $s < 0$ the main contribution is made by the partial-amplitude poles; then, in accordance with (5), these poles exist as complex-conjugate pairs. Retaining the contribution of a single, outermost pole we obtain

$$g_1 \equiv f_{1s}/2\sin\frac{\theta_s}{2} + f_{2s}/2\cos\frac{\theta_s}{2} = \chi_1{}^{\pm}(\sqrt{s})D(j),$$

$$g_2 \equiv f_{1s}/2\sin\frac{\theta_s}{2} - f_{2s}/2\cos\frac{\theta_s}{2} = (\chi_1{}^{\pm}(\sqrt{s}))^* D(j^*),$$

$$g_3 \equiv f_{3s}/2\cos\frac{\theta_s}{2} + f_{4s}/2\sin\frac{\theta_s}{2} = -\chi_2{}^{\pm}(\sqrt{s})D(j),$$

$$g_4 \equiv f_{3s}/2\cos\frac{\theta_s}{2} - f_{4s}/2\sin\frac{\theta_s}{2} = (\chi_2{}^{\pm}(\sqrt{s}))^* D(j^*), \tag{6}$$

where $\chi_1^{\pm}$ and $\chi_2^{\pm}$ are the residues of the amplitudes $h_1^j(\sqrt{s})$ and $h_3^j(\sqrt{s})$ at the poles with signatures ($\pm$), and the quantities $D(j)$ and $D(j^*)$ depend on the function $j = j(\sqrt{s})$, which describes the trajectory of the leading pole,

$$D(j) = \frac{t^{j-1/2} \pm (-t)^{j-1/2}}{\cos\pi j}, \qquad D(j^*) = \frac{t^{j^*-1/2} \pm (-t)^{j^*-1/2}}{\cos\pi j^*}.$$

Using (6), we obtain for the asymptotic invariant amplitudes

$$A_1 = \frac{(g_1+g_3)v(\sqrt{s}+M_1) + (g_4-g_2)v'(\sqrt{s}-M_1)}{4(s-M_1{}^2)},$$

$$A_2 = \frac{g_1 v - g_2 v'}{t\sqrt{s}}$$

$$A_3 = \frac{-(g_1+g_3)v + (g_4-g_2)v'}{2(s-M_1{}^2)} - \frac{t}{s-M_1{}^2}A_4,$$

$$A_4 = -\frac{g_1(\sqrt{s}-M_1)v + g_2(\sqrt{s}+M_1)v'}{2t\sqrt{s}} \tag{7}$$

where

$$v = 16\pi\left(\frac{w_s}{E_{2s}-M_2}\right)^{1/2}, \quad v' = 16\pi\left(\frac{w_s}{E_{2s}+M_2}\right)^{1/2}$$

In spite of the fact that the amplitudes A_2 and

A_4 are asymptotically smaller than A_1 and A_3, they make a comparable contribution to the observed quantities. This difference between the asymptotic behavior is the result of the choice of invariant amplitudes in (1).

For the version with positive (KNY) parity, the invariant amplitudes, have a structure analogous to (7), except that v and v′ must be interchanged.

4. CROSS SECTION AND POLARIZATION COEFFICIENTS

Let us consider first the domain $t \to \infty$, s = const. We calculate with the aid of the asymptotic invariant amplitudes (7) the differential cross section for annihilation

$$\frac{d\sigma}{d\Omega} = \frac{1+\alpha_\pm^2}{8\pi^2}(\rho_1^2+\rho_2^2)t^{2j'-1}. \quad (8a)$$

The degree of polarization of the photon produced upon annihilation of an unpolarized baryon-antibaryon pair is given by the expression

$$\xi_1 = \frac{\alpha_\pm^2-1}{\alpha_\pm^2+1}\,\frac{\rho_1^\pm\rho_2^\pm}{\rho_1^2+\rho_2^2}\cos(\varphi_1^\pm-\varphi_2^\pm), \quad (8b)$$

where ξ_1—Stokes parameter

$$\rho_1^\pm e^{i\varphi_1^\pm} = v\sqrt{-s}(\chi_1^\pm M_1 - \chi_2^\pm\sqrt{s})\,/\,2\sqrt{2}(s-M_1^2),$$
$$\rho_2^\pm e^{i\varphi_2^\pm} = v\sqrt{-s}(\chi_1^\pm\sqrt{s} - \chi_2^\pm M_1)\,/\,2\sqrt{2}(s-M_1^2)$$

(for negative (KNY) parity; in the case of positive parity v must be replaced by v′);*

$$\alpha_\pm^2 = (\mathrm{ch}\,\pi j'' \pm \sin\pi j')\,/\,(\mathrm{ch}\,\pi j'' \mp \sin\pi j'),$$

j′—real part of the function $j(\sqrt{s})$ describing the trajectory of the leading pole, and j″—imaginary part.

If the antibaryons are polarized, then the asymmetries take the form

$$A(\bar{s}_y)\frac{d\sigma}{d\Omega} = \frac{\alpha_\pm\sin\beta}{4\pi^2}\rho_1^\pm\rho_2^\pm\cos(\varphi_1^\pm-\varphi_2^\pm)t^{2j'-1}\cos\Phi;$$

$$A(\bar{s}_x)\frac{d\sigma}{d\Omega} = -\frac{\alpha_\pm\sin\beta}{4\pi^2}\rho_1^\pm\rho_2^\pm\cos(\varphi_1^\pm-\varphi_2^\pm)t^{2j'-1}\sin\Phi, \quad (8c)$$

where Φ—azimuthal angle of production of the γ quantum, and $\tan\beta = \sinh\pi j''/\cos\pi j'$.

For polarized target nucleons we obtain for the asymmetries the relations

$$A(s_y)\frac{d\sigma}{d\Omega} = \frac{\alpha_\pm\sin\beta}{4\pi^2}(\rho_1^{\pm2}+\rho_2^{\pm2})t^{2j'-1}\cos\Phi,$$

$$A(s_x)\frac{d\sigma}{d\Omega} = \frac{\alpha_\pm\sin\beta}{4\pi^2}(\rho_1^{\pm2}+\rho_2^{\pm2})t^{2j'-1}\sin\Phi. \quad (8d)$$

*ch = cosh.

We present also expressions for the Stokes parameters characterizing the photon polarization as a function of the polarizations of the initial particles:

$$\xi_1(\bar{s}_x) = \frac{\alpha_\pm\sin\beta(\rho_1^2\sin\Phi-\rho_2^2\sin3\Phi)}{(1+\alpha_\pm^2)(\rho_1^2+\rho_2^2)-\alpha_\pm\sin\beta\rho_1^\pm\rho_2^\pm\cos(\varphi_1^\pm-\varphi_2^\pm)\sin\Phi}$$

$$\xi_2(\bar{s}_x) = \frac{\alpha_\pm\sin\beta(\rho_1^2\cos\Phi-\rho_2^2\cos3\Phi)}{(1+\alpha_\pm^2)(\rho_1^2+\rho_2^2)-\alpha_\pm\sin\beta\rho_1^\pm\rho_2^\pm\cos(\varphi_1^\pm-\varphi_2^\pm)\sin\Phi},$$

$$\xi_3(\bar{s}_x) = \xi_3(\bar{s}_y) = 0, \quad \xi_3(\bar{s}_z) = (\rho_1^2-\rho_2^2)/(\rho_1^2+\rho_2^2). \quad (8e)$$

It follows from these formulas that only simplest characteristics of annihilation into a photon and a meson are monotonic functions of the energy and of the angle, in spite of the oscillating behavior of the invariant amplitudes.

The only oscillating quantities are those which characterize the annihilation of polarized antibaryons by polarized nucleons. For example, the asymmetry in this case is described by the expression

$$A(\bar{z}x)\frac{d\sigma}{d\Omega} = \frac{t^{2j'-1}}{16\pi^2}\Big\{\alpha_\pm\sin\beta(\rho_1^2+\rho_2^2)\sin\Phi + \frac{\cos\Phi}{2}$$
$$\times[\rho_1^2(\alpha_\pm^2\sin2(j''\zeta+\varphi_1^\pm\pm\beta)-\sin2(j''\zeta+\varphi_1^\pm))$$
$$+\rho_2^2(\alpha_\pm^2\sin2(j''\zeta+\varphi_2^\pm\pm\beta)-\sin2(j''\zeta+\varphi_2^\pm))]\Big\},$$
$$\zeta = \ln(s/s_0). \quad (8f)$$

All the conclusions obtained above remain in force also in the region $t \to \infty$, u = const. The oscillating functions of the energy and angle will likewise be only those quantities which characterize the annihilation of the polarized baryon-antibaryon pair.

5. ISOTOPIC RELATIONS

Let us consider the isotopic relations between the cross sections of different antibaryon annihilation processes, resulting from the fact that a Reggeon with definite value of the isotopic spin is exchanged. The nucleon-antinucleon pair is transformed into a pion and γ quantum in the following reactions:

$$\bar{p}+p\to\pi^0+\gamma \quad \sigma(\bar{p}p), \qquad \bar{n}+n\to\pi^0+\gamma \quad \sigma(\bar{n}n),$$
$$\bar{n}+p\to\pi^++\gamma \quad \sigma(\bar{n}p), \qquad \bar{p}+n\to\pi^-+\gamma \quad \sigma(\bar{p}n).$$

The leading pole in these reactions can have isotopic spin ½ and 3/2, with the isotopic relations depending essentially on the asymptotic region. If the quantum is emitted in the direction of the antinucleon momentum, then in the case of a pole with isotopic spin $1/2$ we have

$$\sigma(\bar{p}p) = 2\sigma(\bar{p}n), \quad \sigma(\bar{n}n) = 2\sigma(\bar{n}p), \tag{9a}$$

In the case of a pole with isospin $3/2$ we have

$$2\sigma(\bar{p}p) = 2\sigma(\bar{n}n) = \sigma(\bar{p}n) = \sigma(\bar{n}p). \tag{9b}$$

If the photon is emitted in the direction of the nucleon momentum, and the pion in the direction of the antinucleon momentum, then the following relations are satisfied when a Reggeon with isotopic spin $1/2$ is exchanged

$$2\sigma(\bar{n}n) = \sigma(\bar{p}n), \quad 2\sigma(\bar{p}p) = \sigma(\bar{n}p), \tag{9c}$$

and for exchange of a Reggeon with isospin $3/2$ we get

$$\sigma(\bar{p}p) = \sigma(\bar{n}n) = 2\sigma(\bar{p}n) = 2\sigma(\bar{n}p). \tag{9d}$$

The annihilation of antihyperons into a K meson and a γ quantum is realized in the reactions

$$\begin{aligned} &\bar{\Lambda} + p \to K^+ + \gamma \ \ \sigma(\bar{\Lambda}p), && \bar{\Sigma}^0 + p \to K^+ + \gamma \ \ \sigma(p^0), \\ &\bar{\Sigma}^+ + p \to K^0 + \gamma \ \ \sigma(p^+), && \bar{\Sigma}^- + n \to K^+ + \gamma \ \ \sigma(n^-), \\ &\bar{\Lambda} + n \to K^0 + \gamma \ \ \sigma(\bar{\Lambda}n), && \bar{\Sigma}^0 + n \to K^0 + \gamma \ \ \sigma(n^0). \end{aligned}$$

In these processes the dependence of the isotopic relations on the asymptotic region is even more manifest. If the γ quantum is emitted in the direction of the antihyperon momentum, then the isospin of the leading pole has values 0 or 1, with the Reggeon having strangeness. If the γ quantum is produced in the direction of the nucleon momentum, then the isotopic spin of the Reggeon assumes half-integer values.

For zero isospin (i.e., in the region $t \to \infty$, u = const) we have

$$\sigma(\Lambda p) = \sigma(\bar{\Lambda}n), \ \sigma(p^+) = \sigma(n^-) = 0, \ \sigma(p^0) = \sigma(n^0);$$

and for unity isospin

$$\sigma(\bar{\Lambda}p) = \sigma(\bar{\Lambda}n), \quad \sigma(p^0) = \sigma(n^0),$$

with $\sigma(n^-)$ and $\sigma(p^+)$ different from zero in this case and having in general different values.

In the asymptotic region $t \to \infty$, s = const we have for isospin $1/2$

$$\sigma(\bar{\Lambda}p) = \sigma(\bar{\Lambda}n), \quad \sigma(p^0) = 2\sigma(p^+), \quad \sigma(n^0) = 2\sigma(n^-);$$

for the case corresponding to exchange of a Reggeon with isospin $3/2$ we have

$$\sigma(\bar{\Lambda}p) = \sigma(\bar{\Lambda}n) = 0, \ \ 2\sigma(p^0) = \sigma(p^+), \ \ 2\sigma(n^0) = \sigma(n^-).$$

6. CONCLUSION

All the results obtained in the present article are based on the pole asymptotics. However, as already noted in the introduction, many features of such asymptotic behavior are retained also when the situation is much more complicated.

Thus, the fact that the amplitudes oscillate at high energies in the corresponding region of angles remains correct also in the case when the outermost singularity in the complex j-plane is a branch point. In this case the asymptotic spin structure is also retained, and as a consequence the deductions concerning the polarization coefficients that are oscillating functions of the energy and angle for the process in question also remain in force. In this connection we can formulate the following general premise: in scattering processes at high energies and at c.m.s. angles $\sim 180°$, and also in annihilation processes, the only oscillating quantities are those [6] characterizing the annihilation of two polarized fermions or else the polarization of the recoil fermion as a function of the polarization of the initial fermion in scattering processes. All other characteristics that are not connected with simultaneously polarized fermions are monotonic functions of the energy and of the scattering angle.

As regards the isotopic relations, they are valid not only for the case of pole asymptotics; they are the consequence of the fact that a state with definite isotopic spin is exchanged [7], regardless of the nature of the state, which can be either a pole, a branch point, or in general some aggregate of singularities.

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