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CONTRIBUTION TO THE FIELD THEORY OF WEAK INTERACTIONS

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The weak interaction between bosons and leptons is investigated on the basis of the Feinberg-Pais procedure. It is shown that the contact terms present in the Born term drop out from the amplitude. These terms reappear, however, if the form factors for strong interactions are taken into account.

1. INTRODUCTION

RECENTLY as a result of the work of Feinberg and Pais[1-3], a new approach has been noted in the theory of weak interactions and, in particular, in the elucidation of the role of effects of higher order in the coupling constant. By investigating lepton reactions involving the exchange of an intermediate vector boson, on the basis of the ladder approximation to a Bethe-Salpeter equation with a kernel that retains only the most strongly divergent terms, Feinberg and Pais[1] have obtained an interesting result: in spite of the smallness of the coupling constant g that describes the coupling of leptons to the $W^{\pm}$ mesons, the sum of divergent diagrams involving the exchange of many mesons can give a finite contribution of the order of magnitude of the contributions given by the Born approximation. In other words, a definite procedure of summation of the most strongly divergent terms of the perturbation theory diagrams corresponding to such a modified Bethe-Salpeter equation leads to a finite effect with respect to many-meson exchange. This occurs because of those very singularities of the nonrenormalizable theory which make the utilization of perturbation theory completely meaningless. (A similar procedure has been applied by Domokos et al[4] to the Bethe-Salpeter equation with a kernel containing a lepton loop. In these papers the starting point is the corresponding differential equation with a singular potential.)

As a result of this an iteration scheme arises in which the principal contribution (in g^2) is given by the most strongly divergent part of the kernel of the Bethe-Salpeter equation, and the remaining part is regarded as a perturbation in the coupling constant g. Although the question of the convergence of such an iteration method remains open, Feinberg and Pais[1] have shown that in each order a finite result is obtained, and the first iteration gives corrections of higher order in g^2 compared to the solution obtained (cf., also [2]).

The Feinberg-Pais procedure has received further justification in Kummer's paper[5] for the case of lepton interactions in the zero energy limit.

With respect to weak processes involving strongly interacting particles (the so-called quasilepton and nonlepton processes) we can state that the applicability to them of the Feinberg-Pais procedure can certainly encounter definite objections, which in the final analysis reduce, in any case for quasilepton processes, to a lack of knowledge of the effect of the strong interaction on vertices in which strongly interacting particles emit an intermediate boson (form factors with all the particles lying off the mass shell). However, an objection of the same kind arises also in purely leptonic processes with respect to taking into account the electromagnetic corrections to the boson-lepton processes (cf., for example, [6]).

At the same time, taking into account a number of the attractive features of the Feinberg-Pais theory which we have mentioned previously, and also the possibility in principle of phenomenologically taking into account corrections of a similar kind, we shall consider in this article the Feinberg-Pais procedure as applied to the boson-lepton weak interactions.

The example of boson-lepton interactions renders more transparent the mechanism of the "renormalization" of the amplitude by higher order effects: the summation of all the more strongly divergent terms of the diagrams (which corresponds to the solution of the Bethe-Salpeter equation with the most strongly divergent kernel) leads in the lowest order in g^2 to a reconstruction of the Born term from which all the contact interactions drop out. While in the case of the lepton-lepton interaction this leads (at low energies) only to a renormalization of the constant $g^2(3g^2/4)$ with respect to the Born term, in the case of the boson-lepton interaction the renormalized amplitude undergoes a more radical change compared to the Born amplitude. Taking into account the "smearing-out" of the contact term by the strong interactions leads to the reappearance in the renormalized amplitude of all the terms appearing in first-order perturbation theory, although some of them are not renormalized at all compared to the Born term.

2. THE INTEGRAL EQUATION AND ITS SOLUTION FOR BOSON-LEPTON SCATTERING

Thus, on the basis of the ladder approximation to the Bethe-Salpeter equation we consider first of all the weak interactions of leptons with mesons of zero spin and of the same parity which are brought about by an exchange of a vector meson $W^{\pm}$. For the time being we shall completely ignore the effects of strong interactions, and we shall assume that the weak interaction vertex for strongly interacting bosons φ, φ' with $W^{\pm}$ is induced by a Lagrangian of the type

$$ih\left[\frac{\partial\varphi'^{+}}{\partial x_{\mu}}\varphi-\varphi'^{+}\frac{\partial\varphi}{\partial x_{\mu}}\right]W_{\mu}, \tag{1}$$

where h is a constant. For particles of the same isotopic multiplet (1) corresponds to the conserved current of bosons $J_{\mu}\sim(p'+p)_{\mu}$. The Lagrangian for the interaction of W-mesons with leptons has the usual form:

$$ig\{\bar{\mu}\gamma_{\alpha}(1+\gamma_5)\nu_{\mu}+\bar{e}\gamma_{\alpha}(1+\gamma_5)\nu_e\}W_{\alpha}+\text{Herm. conj.} \tag{2}$$

Just as in [1,2] the starting point of our investigation is the Bethe-Salpeter equation for the amplitude for the scattering of leptons by bosons $M(p_1,p_2;p_1',p_2')$ (p_1 is the initial four-momentum of the lepton, p_2 is the initial four-momentum of the boson; primes correspond to final momenta):

$$M=M^{(1)}+M^{(1)}G_0M, \tag{3}$$

where G_0 is the free two-particle Green's function for the lepton and the boson, $M^{(1)}$ is the kernel which in the case of the ladder approximation involving the exchange of a $W^{\pm}$ meson has, in accordance with (1) and (2), the form

$$M^{(1)}=gh\gamma_{\mu}(1+\gamma_5)(p_2+p_2')_{\nu}(\delta_{\mu\nu}+q_{\mu}q_{\nu}m^{-2})\,/\,\langle q^2+m^2\rangle, \tag{4}$$

where the Dirac matrices are assumed to be evaluated between lepton spinors; m is the mass of the W-meson, $q=p_1'-p_1=p_2-p_2'$. The angular brackets denote that, wherever necessary, a regularization is carried out which ascribes a definite meaning to non-unique expressions. A small negative imaginary increment is assumed to be present in the propagators.

If, as in [1], we introduce the amplitudes $M_{\pm}=M_o\pm M_e$, where $M_{o(e)}$ corresponds to an amplitude with an odd (even) number of vector mesons ("allowed" and "forbidden" amplitudes), then we can write the following integral equation (cf., diagram):

$$\begin{aligned}M_{\pm}(p_1,p_2;\,p_1',p_2') &= M^{(1)}\pm\frac{igh}{(2\pi)^4}\int\gamma_{\mu}(1+\gamma_5)\\ &\times\frac{1}{\bar{p}_1''}\frac{1}{\bar{p}_2''^2}\frac{1}{\langle(q'-q)^2+m^2\rangle}(p_2'+p_2'')_{\nu}\\ &\times\left(\delta_{\mu\nu}+\frac{(p_1''-p_1')_{\mu}(p_2'-p_2'')_{\nu}}{m^2}\right)\\ &\times M_{\pm}(p_1,p_2;\,p_1'',p_2'')\,d^4p_1''\end{aligned} \tag{5}$$

($q'-q=p_1''-p_1'=p_2'-p_2''$; in the propagators for the lepton and for the boson-hadron the masses of the particles have been omitted in accordance with [1,2]; $\hat{p}=\gamma_{\mu}p_{\mu}$).

Following [1,2] we retain under the integral only the most strongly divergent terms of the kernel. They arise from the term $-p_2''^2p_{1\mu}''$ in the numerator, so that we obtain for the "principal" term of the amplitude $M_{\pm}^{(0)}$ the following equation with a difference kernel:

$$\begin{aligned}M_{\pm}^{(0)}(p_1,p_2;\,p_1',p_2') &= M^{(1)}\mp\frac{igh}{(2\pi)^4m^2}\int(1-\gamma_5)\\ &\times\frac{M_{\pm}^{(0)}(p_1,p_2;\,p_1'',p_2'')\,d^4q'}{\langle(q'-q)^2+m^2\rangle}.\end{aligned} \tag{6}$$

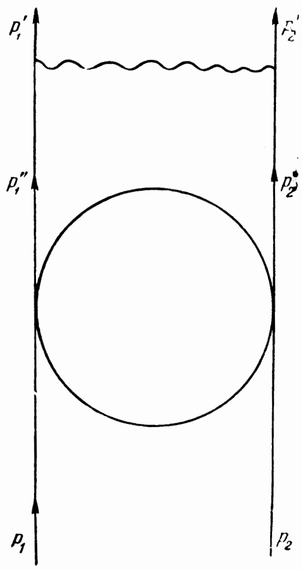
Noting that $M^{(1)}$ actually depends on p_2 and q ($p_2+p_2'=2p_2-q$ in virtue of the law of momentum conservation) while the kernel of equation (6) depends on $q'-q$, we see that it is convenient to choose for the three independent vectors in $M_{\pm}^{(0)}$ the quantities p_1, p_2, q; in this case p_1, p_2 will play the role of parameters in the integral equation. By separating out in $M_{\pm}$ the matrices γ by means of introducing the vector $A_{\mu}^{\pm}(p_1,p_2,q)$

$$M_{\pm}^{(0)}\equiv\gamma_{\mu}(1+\gamma_5)A_{\mu}^{\pm}(p_1,p_2,q),$$

we obtain for it the equation

$$\begin{aligned}A_{\mu}^{\pm}(p_1,p_2,q) &= gh(2p_2-q)_{\nu}\frac{\delta_{\mu\nu}+q_{\mu}q_{\nu}m^{-2}}{\langle q^2+m^2\rangle}\mp\frac{2igh}{(2\pi)^4m^2}\\ &\times\int\frac{A_{\mu}^{\pm}(p_1,p_2,q')\,d^4q'}{\langle(q'-q)^2+m^2\rangle}\end{aligned} \tag{7}$$

It should be noted that in the Bethe-Salpeter equation (3), and consequently also in (7), we are dealing with the amplitude outside the mass shell. Therefore, in the inhomogeneous term of (7) in addition to the ordinary pole we also have a contact term which originates from the second term $q_{\mu}q_{\nu}m^{-2}$ in the numerator of the propagator of the



vector boson. This is a term of the type $q_\mu(1 + q^2m^{-2})/\langle q^2 + m^2\rangle$, which goes over into $q_\mu m^{-2}$ if we formally regard the propagator as unregularized.

The Fourier transform of this term is the derivative of the $\delta^{(4)}$-function, and it can be easily seen that because of the occurrence in the solution for $A^\pm_\mu(p_1, p_2; y)$ of the singular function $\Delta_F(y)$ in the denominator, this term drops out. Indeed, taking the Fourier transform of (7) (in which regularization has been formally omitted) with respect to q, we obtain

$$A_\mu^\pm(p_1, p_2;\ y) = gh\frac{2p_{2\nu}(\delta_{\mu\nu} - m^{-2}\partial_\mu\partial_\nu)\Delta_F(y) - m^{-2}\partial_\mu\delta^{(4)}(y)}{1 \pm 2m^{-2}\,igh\,\Delta_F(y)}$$
$$\left(A_\mu^\pm(p_1, p_2;\ y) \equiv \frac{1}{(2\pi)^4}\int e^{-iqy}A_\mu^\pm(p_1, p_2, q)\,d^4q\right). \tag{8}$$

If now we again go over to $A^\pm_\mu(p_1, p_2, q)$, then it can be easily seen that the second term in the numerator of (8) will give zero contribution because $\Delta_F(y)$ is singular at the origin. If $\Delta_F(y)$ did not occur in the denominator, then, of course, we would again have a contact Born term. In the Appendix the vanishing of this contact term is demonstrated in detail, with account taken of the regularization required to make the formulas meaningful.

Regarding the first term in the numerator of (8), we can say that it is exactly analogous to formula (4.30) of [1], so that we can immediately write down the expression for the "allowed" amplitude (in the region $q\lambda \ll 1$, $\lambda^2 = gh/2\pi^2m^2$, $q^2 \gtrless 0$):

$$A_{\mu,o} = (A_\mu^+ + A_\mu^-)/2 = \frac{3gh}{4}\left[2p_{2\mu}\left(1 - \frac{q^2}{3m^2}\right)\right.$$
$$\left. + \frac{4}{3}\frac{2(p_2q)}{m^2}q_\mu\right]\frac{1}{q^2 + m^2} + O(g^2h^2). \tag{9}$$

The expression for $A^{(1)}_{\mu,o}$ corresponding to the usual Born amplitude (taken off the mass shell), would have the form

$$A^{(1)}_{\mu,o} = gh\left[2p_{2\mu} + \frac{2(p_2q)}{m^2}q_\mu - \left(1 + \frac{q^2}{m^2}\right)q_\mu\right]\frac{1}{q^2 + m^2}. \tag{10}$$

3. THE CONTACT TERMS IN THE FEINBERG-PAIS PROCEDURE

We see that the contact term [the third term in (10)] is not reestablished, although in the limit of small q the change in comparison with (10) again consists of the renormalization of the constant by the factor $^3/_4$.

We now turn to the lepton-lepton case discussed in [1]. The whole effect of taking into account many-meson exchanges for small q consists here of the fact that one can again effectively consider the single meson diagram, but use the modified propagator [7]

$$-i\frac{3}{4}\left[\delta_{\mu\nu}\left(1 - \frac{q^2}{3m^2}\right) + \frac{4}{3}\frac{q_\mu q_\nu}{m^2}\right]\frac{1}{q^2 + m^2} \tag{11}$$

In the case of the boson-lepton interaction it follows from (9) that this "prescription" should be interpreted in the sense that (in the absence of form factors due to strong interactions) in using (11) one should discard the contact term[1)] $-3q_\mu m^{-2}/4$ arising in the amplitude in the course of this calculation. We shall see below that the phenomenological method of taking into account the form factors due to the strong interactions which effectively lead to the "smearing-out" of the contact term contained in (7), enables us to reestablish in the amplitude a term of the type of the third term in (10).

We note that the effect of the disappearance of the contact terms as a result of the renormalization is also evident in [1]. The appearance of the factor $^3/_4$ in the trace of the amplitude $M_{\mu\mu}$ of the lepton-lepton interaction for small q results just from the segregation in the propagator for the W-meson of the contact term (δ-function in the coordinate representation) and from its vanishing due to the appearance in the denominator (as a result

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[1)]Only in this sense can one accept the assertion made without proof at the end of the paper by Bouchiat, d'Espagnat and Prentki [7].

of the summation of an infinite number of diagrams) of the singular function Δ_F.

The same situation also exists in the second paper by Feinberg and Pais [2] where the equation (2.26) for $T_1^{\pm}(p, M)$ —the "high energy" part of the trace of the amplitude for lepton-lepton scattering —contains the inhomogeneous term $-ig^2/m^2$. As M —the regularizing mass of the lepton propagators—tends to infinity the solution of this equation tends to zero as $1/M^2$.

In this sense the situation in the case of boson-lepton processes does not differ from that in the case of lepton-lepton processes. The disappearance of the contact terms in the former case simply leads to a more radical reconstruction of the Born amplitude than was the case in purely lepton-lepton processes where taking higher diagrams into account reduced for small q to single meson exchange with the modified propagator (11). If the contact terms mentioned above were not segregated, as is the case for a spinless propagator, then it can be easily shown that for small q the renormalization procedure would in the final analysis lead to the initial Born term (the contribution from the external region in accordance with the terminology of [1]) with an accuracy up to higher order terms in the coupling constant. Thus, the reconstruction of the amplitude for small q is here a manifestation of the properties of the propagator for a particle with higher spin. In this connection it should be noted that an analogous situation of the dropping out from the amplitude of contact (or subtractive) terms also occurred in the papers of Domokos et al [4]. In those papers these terms arose because of divergences in the theory of the four-fermion interaction which lead to the necessity of two subtractions in the kernel $M^{(1)}$ of (3) (or in the corresponding equation for the Green's function).

Finally, we note that the effect considered above of the dropping out of the contact terms is nothing other than a manifestation of the result established in the paper of Landau and collaborators [8] that in the case of a local four-fermion theory the interaction between fields disappears. The nonvanishing result in the Feinberg-Pais procedure is directly related to the introduction of a physically nonlocal expression in the form of a W-meson which separates lepton pairs (at the same time in the renormalized equation locality is retained between members of each lepton pair [2)]). The transition to a local four-fermion theory [2] leads in accordance with [8] to the vanishing of the amplitude. The contact terms correspond just to the purely local vertices and the fact that they vanish is quite natural. Clearly the introduction of form factors for strong interactions enables us to retain terms which if this were not done would disappear as a result of summing an infinite number of diagrams.

It should be kept in mind that the dropping out from $(q^2 + m^2)A_\mu$ [cf. (9)] of the term linear in $q_\mu = (p_2 - p_2')_\mu$ is undesirable, since even in the case of not very great values of q which are realized, for example, in $K_{\mu 3}$ decay, this term occurs in the amplitude with a relative weight approximately equal to that of the term containing $p_{2\mu}$ [10].

[2)]In the boson-lepton case the renormalization leaves the lepton pair local, since it effectively reduces to a consideration of diagrams in which all the lines for W-mesons are drawn to a lepton vertex. Therefore, in this case the "local action" theorems hold [9].

4. FORM FACTORS FOR STRONG INTERACTIONS

There exists a natural method of removing this difficulty by means of introducing form factors for strong interactions which smear out the interaction between bosons-hadrons and the W meson.

From the outset we shall consider the more general case of a current of strongly interacting bosons with two form factors which also includes the generalization of the interaction (1) where only one form factor $f_+(q)$ appears:

$$J_\mu = f_+(q)(p_2 + p_2')_\mu + f_-(q)(p_2 - p_2')_\mu. \qquad (12)$$

We realize that in this problem, generally speaking, we must know the form factors $f_\pm$ which correspond to both strongly interacting particles outside the mass shell $f_\pm(q, p_2, p_2')$. Of course, we do not know the asymptotic behavior of $f_\pm$ with respect to q^2, when p_2^2, $p_2'^2$ lie off the mass shell. Below we shall assume that as $q^2 \to \infty$ their behavior is effectively such that the corresponding equations are meaningful [cf. (16) below].

The equation analogous to (5) can now be written in the form

$$M_\pm(p_1, p_2; q) = M^{(1)}(p_1, p_2; q) \pm \frac{2igh}{(2\pi)^4}\int [(K_1^{(0)} + K_1^{(1)})f_+((q'-q)^2, p_2''^2, p_2'^2) + K_2 f_-((q'-q)^2, p_2''^2, p_2'^2)]M_\pm(p_1, p_2; q')d^4q', \qquad (13)$$

where

$$M^{(1)}(p_1, p_2; q) = gh\gamma_\mu(1+\gamma_5)[f_+(q^2, p_2^2, p_2'^2)(2p_2 - q)_\nu + f_-(q^2, p_2^2, p_2'^2)q_\nu](\delta_{\mu\nu} + q_\mu q_\nu m^{-2}) / (q^2 + m^2), \qquad (14)$$

$$K_1^{(0)} = -(1-\gamma_5)m^{-2} / \langle (q'-q)^2 + m^2\rangle, \qquad (15a)$$

$$K_1^{(1)} = \gamma_\mu(1+\gamma_5)\frac{1}{\bar{p}_1''p_2''^2}\left[(p_2' + p_2'')_\mu + \frac{p_{1\mu}'p_2''^2 + p_2'^2 p_{1\mu}'' - p_2'^2 p_{1\mu}'}{m^2}\right]\frac{1}{(q'-q)^2 + m^2} \qquad (15b)$$

$$K_2 = \gamma_\mu(1+\gamma_5)(\bar{p}_1''p_2''^2)^{-1}(q'-q)_\mu/m^2. \qquad (15c)$$

We assume that f_+ approaches a constant value as $q^2 \to \infty$, while f_- falls off sufficiently rapidly:

$$f_+(q^2, p_2^2, p_2'^2) \xrightarrow[q^2\to\infty]{} Z, \quad f_-(q^2, p_2^2, p_2'^2) \xrightarrow[q^2\to\infty]{} 0. \qquad (16)$$

The asymptotic behavior indicated above corresponds to the fact that the term with the form factor f_- is induced exclusively by strong interactions. The expression (16) on the mass shell $(-p_2^2 = m_2^2,\ -p_2'^2 = m_2'^2)$ can be related to the conservation of the current of particles belonging to a unitary multiplet. Moreover, we note that under the same conditions $2f_+$ corresponds to a P-wave, while $f_- + (m_2^2 - m_2'^2)q^{-2}f_+$ corresponds to an S-wave of the system of hadrons [11]. It can be easily shown that it is just this last local case that corresponds in (9) to the dropping out of terms from the lowest approximation in gh as a result of renormalization. Here we once again encounter the general situation: the local interaction disappears in the theory which attempts to sum the contribution from the higher order diagrams by means of segregating the principal terms which diverge more strongly than all the others [8].

Returning to our problem, we consider the auxiliary "asymptotic" equation

$$\bar{M}_\pm = ghZ\,\gamma_\mu(1+\gamma_5)(2p_2-q)_\nu \frac{\delta_{\mu\nu}+q_\mu q_\nu m^{-2}}{\langle q^2+m^2\rangle}$$
$$\pm \frac{2ighZ}{(2\pi)^4}\int (K_1^{(0)}+K_1^{(1)})\bar{M}_\pm(p_1, p_2; q')\,d^4q'. \qquad (17)$$

This equation is, of course, solved in the spirit of renormalization (i.e., $K_1^{(1)}$—the less singular part—is discarded). The allowed amplitude $\bar{M}_0$ is obtained directly by multiplying the solution (9) by $Z\gamma_\mu(1+\gamma_5)$.

Subtracting (17) from (13) (cf. [12]) we obtain

$$M_\pm - \bar{M}_\pm = gh\,\gamma_\mu(1+\gamma_5)\Big[(f_+-Z)(2p_2-q)_\nu$$
$$\times \frac{(\delta_{\mu\nu}+q_\mu q_\nu m^{-2})}{q^2+m^2} + f_- q_\nu \frac{\delta_{\mu\nu}+q_\mu q_\nu m^{-2}}{q^2+m^2}\Big]$$
$$\pm \frac{2igh}{(2\pi)^4}\int\{[(K_1^{(0)}+K_1^{(1)})[f_+((q'-q)^2, p_2''^2, p_2'^2)-Z]$$
$$+K_2 f_-((q'-q)^2, p_2''^2, p_2'^2)]M_\pm(p_1; p_2, p_1'', p_2'')$$
$$+Z(K_1^{(0)}+K_1^{(1)})[M_\pm(p_1, p_2, p_1'', p_2'')$$
$$-\bar{M}_\pm(p_1, p_2, q')]\}\,d^4q'. \qquad (18)$$

From (15a)—(15c) it follows that if $f_+ - Z$ and f_- tend to zero faster[3)] than $1/q^3$ as $q^2 \to \infty$ then the solution of (18) will be given up to terms $O(g^2h^2)$ by

$$M_\pm = \bar{M}_\pm + \text{inhomogeneous term in (18)},$$

since the (convergent) integral term is of order $(gh)^2$.

Finally we obtain for the allowed amplitude M_0

$$M_0 = gh\gamma_\mu(1+\gamma_5)\{[2p_{2\mu}(f_+(q) - {}^1\!/\!_4 Z(1+q^2m^{-2}))$$
$$+ f_+ 2(p_2q)q_\mu m^{-2}](q^2+m^2)^{-1}$$
$$-(f_+ - f_- - Z)q_\mu m^{-2}\} + O((gh)^2). \qquad (19)$$

The value $Z = 0$, of course, leads to the usual Born amplitude.

We see that when strong interactions are taken into account renormalization reduces to the renormalization of the "initial" terms induced not only by the strong interactions (form factor f_+). The rapidly falling off terms (f_-) do not differ from the Born terms. The magnitude of the renormalization (for small q) is related to the value of the form factors at infinity $f_+(\infty) = Z$.

If we had utilized in the renormalized equation the conserved current J_μ (β-decay of the π meson), then in (19) we would have had $f_- = 0$, $h = g\sqrt{2}$,[4)] $f_+(0) = 1$, and since for the β-decay of the pion we have $q_\mu^2 \ll m^2$, we would have obtained the matrix element

$$M_0 \approx \sqrt{2}g^2m^{-2}(1-{}^1\!/\!_4 Z)\gamma_\mu(1+\gamma_5)2p_{2\mu}. \qquad (20)$$

From here we see that in order to guarantee the equality of the constants for the β decay of the pion and for the decay of the μ meson we must have $Z = 1$.[5)] A similar result is obtained in [12] with respect to the β-decay constant, taking the nucleon form factor into account. One should keep in mind that after higher order corrections with respect to the weak interactions are taken into account expression (20) does not correspond to the form of the matrix element which is proportional to $\gamma_\mu(2p_{2\mu} - q_\mu) = \gamma_\mu(p_{2\mu} + p_{2\mu}')$ and which corresponds to conservation of current. However, since the mass of the electron is small this difference is hardly likely to be of experimental interest. Therefore, a detailed study of all the characteristics of $K_{\mu 3}$-decay carried out under conditions of good statistics is indubitably of interest. Of even greater interest, but for the time being less easily realized, is the reaction $K_2^0 + e^- \to \pi^- + \nu$.

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3)This rate of falling off could be reduced if the appropriate regularizations are carried out in K_1 and K_2.

4)The coefficient $\sqrt{2}$ is related to the isotopic structure of the conserved current.

5)It is not entirely clear as to what physical meaning is contained in the equality $f_+(0) = f_+(\infty)$ which essentially replaces the hypothesis of the conservation of vector current.

5. CONCLUSIONS

In this paper we have emphasized the dropping out from the amplitude of contact terms when higher-order effects are taken into account because this is in agreement with the fact that the amplitude vanishes for local interactions [8]. One can suppose that this effect of the suppression of the "influence of the light-cone" as a result of a definite method of summation of higher order graphs is characteristic not only of ladder diagrams.

In connection with this the following question arises: what can be said about the vertex itself describing the interaction of the $W^{\pm}$-meson, for example, with leptons where there are no strong interactions? Is it local or not? And if it is, then would this not lead to its disappearance when higher orders are taken into account? In other words, could one in a model where the leptons interact only via $W^{\pm}$ mesons have a local W-lepton vertex? Are we not going to be forced once again to introduce some sort of nonlocality? These questions will be investigated in a subsequent paper.

APPENDIX

We shall show that a term of the type $q_{\mu}(1 + q^2/m^2)\langle q^2 + m^2\rangle^{-1}$ makes no contribution to the solution of (7). Denoting its contribution to $A_{\mu}^{\pm}$ by $\widetilde{A}_{\mu}^{\pm}$, performing a Fourier transformation with respect to q, and returning again to $\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q)$ we obtain

$$\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q) = -igh\int \frac{e^{i(qy)}}{D^{\pm}}\left(\delta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}}{m^2}\right)\partial_{\nu}\langle\Delta_F(y)\rangle\, d^4y,$$

$$D^{\pm} = 1 \pm 2ighm^{-2}\langle\Delta_F(y)\rangle. \tag{I}$$

Utilizing the fact that $\Delta_F(y)$ is a function of y^2 this expression can be easily brought to the form

$$\widetilde{A}_{\mu}(p_1, p_2; q) = -4ghq_{\mu}\frac{\partial}{\partial q^2}\int \frac{d^4y e^{i(qy)}}{D^{\pm}(y)}$$

$$\times\left[\langle\Delta_F'\rangle - \frac{4}{m^2}\left(3\langle\Delta_F''\rangle + y^2\langle\Delta_F'''\rangle\right)\right] \tag{II}$$

where primes denote differentiation with respect to the argument.

For small values of y the function $D^{\pm}(y) \sim y^{-2}$, while the expression in square brackets has a singularity at the origin not worse than y^{-4}, and therefore one can apply to the integral (II) the reduction formula utilized by Feinberg and Pais [1] with all the consequences arising thereform (vanishing of the contour integrals, etc.). Utilizing this formula for $q^2 > 0$ [cf. (5.2) of [1]] and replacing in the spirit of renormalizations Δ_F and its derivatives by modified Bessel functions of the second kind, we obtain

$$\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q) = -ghq_{\mu}\int_0^{\infty}\frac{dy}{D^{\pm}(y)}\frac{J_2(qy)}{q^2}$$

$$\times[y\langle m^2K_2(my)\rangle + m^{-2}(6\langle m^3K_3(my)\rangle - y\langle m^4K_4(my)\rangle)],$$

$$D^{\pm} = 1 \pm \lambda^2 y^{-1}\langle mK_1(my)\rangle.$$

Utilizing the recurrence formula $6K_3(z) = zK_4(z) - ZK_2(z)$ we obtain

$$\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q) = -ghq_{\mu}\int_0^{\infty}\frac{ydy}{D^{\pm}}\frac{J_2(qy)}{q^2}$$

$$\times[\langle m^2K_2(my)\rangle - m^{-2}\langle m^4K_2(my)\rangle]. \tag{III}$$

The origin of the factor m^{-2} in the second term in the integrand is related to the numerator of the propagator of the vector boson, and, therefore, it is taken outside the symbol denoting regularization.

Regularizing (III) and having in mind that the regularization parameter $M \gg m$ we obtain

$$\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q) = -ghq_{\mu}\frac{M^4}{m^2}\int_0^{\infty} y\, dy\frac{J_2(qy)}{q^2}$$

$$\times\frac{K_2(My)}{1 \pm \lambda^2 y^{-1}(mK_1(my) - MK_1(My))}$$

It is evident that, owing to the occurrence of the Bessel function of argument My in the integrand, a contribution which is not exponentially small (as $M \to \infty$) can come only from the region near the origin: $0 \lesssim y \lesssim \alpha/M$, where α can be chosen to be a small quantity. Then, expanding both types of Bessel functions for small values of their arguments we obtain

$$\widetilde{A}^{\mu\pm}(p_1, p_2; q)$$

$$= -ghq_{\mu}\lim\frac{M^2}{m^2}\int^{\alpha/M}\frac{y\, dy}{1 \pm (gh/4\pi^2)(M/m)^2\ln My},$$

from where it can be seen that $\widetilde{A}_{\mu}^{\pm}(p_1, p_2; q)$ tends to zero as a $1/M^2$.

We point out that the linear increase in the contact term with increasing q presents no obstacle to the application of the Feinberg-Pais procedure, since it can be easily shown that a sufficient condition for the convergence of an integral of the type

$$\int \frac{e^{iqv}\widetilde{A}_{\mu}^{(1)}(p_1, p_2, y)d^4y}{D^{\pm}}$$

corresponding to the solution of (7) is that the Fourier transform $\widetilde{A}_{\mu}^{(1)}(p_1, p_2, y)$ of the Born term should behave for small y like y^{α}, where $\alpha > -6$, i.e., that the Born term should behave for large values of q like q^{β}, where $\beta < 2$. In this sense the weak process of the scattering of a boson by a boson as a result of the exchange of a $W^{\pm}$ meson will in the absence of form factors lie outside the sphere of applicability of this procedure because, as can be shown, the inhomogeneous term of the corresponding integral equation contains contact terms which increase at infinity respectively like q and q^2 (in greater detail cf. [13]).

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