RAYLEIGH LIGHT SCATTERING DURING SOUND INSTABILITY

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A theory is developed of Rayleigh light scattering in piezoelectric semiconductors during sound instability. The case of relatively weak growth of sound fluctuations is considered, so that the linear theory is applicable. However, even in this case, it is shown that the intensity of scattering can exceed the scattering intensity from sound fluctuations in thermodynamic equilibrium by two or three orders of magnitude. The spectral distribution of scattered light is investigated and the width of the Rayleigh line is determined. For this purpose, the theory of increasing fluctuations developed in [8,9] is generalized to include time correlation of the fluctuations.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

RECENTLY, the phenomenon of sound instability in piezoelectric semiconductors in a constant electric field has been discovered and investigated in a series of researches.[2-7] As shown in the works of one of the authors [8,9] 1) (see also[10]), the sound instability in the simplest case is convective:2) the sound fluctuations are stationary in time, but increase in space "along the flow." If the finite dimensions of the crystal limit the intensity of the increasing fluctuations to such a level in which the nonlinear effects no longer play a role (the "linear region"), then the intensity of the fluctuations is determined by the linear theory developed in I and II. However, if the intensity of the fluctuations exceeds some critical level, their further growth is limited by the nonlinear effects.

The role of nonlinear effects in the conditions for the amplification of a single sound wave has previously been investigated.[12-14] In the problem involving increasing fluctuations, the nonlinear effects consist in the interaction of many waves. The corresponding theory has not yet been developed. There are only the qualitative considerations of Hutson,[15] which, unfortunately, do not permit us to obtain quantitative characteristics of this unique and hitherto almost uninvestigated state, which arises in the case of sound instability in the nonlinear region.

What sort of experiments could be used to study this state? For transparent crystals, experiments on Rayleigh light scattering give sufficiently complete information on the intensity of sound fluctuations. Under normal conditions, the Rayleigh scattering of light by sound fluctuations is very small, because of the low intensity of the fluctuations in the state of thermodynamic equilibrium.

Under conditions of sound instability, the intensity of the longwave fluctuations, which correspond to light scattering, can increase, relative to the equilibrium value, by a hundred- or thousand-fold. Moreover, this intensity depends very critically on an external parameter—the electric field—which can easily be changed. The first circumstance should guarantee the possibility of observation of light scattering during sound instability in the "linear" region. Not only is the total intensity of light scattering determined in a given direction, but also the shape of the Rayleigh scattering line. As is well known,[16] the line shape is Lorentzian in the case of spatially homogeneous fluctuations, while its width is proportional to the damping coefficient of sound vibrations γ. Therefore, it is natural to raise the question as to how the width of the Rayleigh line behaves when the damping coefficient γ decreases to zero and then changes sign. It is shown that as γ approaches zero, the line loses its Lorentz shape and its form begins to be determined by the dimensions of the semiconductor in the direction in which the growth of fluctuations takes place. When γ, after changing sign, begins to increase in absolute value, the shape of the line again becomes Lorentzian, and its width is determined by the modulus of γ.

1)The papers[8,9] will be denoted below by I and II, and these numbers will be written in references to the formulas.

2)The concept of convective instability was first introduced in the book of Landau and Lifshitz.[11]

In order to determine the shape of the Rayleigh line, i.e., to investigate the change in the light frequency upon scattering, it is necessary to obtain an expression for the time correlation of the fluctuations. Therefore, we shall, in the second section, construct a linear phenomenological theory of the time correlation of increasing sound fluctuations. From this point of view, the second section of the present paper must be regarded as the direct continuation of reference II, in which fluctuations are considered at a single instant of time, and we shall simply refer to the formulas and notation of that paper without rewriting them.

In the following section, the scattering of light by piezoelectrics will be investigated on the basis of this theory. Expressions will be obtained for the extinction coefficient, which can be compared with experiment. By means of such a comparison, one can establish the conditions under which the nonlinear effects begin. (No theory for this exists at the present time.) The experimental investigation of these effects with the help of the sensitive method of Rayleigh light scattering is a very interesting and attractive task.

2. TIME CORRELATION OF THE FLUCTUATIONS

We begin with the consideration of spatially homogeneous fluctuations. We shall consider any of the random variables $\xi_{q'}^{(m)}$ $(m = 1, 2, 3)$ introduced in II, taken at the time $t + \tau$ $(\tau > 0)$. It satisfies the equation

$$\frac{\partial}{\partial\tau}\xi_{q'}^{(m)}(t+\tau) + (i\omega_{mq'} + \gamma_{mq'}/2)\,\xi_{q'}^{(m)}(t+\tau) = y_{q'}^{(m)}(t+\tau), \tag{2.1}$$

while

$$\overline{y_{q}^{(n)*}(t)\,y_{q'}^{(m)}(t')} = \frac{Y_{mn}}{V_0}\,\delta_{q'q}\,\delta(t'-t). \tag{2.2}$$

We multiply (2.1) by $\xi_q^{(n)*}(t)$ and average. Taking it into account that

$$\overline{\xi_{q}{}^{(n)*}(t)\,y_{q'}{}^{(m)}(t+\tau)} = 0$$

(inasmuch as the value of a random quantity at a much earlier moment cannot depend on the value of the random force at a much later moment), we get an equation for the function

$$\mathfrak{A}_{q'q}^{mn}(t,\tau) = \overline{\xi_{q}^{(n)*}(t)\,\xi_{q'}^{(m)}(t+\tau)},$$

which characterizes the correlation of the fluctuations:

$$\frac{\partial}{\partial\tau}\mathfrak{A}_{q'q}^{mn}(t,\tau) + (i\omega_{mq'} + \gamma_{mq'}/2)\,\mathfrak{A}_{q'q}^{mn}(t,\tau) = 0. \tag{2.3}$$

As an initial condition for (2.3), we set

$$\mathfrak{A}_{q'q}^{mn}(t,\tau)\big|_{\tau=0} = A_{q'q}^{mn}(t) = \overline{\xi_{q}^{(n)*}(t)\,\xi_{q'}^{(m)}(t)}. \tag{2.4}$$

The function (2.4) is determined with the help of the theory of fluctuations at a single instant of time, as developed in I and II.

The value of the correlator $\mathfrak{A}_{q'q}^{mn}(t, -\tau)$ for $-\tau < 0$ is found with the help of the relation

$$\mathfrak{A}_{q'q}^{mn}(t, -\tau) = [\mathfrak{A}_{qq'}^{nm}(t-\tau, \tau)]^*. \tag{2.5}$$

In the stationary case, in which neither of the quantities entering into the problem depends explicitly on t, it follows from (2.3)—(2.5) that $(\tau > 0)$:

$$\mathfrak{A}_{q'q}^{mn} = A_{q'q}^{mn}\exp(-i\omega_{mq'}\tau - \gamma_{mq'}\tau/2). \tag{2.6}$$

We proceed to the consideration of the time correlation of spatially inhomogeneous fluctuations, including those arising during convective instability of the system. We shall start out from the equation for the random quantity $\xi^{(m)}(\mathbf{r}', t+\tau)$ in the coordinate representation:

$$\frac{\partial}{\partial\tau}\xi^{(m)}(\mathbf{r}', t+\tau) - \int d^3r_1\,\alpha_{mm}(\mathbf{r}'-\mathbf{r}_1)\,\xi^{(m)}(\mathbf{r}_1, t+\tau) = y^{(m)}(\mathbf{r}', t+\tau). \tag{2.7}$$

We multiply (2.7) by $\xi^{(n)}(\mathbf{r}, t)$ and average:

$$\frac{\partial}{\partial\tau}\overline{\xi^{(n)}(\mathbf{r}, t)\,\xi^{(m)}(\mathbf{r}', t+\tau)} - \int d^3r_1\,\alpha_{mm}(\mathbf{r}'-\mathbf{r}_1) \times \overline{\xi^{(n)}(\mathbf{r}, t)\,\xi^{(m)}(\mathbf{r}_1, t+\tau)} = 0. \tag{2.8}$$

Our purpose is to obtain an expression for the correlator

$$\mathfrak{B}_{Q'R', QR}^{mn} = \overline{\xi_{QR}^{(n)*}(t)\,\xi_{Q'R'}^{(m)}(t+\tau)}$$

in the wave packet representation considered in II. For this purpose, we multiply (2.8) by $V_0^{-2}\psi_{QR}(\mathbf{r})\psi^*_{Q'R'}(\mathbf{r}')$ and integrate over $\mathbf{r}$ and $\mathbf{r}'$. The integral of the first component is obviously $\partial/\partial\tau\,\mathfrak{B}^{mn}_{Q'R',QR}$. In the second component, we expand the functions $\psi_{QR}(\mathbf{r})$ and $\psi^*_{Q'R'}(\mathbf{r}')$ and the kernel $\alpha_{mm}(\mathbf{r}'-\mathbf{r}_1)$ in Fourier series, after which we carry out integration over $\mathbf{r}$, $\mathbf{r}'$ and $\mathbf{r}_1$. As a result, we obtain an equation for the function $\mathfrak{B}$:

$$\frac{\partial}{\partial\tau}\mathfrak{B}_{Q'R', QR}^{mn}(t,\tau) + \frac{a^3}{V_0}\sum_{kk'}\mathfrak{A}_{Q'+k', Q+k}^{mn}(t,\tau) \times [\exp(i\mathbf{k}'\mathbf{R}' - i\mathbf{k}\mathbf{R})]\,(i\omega_{m, Q'+k'} + \gamma_{m, Q'+k'}/2) = 0. \tag{2.9}$$

The solution of this must satisfy the initial condition

$$\mathfrak{B}_{Q'R', QR}^{mn}(t,\tau)\big|_{\tau=0} = B_{Q'R', QR}^{mn}(t) \equiv \overline{\xi_{QR}^{(n)*}(t)\,\xi_{Q'R'}^{(m)}(t)}. \tag{2.10}$$

The values of the function $\mathfrak{B}$ for $-\tau < 0$ are determined with the aid of the relation

$$\mathfrak{B}_{Q'R', QR}^{mn}(-\tau) = [\mathfrak{B}_{QR, Q'R'}^{nm}(\tau)]^*. \tag{2.11}$$

In the stationary case, the solution of Eq. (2.9), with account of (2.10), has the following form, as is easy to prove:

$$\mathfrak{B}^{mn}_{Q'R',\,QR}(\tau)=\frac{a^3}{V_0}\sum_{kk'}A^{mn}_{Q'+k',\,Q+k}\exp(-i\mathbf{kR}+i\mathbf{k'R'})$$
$$\times\exp(-i\omega_{m,\,Q'+k'}\tau-\gamma_{m,\,Q'+k'}\tau/2).$$

The coefficient of light scattering, as we shall see below, is expressed in terms of the diagonal ($n=m$, $Q=Q'$) elements of the matrix $\mathfrak{B}$, which we shall therefore compute. Expanding $\omega_{m,Q'+k'}$ and $\gamma_{m,Q+k'}$ in powers of $\mathbf{k}'$, and limiting ourselves in the first case to the zero and first, and in the second, only to the zero, terms of the expansion, we get, for $\tau>0$:

$$\mathfrak{B}^{nn}_{QR',\,QR}(\tau)=a^3V_0^{-1}\exp(-i\omega_{nQ}\tau-\gamma_{nQ}\tau/2)$$
$$\times\sum_{kk'}A^{nn}_{Q+k',\,Q+k}\exp(-i\mathbf{kR}+i\mathbf{k'R'}-i\mathbf{k'w}_n\tau). \qquad (2.12)$$

By considering that $A^{nn}_{q'q}$ is a function of the half-sum and difference of its arguments, as was done in II, and transforming in (2.12) to summation over $\mathbf{k}_0=(\mathbf{k}+\mathbf{k}')/2$ and $\Delta\mathbf{k}=\mathbf{k}-\mathbf{k}'$, we get, with the stated accuracy,

$$\mathfrak{B}^{nn}_{QR',\,QR}=\exp(-i\omega_{nQ}\tau-\gamma_{nQ}\tau/2)\sum_{\Delta k}A^{nn}(\mathbf{Q},\Delta\mathbf{k})$$
$$\times\exp[-i\Delta\mathbf{k}(\mathbf{R}_0-\mathbf{w}_n\tau/2)]\,D(\Delta\widetilde{\mathbf{R}}), \qquad (2.13)$$

where

$$\widetilde{\mathbf{R}}'=\mathbf{R}'-\mathbf{w}_n\tau,\quad \Delta\widetilde{\mathbf{R}}=\mathbf{R}-\widetilde{\mathbf{R}}',\quad \mathbf{R}_0=(\mathbf{R}+\mathbf{R}')/2,$$
$$D(\Delta\widetilde{\mathbf{R}})=\frac{a^3}{V_0}\sum_{k_0}\exp(-i\mathbf{k}_0\Delta\widetilde{\mathbf{R}}). \qquad (2.14)$$

The vector $\widetilde{\mathbf{R}}'$, generally speaking, is not a vector of the $\mathbf{R}$ lattice (i.e., its components are not multiples of the elementary length a). Therefore, the function $D(\Delta\widetilde{\mathbf{R}})$ is not equal to $\delta_{\Delta\mathbf{R},0}$. However, if, completing the sum, we use the explicit expression for this function

$$D(\Delta\mathbf{R})=\frac{a^3}{\pi^3\Delta\widetilde{X}\Delta\widetilde{Y}\Delta\widetilde{Z}}\sin\frac{\pi\Delta\widetilde{X}}{a}\sin\frac{\pi\Delta\widetilde{Y}}{a}\sin\frac{\pi\Delta\widetilde{Z}}{a}, \qquad (2.15)$$

then it is not difficult to see that it possesses the following two properties: 1) it falls off sufficiently rapidly for $|\Delta\widetilde{\mathbf{R}}|\gg a$; 2) $\sum_{R}D(\Delta\widetilde{\mathbf{R}})=1$, while, by virtue of the property 1), summation is actually carried out over such vectors of the $\mathbf{R}$ lattice which are not appreciably different from $\widetilde{\mathbf{R}}'$.

As will be seen below, the expression for the quantities that characterize the light scattering is obtained by multiplication of (2.13) by some smooth function of $\Delta\widetilde{\mathbf{R}}$ (in the simplest case, by a constant) and by subsequent summation over $\Delta\widetilde{\mathbf{R}}$. But, the function $D(\Delta\widetilde{\mathbf{R}})$ behaves like a δ-symbol relative to such operations. Therefore, with the given accuracy, one can replace it by $\delta_{[\Delta\widetilde{R}],0}$, where $[\Delta\widetilde{\mathbf{R}}]$ is the R-lattice vector closest to the $\Delta\widetilde{\mathbf{R}}$. Then, taking (2.11. II) into account, we obtain

$$\mathfrak{B}^{nn}_{QR',\,QR}=\exp[-i\omega_{nQ}\tau-\gamma_{nQ}\tau/2]\,B^n_Q(\mathbf{R})\,\delta_{\mathbf{R},\,[\mathbf{R}'-\mathbf{w}\tau]}. \qquad (2.16)$$

This is the final expression for the time correlation of increasing fluctuations.

The inverse transition from the function $\mathfrak{B}$ to the function $\mathfrak{A}$ is carried out according to the following general formula:

$$\mathfrak{A}^{mn}_{Q'+k',\,Q+k}=\frac{a^3}{V_0}\sum_{RR'}\mathfrak{B}^{mn}_{Q'R',\,QR}\exp(i\mathbf{kR}-i\mathbf{k'R'}). \qquad (2.17)$$

The coefficient of light scattering is expressed, as we shall see below, in terms of the function $\mathfrak{A}^{nn}_{Q+k,Q+k}$. Evidently, the vector $\mathbf{k}$ can be taken as equal to zero without loss of generality. Transforming (2.17) from summation to integration in this case, and replacing the Kronecker symbol by the δ function, we get, for $\tau>0$,

$$\mathfrak{A}^{nn}_{qq}(\tau)=\frac{1}{V_0}\int d^3R\int d^3R'B_q{}^n(\mathbf{R}')\,\delta(\mathbf{R}-\mathbf{R}'-\mathbf{w}_{nq}\tau)$$
$$\times\exp(-i\omega_{nq}\tau-\gamma_{nq}\tau/2)=[\mathfrak{A}^{nn}_{qq}(-\tau)]^*. \qquad (2.18)$$

3. CALCULATION OF THE EXTINCTION COEFFICIENT

The differential extinction coefficient $dh=h(o)do$ is the ratio of the light intensity scattered in a given range of angle do to the size of this interval, the density of the incident light flux and the volume of the scattering medium (see [16], p. 495). In the case of a spatial growth in the fluctuations, the scattering ability of the crystal is a function of the coordinate. Therefore, we agree to interpret the extinction coefficient as a quantity averaged over the entire volume of the crystal.

For Rayleigh light scattering, the change in the light frequency ω_0 is very small. Therefore, one can consider the light scattering by assuming that the fluctuation with wave vector $\mathbf{q}$, equal to the difference in the wave vectors of the scattered and incident light, makes a contribution to the dielectric permittivity $\epsilon^{(0)}_{ik}(\omega_0)$, equal to [3)]

$$\delta\varepsilon_{ik}(\omega_0,\mathbf{r},t)=\alpha_{iklm}u_{lm}(\mathbf{q})+\gamma_{l,\,ik}\mathscr{E}_{lq}-4\pi m_{ik}^{-1}e^2n_{\mathbf{q}}/\omega_0^2. \qquad (3.1)$$

[3)]The last component in (3.1) is described in the effective mass approximation. This does not affect the subsequent estimates called upon to demonstrate the smallness of this quantity.

Here ϵ is the charge on the electron, m_{ik}^{-1} is the tensor of its reciprocal effective mass, $u_{lm}(q) = (i/2)(u_{lq}q + u_{mq}q_l)$ is the fluctuation deformation (u_{lq} is the l-th component of the amplitude of the displacement vector in the sound wave with wave vector **q**), $\mathscr{E}_{lq}$ is the fluctuation electric field, and n_q is the fluctuating concentration. Each of these three quantities is expressed linearly in terms of three independent random functions $\xi_q^{(1)}$, $\xi_q^{(2)}$ and $\xi_q^{(3)}$ which were introduced for the piezoelectric in II. Finally, the extinction coeffcient h is expressed in terms of the average of the square of $\delta\epsilon_{ik}$, i.e., in terms of the function $\mathfrak{A}_{qq}^{mn}$ which can exceed the thermodynamic average by a factor of a hundred or a thousand during instability.

For $\mathbf{q}\cdot\mathbf{V} > 0$ (**V** is the drift velocity of the electron conductivity), only $\xi_q^{(1)}$ grows as a consequence of the instability of the three functions, $\xi_q^{(n)}$.[4)] Correspondingly, in linear relations which express $u_{lm}(q)$, $\mathscr{E}_{lq}$ and n_q in terms of the $\xi_q^{(i)}$, we keep only terms proportional to $\xi_q^{(1)}$. As a result, the extinction coefficient is expressed in terms of the average $\overline{\xi_q^{(1)*}\xi_q^{(1)}} = A''_{qq}$, which, by virtue of (2.18), is represented in the form of an integral of the function $B_Q^{11}(R) \equiv U_Q(R)$.

We estimate the relative order of the different components in (3.1). The second component in (3.1) can be neglected in comparison with the first by virtue of the inequalities

$$|\beta_{i,\,kl}| \ll \sqrt{\varepsilon^{(0)}\lambda/4\pi} \sim E_a, \qquad |\gamma_{i,\,kl}| \lesssim 1/E_a \tag{3.2}$$

(λ is the modulus of elasticity, E_a is an electric field of atomic order), which are satisfied for most piezoelectrics. The third component in (3.1) can be neglected if $\omega_0 \gg 4\pi\beta e/\epsilon mc \approx 10^{13}\ \text{sec}^{-1}$; this condition is satisfied for visible light.

We also note that the contribution to $\delta\epsilon$ from $\xi_q^{(3)}$ is small in every case if $n_0 \lesssim m^2\omega_0^4 T/e^4\lambda$. This inequality will also be assumed to be satisfied below.

We denote by $h_0(o)$ the extinction coefficient at thermodynamic equilibrium. The problem of the computation of h_0 in crystals is considered in the book of Vol'kenshteĭn.[17] During sound instability, we have

$$\frac{h}{h_0} = \int d^3R\, U_Q(\mathbf{R})/2U_{QT}. \tag{3.3}$$

Here $U_{QT} = T/\rho V_0\omega_{Q0}^2$ is the mean square amplitude of the sound wave with wave vector **Q** in the state of thermodynamic equilibrium; the factor 2 in the denominator of (3.3) is brought about in the final analysis by the fact that of the two waves traveling in opposite directions, only one is amplified in the electric field.

The spectral distribution of the light can be characterized by the function $I(\omega)$, which satisfies the normalization condition $\int_{-\infty}^{\infty} I(\omega)\,d\omega = 1$. In the case under consideration,

$$I(\omega) = [\pi \int d^3R\, U_Q(\mathbf{R})]^{-1}\,\mathrm{Re}\int_0^\infty d\tau e^{-i\Delta\omega\tau - \gamma_q\tau/2}\int d^3R \times \int d^3R'\,\delta(\mathbf{R}' - \mathbf{R} - \mathbf{w}\tau)\, U_Q(\mathbf{R}). \tag{3.4}$$

Here $\Delta\omega = \omega - \omega_0 - \omega_{q0}$ for $\mathbf{q}\cdot\mathbf{V} > 0$ and $\Delta\omega = \omega - \omega_0 + \omega_{q0}$ for $\mathbf{q}\cdot\mathbf{V} < 0$. The integral over τ converges even for $\gamma_q < 0$, since integration over R and R′ is carried out over the finite volume of the crystal.

We consider these expressions in the simplest case, where the piezoelectric has the shape of a plate of thickness L, the plane of which is perpendicular to the X axis. We shall also neglect the lattice sound absorption and assume that $\gamma_0/|\gamma| \gg 1$, and that the electron noise temperature T_e is identical with the lattice temperature T. Then [see (6.3. I) and (2.26. II)]

$$U_Q = U_{QT}(\gamma_0/\gamma)\,[1 - \exp(-\gamma X/w_x)], \tag{3.5}$$

$$h/h_0 = (\gamma_0/2\gamma)\,\{1 - (w_x/\gamma L)\,[1 - \exp(-\gamma X/w_x)]\}, \tag{3.6}$$

$$I(\omega) = \frac{1}{2\pi}\left\{1 - \frac{w_x}{\gamma L}\left[1 - \exp\left(-\gamma\frac{L}{w_x}\right)\right]\right\}^{-1}\frac{\gamma}{\Delta\omega^2 + \gamma^2/4} \times\left\{1 + \frac{2w_x}{\gamma L}\frac{\Delta\omega^2 - \gamma^2/4}{\Delta\omega^2 + \gamma^2/4} - \frac{w_x}{\gamma L}\left[1 + \exp\left(-\frac{\gamma L}{w_x}\right)\right] + \frac{w_x}{\gamma L}\frac{\gamma^2}{\Delta\omega^2 + \gamma^2/4}\cos\frac{\Delta\omega L}{w_x}\exp\left(-\frac{\gamma L}{2w_x}\right) - \frac{2w_x}{\gamma L}\frac{\gamma\Delta\omega}{\Delta\omega^2 + \gamma^2/4}\sin\frac{\Delta\omega L}{w_x}\exp\left(-\frac{\gamma L}{w_x}\right)\right\}. \tag{3.7}$$

Let us consider how this expression behaves in the various limiting cases. For $\gamma > 0$ and $\gamma L \gg w_x$, the theory of spatially inhomogeneous fluctuations is appropriate, and (3.7) transforms into the well known[16] expression which gives the Lorentz line shape:

$$I(\omega) = \frac{1}{2\pi}\frac{\gamma}{\Delta\omega^2 + \gamma^2/4}. \tag{3.8}$$

For $|\gamma|\,L/w_x \ll 1$,

$$I(\omega) = \frac{2}{\pi}\frac{w_x}{L}\frac{1}{(\Delta\omega)^2}\left(1 - \frac{w_x}{\Delta\omega L}\sin\frac{\Delta\omega L}{w_x}\right). \tag{3.9}$$

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4)Conversely, for $\mathbf{q}\cdot\mathbf{V} < 0$, $\xi_q^{(2)}$ increases. The case $\mathbf{q}\cdot\mathbf{V} < 0$ is considered in analogous fashion, and we limit ourselves to stating the final results.

In this case, to the generally smooth change in the function $I(\omega)$ are added higher-frequency oscillaations with period $2\pi w_x/L$. These oscillations actually take place if the scatter in the values of the length L over the cross section of the specimen is much less than the sound wavelength.

Finally, for $\gamma < 0$ and $|\gamma| L/w_x \gg 1$,

$$I(\omega) = \frac{1}{2\pi} \frac{|\gamma|}{\Delta\omega^2 + \gamma^2/4} \tag{3.10}$$

In this case, the shape of the line is Lorentzian as before, while the intensity of the scattered light increases exponentially. It is interesting to note that inasmuch as only the traveling waves which are propagated in one direction are amplified, the intensity of only one component of the Mandel'shtam-Brillouin doublet increases, namely, the anti-Stokes line, if the light scattering takes place in the direction of the sound amplifidcation, and the Stokes line if the scattering takes place in the opposite direction.

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