SPECTRUM OF COUPLED MAGNETOELASTIC OSCILLATIONS IN A THIN MAGNETIC FILM

V. A. IGNATCHENKO and E. V. KUZ'MIN

Institute of Physics, Siberian Division, Academy of Sciences, U.S.S.R.

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A phenomenological derivation is given of the boundary conditions for a magnetoelastic system. The discrete spectrum of characteristic frequencies of a thin magnetic film, dependent on exchange and magnetoelastic interactions, is calculated. In the spectrum it is possible to distinguish a Kittel exchange part (somewhat modified by magnetoelastic interaction), a mixed part, and a part due to magnetoelastic interaction (and somewhat modified by exchange interaction). Oscillations of all three types can be excited in a thin film by a uniform microwave field.

INTRODUCTION

It was shown by Kittel[1] that the boundary conditions for the magnetization vector $\mathbf{M}$ at the surface of a thin magnetic film, magnetized perpendicular to its surface, lead to a discrete spectrum of spin-wave frequencies,

$$\omega_n = \omega_0 + \alpha g M_0 k_n^2,$$

where $\omega_0 = g(H_0 - 4\pi M_0)$ is the frequency of uniform ferromagnetic resonance, α is an exchange interaction constant, and g is the gyromagnetic ratio. The permissible values of the wave number k_n are determined from the boundary conditions. The boundary conditions for the magnetization vector have been analyzed in a number of works[1-3]. In the general case the boundary conditions lead to transcendental equations for k_n, an approximate solution of which is given by the expression

$$k_n \approx n\pi / 2d, \quad n = 0, 1, 2, \ldots,$$

where $2d$ is the film thickness.

Some of these frequencies (those corresponding to odd values of n) can be excited by a uniform microwave field. Such spin-wave resonance in thin films was observed in a series of investigations in the centimeter-wavelength range of frequencies[4,5] (cf. also the review by Frait[6]). From the distance between spin-wave resonance peaks it is possible to determine the exchange-interaction constant α, one of the fundamental constants of the theory of ferromagnetism.

It seemed of interest to investigate how the spectrum cf characteristic frequencies of a magnetic film would be affected by inclusion of magnetoelastic interaction. It is known[7] that in an infinite medium, magnetoelastic interaction leads to an important modification of the equations of motion and, consequently, of the dispersion relation. It is clear that taking account of magnetoelastic interaction in a thin film should lead both to modification of the equations of motion (the dispersion relation) and to modification of the boundary conditions for the magnetization vector $\mathbf{M}$ and the elastic displacement vector $\mathbf{u}$.

1. THE HAMILTONIAN OF THE SYSTEM

For an infinite ferromagnetic crystal, a phenomenological Hamiltonian[1)], with magnetoelastic-interaction terms included, has been presented in a number of works (cf., for example, [7]). For a thin magnetic film, however, it is important to separate out the surface part of the Hamiltonian. We will describe the deviation of the system from the magnetoelastic ground state by the normalized components of the vector magnetization, $\mu_i = M_i/M_0$, and by the components of the elastic displacement vector, u_i. (As ground state we choose $\mu_x = \mu_y = 0$, $\mu_z = 1$, $u_i = 0$; z is the axis of easy magnetization.) Since the generalized coordinates μ_i and u_i of the system are functions of the Cartesian coordinates, the Hamiltonian is written in the form of a series in the generalized coordinates and their spatial derivatives. On imposing the usual requirements (invariance of $\mathcal{H}$ with respect

1)What is being considered is the potential-energy part of the Hamiltonian.

to inversion of the sign of time, a minimum of $\mathcal{H}$ in the ground state), we get for the elastic part of the Hamiltonian, $\mathcal{H}_p$, the expression

$$\mathcal{H}_p = \int_V \Big\{ A_{ij}^p u_i u_j + aB_{ijk}^p u_i \frac{\partial u_j}{\partial x_k} + a^2 C_{ijkl}^p \frac{\partial u_j}{\partial x_k} \frac{\partial u_j}{\partial x_l} + a^2 D_{ijkl}^p u_i \frac{\partial^2 u_j}{\partial x_k \partial x_l} + \ldots \Big\} dV \tag{1.1}$$

and for the magnetic part of the Hamiltonian, $\mathcal{H}_m$, a formally analogous expression; for the part of $\mathcal{H}$ that describes the magnetoelastic interaction, $\mathcal{H}_{mp}$, we get

$$\mathcal{H}_{mp} = \int_V \Big\{ E_{ijk} \mu_i \mu_j u_k + aF_{ijkl} \mu_i \frac{\partial \mu_j}{\partial x_k} u_l + aG_{ijkl} \mu_i \mu_j \frac{\partial u_l}{\partial x_k} + \ldots \Big\} dV. \tag{1.2}$$

On transforming certain terms in (1.1) and (1.2) and in $\mathcal{H}_m$ to surface and contour integrals (and neglecting the latter), collecting all terms of the same type with new tensor coefficients, and imposing the requirement of invariance of $\mathcal{H}$ with respect to a uniform displacement of the whole crystal, we get

$$\mathcal{H} = \int_V \Big\{ A_{ij} \mu_i \mu_j + a^2 B_{ijkl} \frac{\partial \mu_i}{\partial x_k} \frac{\partial \mu_j}{\partial x_l} + a^2 C_{ijkl} u_{ij} u_{kl} + aD_{ijkl} \mu_i \mu_j u_{kl} \Big\} dV + \int_S \Big\{ A'_{ijk} \mu_i \mu_j - aB'_{ijkk} \mu_i \frac{\partial \mu_j}{\partial n_k} - aC'_{ijkk} u_i \frac{\partial u_j}{\partial n_k} + D'_{ijlk} \mu_i \mu_j u_l \Big\} a\, dS_k, \tag{1.3}$$

where $D_{zzkl} = D'_{zzkl} = 0$, u_{ik} is the elastic deformation tensor, and n_k is the outward normal to the surface of integration. Here and hereafter, primed constants refer to the surface of the crystal.

It is also possible to introduce a phenomenological Hamiltonian of a magnetoelastic system by a method that takes account of the discrete character of the crystal lattice. A simple lattice is formed by translation of three noncoplanar vectors $a^{(k)}$. The position of an atom in the lattice is conveniently described by a vector $\mathbf{n}$ with whole-number components

$$x_i^{\mathbf{n}} = \sum_k a_i^{(k)} n_k \tag{1.4}$$

or, on introduction of the matrix $\hat{a} = \| a_i^{(k)} \|$,

$$\mathbf{r}^{\mathbf{n}} = \hat{a}\mathbf{n}. \tag{1.4a}$$

In such a treatment, the generalized coordinates of the system are the displacements $u_i^{\mathbf{n}}$ of the atoms from the equilibrium position and the values $\mu_i^{\mathbf{n}}$ of the magnetic moments associated with a single atom. The Hamiltonian of the system is written in the form

$$\mathcal{H} = \sum_{\mathbf{m},\mathbf{n}} A_{ij}^{\mathbf{mn}} \mu_i^{\mathbf{m}} \mu_j^{\mathbf{n}} + \sum_{\mathbf{m},\mathbf{n}} \Phi_{ij}^{\mathbf{mn}} u_i^{\mathbf{m}} u_j^{\mathbf{n}} + \sum_{\mathbf{m},\mathbf{n}} G_{ijk}^{\mathbf{mn}} \mu_i^{\mathbf{m}} \mu_j^{\mathbf{m}} u_k^{\mathbf{n}}. \tag{1.5}$$

For $\mathbf{m} = \mathbf{n}$ the coefficients $A_{ij}^{\mathbf{mm}}$ describe the magnetic anisotropy, but $\Phi_{ij}^{\mathbf{mm}} = G_{ijk}^{\mathbf{mm}} = 0$.

The sum over $\mathbf{m}$ can be broken up into two sums by separating out the values $\mathbf{m} = \mathbf{m}'$ that correspond to surface atoms. If the changes of displacements and of magnetic moments between neighboring atoms are sufficiently small, and if the exchange interactions are of sufficiently short range, then the following expansion is valid:

$$u_j^{\mathbf{n}} = u_j^{\mathbf{m}} + (\hat{a}\mathbf{h})_k \frac{\partial u_j^{\mathbf{m}}}{\partial x_k} + \frac{1}{2} (\hat{a}\mathbf{h})_k (\hat{a}\mathbf{h})_l \frac{\partial^2 u_j^{\mathbf{m}}}{\partial x_k \partial x_l} + \ldots, \tag{1.6}$$

where $\mathbf{h} = \mathbf{n} - \mathbf{m}$. A similar expansion holds also for $\mu_j^{\mathbf{n}}$. On substituting these expansions in (1.5), performing the summation over $\mathbf{n}$ for fixed $\mathbf{m}$ (which leads to different results for $\mathbf{m} \neq \mathbf{m}'$ and for $\mathbf{m} = \mathbf{m}'$), and then replacing the summation over $\mathbf{m}$ by integration, we get an expression agreeing with (1.3).

Hereafter we will consider a magnetically uniaxial, elastically isotropic crystal in the form of a thin film (Fig. 1). For such crystal symmetry, the volume density of the Hamiltonian takes the form

$$\tilde{\mathcal{H}}_V = -\frac{1}{2} \beta (\mathbf{Mn})^2 + \frac{1}{2} \alpha \left(\frac{\partial \mathbf{M}}{\partial x_i} \right)^2 + \frac{1}{2} \lambda u_{ii}^2 + \mu u_{ik}^2 + \gamma_{ik}(\mathbf{M}) u_{ik}, \tag{1.7}$$

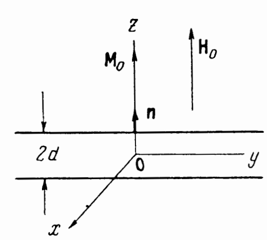

FIG. 1

where β is the uniaxial anisotropy constant, α is the exchange constant, λ and μ are the elastic constants, and

$$\gamma_{ik}(\mathbf{M}) = \gamma_0 M_0^2 \delta_{ik} + \gamma M_i M_k, \quad \gamma_{zz} = 0. \tag{1.8}$$

Instead of a surface density of the Hamiltonian, it is convenient to write the volume density in an infinitely thin layer close to the surface,

$$\widetilde{\mathscr{H}}_S = -\frac{1}{2}\beta'(\mathbf{Mn})^2 - \frac{\alpha'}{a}\mathbf{M}\frac{\partial \mathbf{M}}{\partial n} - \frac{\mu'}{a}\mathbf{u}\frac{\partial \mathbf{u}}{\partial n} + \frac{\gamma'}{a}(\mathbf{Mn})(\mathbf{M}_\perp\mathbf{u}_\perp) + \frac{\gamma_1'}{a}\mathbf{M}_\perp^2(\mathbf{un}). \quad (1.9)$$

In this expression, all quantities are functions only of the surface coordinates (in our case x and y); the constants α, μ', and γ' agree in order of magnitude with α, μ, and γ respectively. For a ferromagnet in a magnetic field, a term $-\mathbf{M}\cdot\mathbf{H}$ must be added to the expressions (1.7) and (1.9).

2. EQUATIONS OF MOTION

The equations of motion for a magnetoelastic system are the Landau-Lifshitz equations, an equation of elasticity, and Maxwell's system of equations (cf., for example, [7]). The effective field $\mathbf{H}^{(e)}$ and force $\mathbf{f}^{(e)}$ that enter in these equations are determined through the volume part of the Hamiltonian (1.7):

$$H_i^{(e)} = -\frac{\partial \widetilde{\mathscr{H}}_V}{\partial M_i} + \frac{\partial}{\partial x_k}\left[\frac{\partial \widetilde{\mathscr{H}}_V}{\partial(\partial M_i/\partial x_k)}\right], \quad (2.1)$$

$$f_i^{(e)} = -\frac{\partial \widetilde{\mathscr{H}}_V}{\partial u_i} + \frac{\partial}{\partial x_k}\left[\frac{\partial \widetilde{\mathscr{H}}_V}{\partial u_{ik}}\right]. \quad (2.2)$$

By neglecting the change of volume with deformation and by carrying out a linearization, it is possible to obtain a system of equations for $\mathbf{m} = \mathbf{M} - \mathbf{M}_0$ and $\mathbf{u}$. In the solution of the problem of characteristic oscillations of a magnetic film, we limit ourselves to the case of oscillations that are uniform in the plane of the film (in this case high-frequency magnetic fields are absent). On introducing the circular components $m^\pm = m_x \pm i m_y$, $u^\pm = u_x \pm i u_y$, we get two independent systems of equations for the temporal Fourier components of m^+, u^+ and of m^-, u^-:

$$agM_0\frac{\partial^2 m^\pm}{\partial z^2} - (\omega_0 \pm \omega)m^\pm - \gamma g M_0^2\frac{\partial u^\pm}{\partial z} = 0,$$

$$s_t^2\frac{\partial^2 u^\pm}{\partial z^2} + \omega^2 u^\pm + \frac{\gamma M_0}{\rho}\frac{\partial m^\pm}{\partial z} = 0, \quad (2.3)$$

where $\omega_0 = g(\beta M_0 - H_0^{(i)})$, $H_0^{(i)} = H_0 - 4\pi M_0$, and $s_t = (\mu/\rho)^{1/2}$ is the transverse speed of sound.

We seek a solution of each system in the form of a plane wave,

$$m^\pm = a^\pm \sin kz + b^\pm \cos kz, \qquad u^\pm = A^\pm \sin kz + B^\pm \cos kz, \quad (2.4)$$

and accordingly obtain two systems of homogeneous equations in the amplitudes. From the vanishing of the determinants of these systems we get the dispersion relations. For m^+ and u^+,

$$[\omega + \omega_0 + agM_0k^2][\omega^2 - s_t^2k^2] + \eta g M_0 s_t^2 k^2 = 0, \quad (2.5)$$

for m^- and u^-,

$$[\omega - (\omega_0 + agM_0k^2)][\omega^2 - s_t^2k^2] - \eta g M_0 s_t^2 k^2 = 0. \quad (2.6)$$

Here $\eta = \gamma^2 M_0^2/\rho s_t^2$ is a dimensionless magnetoelastic coupling parameter.

The relation (2.5) for right-hand polarization describes a weakly modified elastic wave. Hereafter we shall be interested in the dispersion relation (2.6) for left-hand polarization; it describes modified magnetic and elastic oscillations. The permissible values of the wave number k, and consequently the spectrum of characteristic frequencies, are determined by the boundary conditions on the surface of the film.

3. BOUNDARY CONDITIONS

For derivation of the boundary conditions it is necessary to consider the Landau-Lifshitz equation and the equation of elasticity on the surface of the crystal. The effective field $\mathbf{H}^{(e)}$ and force $\mathbf{f}^{(e)}$ that enter in these are determined through the surface part of the Hamiltonian (1.9). Because $\widetilde{\mathscr{H}}_S$ is independent of the coordinate normal to the surface, we have

$$H_i^{(e)} = -\frac{\partial \widetilde{\mathscr{H}}_S}{\partial M_i}, \quad f_i^{(e)} = -\frac{\partial \widetilde{\mathscr{H}}_S}{\partial u_i}. \quad (3.1)$$

On carrying out linearization and on introducing circular components, we get for the Fourier components of $m^\pm$ and $u^\pm$ the modified "magnetic" and "elastic" boundary conditions (at $z = \pm d$)

$$\alpha' g M_0\frac{\partial m^\pm}{\partial n} - a(\omega_0' \pm \omega)m^\pm - \gamma' g M_0^2 u^\pm = 0,$$

$$\frac{\mu'}{\rho}\frac{\partial u^\pm}{\partial n} + a\omega^2 u^\pm - \frac{\gamma' M_0}{\rho}m^\pm = 0, \quad (3.2)$$

where $\omega_0' = g(\beta' M_0 + H_0^{(i)})$.

On substituting in (3.2) the solution for m^- and u^- in the form (2.4), we get a system of four homogeneous equations in the amplitudes. From the vanishing of the determinant of this system we get

$$\{[\omega_e' v \operatorname{tg} v - (\omega - \omega_0')][v \operatorname{tg} v - aD'^2/d] - \eta' dgM_0/a\}\{[\omega_e' v \operatorname{ctg} v + (\omega - \omega_0')][v \operatorname{ctg} v + aD'^2/d] - \eta' dgM_0/a\} = 0,$$

$$\omega_e' = \alpha' g M_0/ad, \quad D' = \omega d/s_t', \quad s_t' = (\mu'/\rho)^{1/2}, \quad \eta' = \gamma'^2 M_0^2/\rho s_t^2, \quad v = kd. \quad (3.3)^*$$

Substitution in (3.3) of the dispersion relation

*tg = tan, ctg = cot.

(2.6) leads to a transcendental equation, which determines the permissible values of v.

In the absence of magnetoelastic coupling ($\eta = \eta' = 0$), we get the known relations determining the discrete spectrum of spin waves [2],

$$\operatorname{tg} v = -\frac{gM_0\Delta\beta}{\omega_e' v} + \frac{\omega_e}{\omega_e'} v, \quad \operatorname{ctg} v = \frac{gM_0\Delta\beta}{\omega_e' v} - \frac{\omega_e}{\omega_e'} v, \tag{3.4}$$

where $\Delta\beta = \beta' - \beta$ and $\omega_e = \alpha g M_0/d^2$, and the equations determining the discrete spectrum of characteristic elastic oscillations of a thin film,

$$\operatorname{tg} v \approx av/d \ll 1, \quad \text{i.e.} \quad v \approx n\pi,$$
$$-\operatorname{ctg} v \approx av/d \ll 1, \quad \text{i.e.} \quad v \approx (n + 1/2)\pi. \tag{3.5}$$

In the presence of magnetoelastic coupling, we have four transcendental equations

$$\operatorname{tg} v = f_{1,2}(v), \quad \operatorname{ctg} v = \varphi_{1,2}(v), \tag{3.6}$$

which can be solved graphically for given values of the parameters. The general character of each solution has the form

$$v_n^{(i)} = (2n-1)\pi/2 - \delta_n^{(i)}, \tag{3.7}$$

where $n = 1, 2, \ldots$, and $\delta_n^{(i)} \le \pi/2$.

4. THE SPECTRUM OF CHARACTERISTIC FREQUENCIES

The dispersion relation (2.6) can be put into the form

$$\sigma_0 = 1 - u + \eta\sigma_M \frac{u}{u - u_k}, \tag{4.1}$$

where

$$\sigma_0 = \omega_0/\omega, \quad \sigma_M = gM_0/\omega, \quad u = \omega_e v^2/\omega,$$
$$u_k = \omega_e D^2/\omega, \quad D = \omega d/s_t.$$

The character of the curve (4.1) is depicted in Fig. 2. The dispersion relation is represented by two hyperbolas with asymptotes $u = u_k$ and $u = 1 - \sigma_0 + \eta\sigma_M$. The quantities u_k and $\eta\sigma_M$ are represented in Fig. 2 not to scale, but in such a way as to emphasize the properties of the spectrum that are due to magnetoelastic coupling. The actual values are $u_k \sim 0.5 \times 10^{-2}$, $\eta\sigma_M \sim 1.6 \times 10^{-2}$ (we use the following values of the constants: $\alpha \sim 10^{-12}$ cm^2, $M_0 \sim 5 \times 10^2$ G, $\omega \sim 5 \times 10^{10}$ sec^{-1}, $s_t \sim 3 \times 10^5$ cm sec^{-1}, $\eta \sim 8 \times 10^{-2}$).

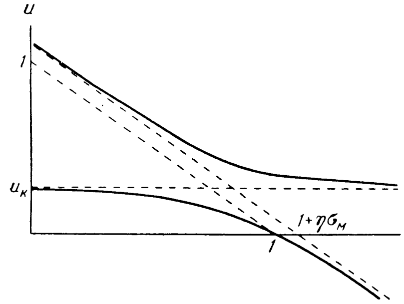

FIG. 2. General form of the dispersion relation (4.1).

From the boundary condition, written in the form (3.7), it follows that the permissible values of u can be expressed in the form

$$u_n = \frac{\omega_e}{\omega} v_n^2 = \sigma_M \frac{\alpha}{d^2}\left[(2n-1)\frac{\pi}{2} - \delta_n\right]^2. \tag{4.2}$$

A uniform microwave field can excite the types of oscillation with small δ_n ($\delta_n = 0$ corresponds to the case in which the film thickness contains an odd number of half-waves; this is the case in which the excitation is most effective). For such types of oscillation,

$$u_n \sim \sigma_M \frac{\alpha\pi^2}{(2d)^2}(2n-1)^2. \tag{4.3}$$

It is easy to show that the maximum number of different wave numbers in the spectrum increases linearly with increase of the film thickness. Simultaneously with the increase of the number of characteristic frequencies, the value of u_1 ($n = 1$) approaches zero, and the "frequency" σ_{01} corresponding to it approaches unity. For thick films, satisfying the relation

$$2d \geqslant \pi s_t/\omega, \tag{4.4}$$

u_1 becomes less than or equal to u_k. For the values of ω and s_t that we have used, the relation (4.4) is satisfied for thicknesses $2d \gtrsim 1.7 \times 10^{-5}$ cm. Thus even in films of thickness 1700 Å the first peak falls in the region of critical u (we shall use the term critical for the range of u values in which $u \sim u_k$). The range of critical u is shown in Fig. 3, from which it is clear that when $u \gtrsim u_k$, the corresponding peak falls in the region $\sigma_0 > 1$, i.e., to the right of the uniform-resonance peak. If some "wave numbers" u_n fall in the critical

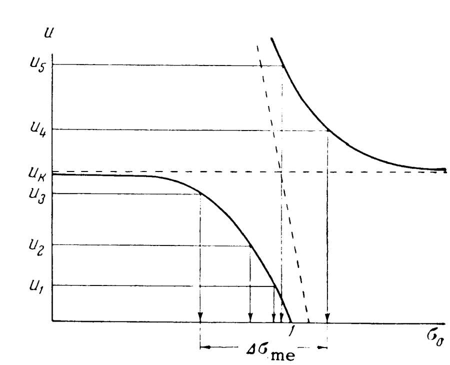

FIG. 3. Character of the magnetoelastic spectrum in the range of critical u_n, $u_n \sim u_k$. The arrows show the "frequencies" σ_{0n} corresponding to the u_n's.

region, then the distribution of peaks can be very peculiar.

The distance between neighboring peaks, in the case in which u_n and u_{n+1} correspond to the same branch of the dispersion relation, is

$$(\Delta\sigma_0)_{n+1,n} = (u_{n+1}-u_n)\left[1+\eta\sigma_M\frac{u_k}{(u_n-u_k)(u_{n+1}-u_k)}\right] = \sigma_M(\alpha/d^2)[2n\pi-(\delta_{n+1}+\delta_n)][\pi-(\delta_{n+1}-\delta_n)]\left\{1+d^2\eta D^2/\alpha\left(\left[(2n+1)\frac{\pi}{2}-\delta_{n+1}\right]^2-D^2\right)\times\left(\left[(2n-1)\frac{\pi}{2}-\delta_n\right]^2-D^2\right)\right\}. \quad (4.5)$$

For u_n sufficiently far from u_k, the contribution of magnetoelastic coupling may be neglected, and the distance between neighboring peaks is

$$(\Delta\sigma_0)_{n+1,\,n} \sim \alpha\sigma_M 8\pi^2 n/(2d)^2, \quad (4.6)$$

that is, in this case measurement of $(\Delta\sigma_0)_{n+1,n}$ gives a possibility of determining the exchange constant α.

For u_n's close to u_k, at sufficiently large η the term unity in the curved brackets in (4.5) may be neglected in comparison with the second term. In this case, the distance between peaks is determined by the magnetoelastic coupling parameter η:

$$(\Delta\sigma_0)_{n+1,\,n} \sim 8\eta\sigma_M P_n/[(2n+1)^2-P][(2n-1)^2-P], \quad (4.7)$$

where $P = (2d\omega/\pi s_t)^2$.

If $u_m < u_k$ and is close to the value of u_k "from below," but $u_{m+1} > u_k$, then at $m+1$ there begins a "transfer" of the spectrum from the left branch to the right (cf. Fig. 3). The distance $(\Delta\sigma_0)_{m+1,m} \equiv (\Delta\sigma_0)_{me}$ is the "width" of the magnetoelastic spectrum,

$$(\Delta\sigma_0)_{me} \sim \eta\sigma_M u_k\frac{u_{m+1}-u_m}{(u_{m+1}-u_k)(u_k-u_m)}, \quad (4.8)$$

which is also determined by the parameter η.

CONCLUSION

The investigation presented has shown that besides the discrete spectrum of characteristic frequencies dependent on exchange interaction, there is in a thin magnetic film a discrete spectrum of characteristic frequencies dependent on magnetoelastic interaction. These spectra are not independent; therefore there can actually be excited in the film mixed exchange-magnetoelastic oscillations. The positions of the resonance peaks of these oscillations depend both on the exchange-interaction parameter α and on the magnetoelastic-interaction parameter η. Somewhat conditionally, however, the whole spectrum can be broken into three parts: (a) the usual Kittel exchange spectrum (but somewhat modified by magnetoelastic interaction); (b) a mixed spectrum; and (c) a spectrum dependent on magnetoelastic interaction (but somewhat modified by exchange interaction).

The magnetoelastic spectrum behaves quite peculiarly. Some of its resonance peaks may lie to the right of the uniform ferromagnetic resonance peak (Fig. 3); monotonic dependence of the position of the peaks on the order-number of the wave number may not be observed (transfer to the second branch of the dispersion relation); etc. The peculiar behavior of this spectrum and its great sensitivity to change of thickness of the film should give a possibility of observing it experimentally.

Measurement of the position of these peaks gives the possibility of determining the magnetoelastic interaction constant η with microwaves.

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