A PHENOMENOLOGICAL ANALYSIS OF SCATTERING OF PHOTONS BY NUCLEONS

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The problem of a complete experiment of scattering of photons by nucleons is considered. Various choices of experiments required for the determination of the amplitude for scattering of photons by nucleons are analyzed.

1. The complete experiment, i.e., the minimal assembly of experiments necessary to determine the scattering amplitude, was discussed many times for nucleon-nucleon scattering[1], and the photoproduction of pions on nucleons was recently considered from the same point of view[2].

In the present paper we consider the complete experiment for the scattering of photons by nucleons.

Up to now, only the differential and total cross sections, averaged over the polarizations of all particles, were measured for this process. It can be thought that other more complicated experiments on the scattering of photons by nucleons will become feasible in the near future. In particular, the availability of polarized photon beams[3] and of polarized nucleon targets[4] greatly extends the circle of problems which can be investigated in experiments on the scattering of γ quanta by nuclei. Measurement of the recoil-nucleon polarization raises no particular problems at high γ-quantum energies (≥ 500 MeV), but at lower energies difficulties arise, connected with the low energy of the recoil nucleon and with the absence of analyzers.

Some individual problems concerning polarization phenomena and the scattering of photons were considered in several papers: Lapidus[5] analyzed experiments on the scattering of polarized photons and measurement of the recoil-nucleon polarization; Lapidus and Chou Kuang-chao[6] determined the polarization of the recoil nucleons; Berkelman[7] calculated the asymmetry in scattering of polarized photons and the polarization of recoil nucleons in a broad energy interval; Frolov[8] obtained expressions for the polarization coefficients of the Compton effect in terms of invariant amplitudes.

2. It is more convenient, however, to examine the complete experiment in terms of scalar amplitudes in the center-of-mass system (c.m.s.). The process of elastic scattering of γ quanta by nucleons is described by six complex amplitudes, which depend on the total energy and on the scattering angle. In the c.m.s. these amplitudes are best defined in the following fashion[1)]:

$$\hat{F} = (\mathbf{em})(\mathbf{e'm})f_1 + (\mathbf{en})(\mathbf{e'n})f_2 + i(\boldsymbol{\sigma}\mathbf{m})(\mathbf{em})(\mathbf{e'm})f_3 + i(\boldsymbol{\sigma}\mathbf{m})(\mathbf{en})(\mathbf{e'n})f_4 + i(\boldsymbol{\sigma}\mathbf{n})[(\mathbf{en})(\mathbf{e'm}) + (\mathbf{em})(\mathbf{e'n})]f_5 + i(\boldsymbol{\sigma}\mathbf{l})[(\mathbf{en})(\mathbf{e'm}) - (\mathbf{em})(\mathbf{e'n})]f_6, \quad (1)$$

where $\hat{F}$ —total amplitude of the process, $\mathbf{e}$ and $\mathbf{e'}$ —polarization vectors of the initial and final photons, and $f_1, \ldots, f_6$ —complex amplitudes. Formula (1) has been written out on the basis of the invariance against space and time reflections or space rotations, but these requirements are not sufficient for a unique selection of the six amplitudes. We have therefore chosen (1) for convenience in calculations and for the simplicity of the various polarization coefficients characterizing the scattering of quanta by nucleons. The vectors $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$ form an orthogonal set of unit vectors, the choice of which is explained in the figure. We have

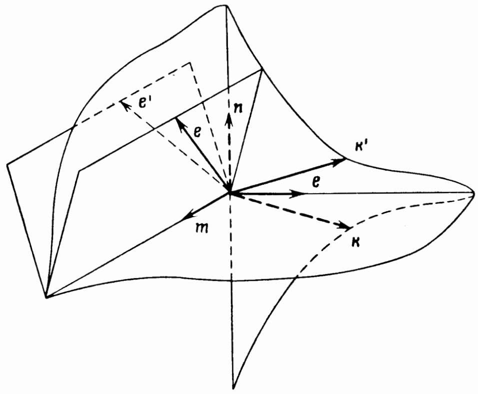
$$\mathbf{l} = (\mathbf{k}+\mathbf{k'})/|\mathbf{k}+\mathbf{k'}|, \quad \mathbf{m} = [\mathbf{kk'}]/|[\mathbf{kk'}]|, \quad \mathbf{n} = [\mathbf{lm}], \quad (2)^*$$

where $\mathbf{k}$ —momentum of the initial photon, $\mathbf{k'}$ —momentum of the final photon in the c.m.s.; $\mathbf{l}$, $\mathbf{n}$ —vectors, $\mathbf{m}$ —pseudovector, a fact taken into account in (1).

Thus, for the complete reconstruction of the scattering amplitudes it is necessary to determine 11 real functions: 6 moduli of the amplitudes f_i and 5 relative phase shifts. Inasmuch as the observed quantities characterizing the γN scattering contain the relative phase shifts as arguments of sinu-

1)Other expressions for the amplitude $\hat{F}$ are also possible (see, for example,[9]).

*$[\mathbf{kk'}] = \mathbf{k} \times \mathbf{k'}$.



soidal functions, for a unique determination of each relative phase shift it is necessary to measure two quantities, with the phase shift the argument of a sine function in one and of a cosine function in the other. It follows therefore that for a unique determination of all the quantities it is necessary to measure 16 different polarization coefficients. If we disregard the possible ambiguity in the determination of the relative phase shifts, 11 experiments are necessary. We can hope that this ambiguity can be gotten around, for example, in the energy region where the two-particle unitarity, which relates the imaginary parts of the amplitudes of the Compton effect with the amplitudes of pion photoproduction, is valid. In this case additional equations appear, and knowledge of the photoproduction amplitudes can help resolve the possible ambiguity.

Repeating the reasoning of Moravcsik[2], we note that 256 different experiments are possible in the scattering of photons by nucleons; of course, not all contain independent information. There exist, however, enough methods of selecting 11 experiments to determine the quantities of interest.

3. With the aid of amplitude (1) we calculate the differential cross section

$$d\sigma / d\Omega = {}^1/_2 \,\mathrm{Sp}\, \hat{F}^+\hat{F}, \tag{3}$$

the polarization of the recoil nucleons

$$\mathbf{P}\, d\sigma / d\Omega = {}^1/_2 \,\mathrm{Sp}\, \hat{F}^+\boldsymbol{\sigma}\hat{F}, \tag{4}$$

the azimuthal asymmetry for polarized target nucleons

$$\mathbf{A}\, d\sigma / d\Omega = {}^1/_2 \,\mathrm{Sp}\, \hat{F}^+\hat{F}\boldsymbol{\sigma}, \tag{5}$$

and the tensor T_{ij} characterizing the change in the nucleon polarization (the initial nucleon is polarized along j and the final along i)

$$T_{ij} d\sigma / d\Omega = {}^1/_2 \,\mathrm{Sp}\, F^+\sigma_i F\sigma_j. \tag{6}$$

The coordinate system is defined by the three vectors $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$; all the quantities calculated below pertain to this system. We shall henceforth use the notation

$$\begin{aligned} |(\mathbf{em})|^2 &= E_1, & |(\mathbf{e'm})|^2 &= E_1', \\ |(\mathbf{en})|^2 &= E_2, & |(\mathbf{e'n})|^2 &= E_2', \\ (\mathbf{em})(\mathbf{e^*n}) &= E_3, & (\mathbf{e'm})(\mathbf{e'^*n}) &= E_3'. \end{aligned} \tag{7}$$

We then obtain for the differential cross section, which depends on the polarization of the initial and final photons,

$$\begin{aligned} d\sigma / d\Omega = {} & (|f_1|^2 + |f_3|^2)E_1E_1' + (|f_2|^2 + |f_4|^2)E_2E_2' \\ & + (|f_5|^2 + |f_6|^2)(E_1E_2' + E_2E_1') \\ & + 2(|f_5|^2 - |f_6|^2)\,\mathrm{Re}\, E_3E_3'^* \\ & + 2\,\mathrm{Re}(f_1f_2^* + f_3f_4^*)\,\mathrm{Re}\, E_3E_3' \\ & - 2\,\mathrm{Im}(f_1f_2^* + f_3f_4^*)\,\mathrm{Im}\, E_3E_3'. \end{aligned} \tag{8}$$

We see therefore that in the general case of polarized initial photons with subsequent measurement of the polarization of the scattered photons, the measurement of the differential cross section enables us to determine six combinations of scalar amplitudes.

For the components of the recoil-photon polarization vector we obtain the following expressions:

$$\begin{aligned} P_l d\sigma / d\Omega = {} & 2\,\mathrm{Im}\,(f_1f_6^* + f_3f_5^*)\,\mathrm{Re}\, E_3E_1' \\ & + 2\,\mathrm{Re}(f_1f_6^* + f_3f_5^*)\,\mathrm{Im}\, E_3E_1' \\ & - 2\,\mathrm{Im}\,(f_1f_6^* - f_3f_5^*)\,\mathrm{Re}\, E_1E_3' \\ & - 2\,\mathrm{Re}\,(f_1f_6^* - f_3f_5^*)\,\mathrm{Im}\, E_1E_3' \\ & + 2\,\mathrm{Im}(f_2f_6^* + f_4f_5^*)\,\mathrm{Re}\, E_2E_3' \\ & + 2\,\mathrm{Re}(f_2f_6^* + f_4f_5^*)\,\mathrm{Im}\, E_2E_3' \\ & - 2\,\mathrm{Im}(f_2f_6^* - f_4f_5^*)\,\mathrm{Re}\, E_3^*E_2' \\ & - 2\,\mathrm{Re}(f_2f_6^* - f_4f_5^*)\,\mathrm{Im}\, E_3^*E_2', \end{aligned} \tag{9a}$$

$$\begin{aligned} P_m d\sigma / d\Omega = {} & 2\,\mathrm{Im}\, f_1f_3^*E_1E_1' + 2\,\mathrm{Im}\, f_2f_4^*E_2E_2' \\ & + 2\,\mathrm{Im}\,(f_2f_3^* + f_1f_4^*)\,\mathrm{Re}\, E_3E_3' \\ & - 2\,\mathrm{Re}\,(f_2f_3^* - f_1f_4^*)\,\mathrm{Im}\, E_3E_3' \\ & - 2\,\mathrm{Im}\, f_5f_6^*(E_1E_2' - E_2E_1') \\ & - 4\,\mathrm{Re}\, f_5f_6^*\,\mathrm{Im}\, E_3E_3'^*. \end{aligned} \tag{9b}$$

The expression for $P_n d\sigma/d\Omega$ we can obtain from (9a), if we make in the latter the substitution

$$f_1 \to -f_3, \quad f_3 \to f_1, \quad f_2 \to -f_4, \quad f_4 \to f_2.$$

In order to obtain an expression for the azimuthal asymmetry, we make use of relations that follow from the time-reversibility requirement

$$\begin{aligned} A_l(\mathbf{e}, \mathbf{e}')d\sigma / d\Omega &= -P_l(\mathbf{e}', \mathbf{e})d\sigma / d\Omega, \\ A_m(\mathbf{e}, \mathbf{e}')d\sigma / d\Omega &= P_m(\mathbf{e}', \mathbf{e})d\sigma / d\Omega, \\ A_n(\mathbf{e}, \mathbf{e}')d\sigma / d\Omega &= P_n(\mathbf{e}', \mathbf{e})d\sigma / d\Omega. \end{aligned} \tag{10}$$

For the tensor T_{ij} we have

$$\begin{aligned} T_{nl} d\sigma / d\Omega = {} & 2\mathrm{Re}\, f_1f_3^*E_1E_1' + 2\mathrm{Re}\, f_2f_4^*E_2E_2' \\ & + 2\mathrm{Re}\, f_5f_6^*(E_2E_1' - E_1E_2') - 4\,\mathrm{Im}\, f_5f_6^*\,\mathrm{Im}\, E_3E_3'^* \\ & + 2\mathrm{Re}\,(f_1f_4^* + f_2f_3^*)\,\mathrm{Re}\, E_3E_3' \\ & - 2\mathrm{Im}\,(f_1f_4^* - f_2f_3^*)\,\mathrm{Im}\, E_3E_3'. \end{aligned} \tag{11}$$

The expression for $T_{ll}\, d\sigma/d\Omega$ is obtained from (8) by making the substitution

$$|f_3|^2 \to -|f_3|^2, \quad |f_4|^2 \to -|f_4|^2,$$
$$|f_5|^2 \to -|f_5|^2, \quad f_3f_4^* \to -f_3f_4^*;$$

if we make the substitution

$$|f_3|^2 \to -|f_3|^2, \quad |f_4|^2 \to -|f_4|^2,$$
$$|f_6|^2 \to -|f_6|^2, \quad f_3f_4^* \to -f_3f_4^*,$$

then we get from (8) an expression for $T_{nn}\, d\sigma/d\Omega$; finally, the expression for $T_{mm}\, d\sigma/d\Omega$ can be obtained from (8) after making the substitutions $|f_5|^2 \to -|f_5|^2$ and $|f_6|^2 \to -|f_6|^2$. To obtain an expression for $T_{lm}\, d\sigma/d\Omega$ we must make in (9a) the substitutions

$$f_1 \to f_3, \quad f_3 \to f_1, \quad f_2 \to f_4, \quad f_4 \to f_2$$

and the imaginary parts of the bilinear combinations of the amplitudes must be replaced by the real parts, while the real parts are replaced by the imaginary parts with change in sign. Then $T_{mn}\, d\sigma/d\Omega$ can be obtained from $T_{lm}\, d\sigma/d\Omega$ by means of the substitution

$$f_1 \to f_3, \quad f_3 \to f_1, \quad f_2 \to f_4, \ f_4 \to f_2.$$

The remaining components of the tensor T_{ij} can be obtained by using the symmetry properties that follow from the invariance with respect to time reflections:

$$\begin{aligned} T_{ln}(\mathbf{e}, \mathbf{e}')\, d\sigma / d\Omega &= -T_{nl}(\mathbf{e}', \mathbf{e})\, d\sigma / d\Omega, \\ T_{nm}(\mathbf{e}, \mathbf{e}')\, d\sigma / d\Omega &= -T_{mn}(\mathbf{e}', \mathbf{e})\, d\sigma / d\Omega, \\ T_{ml}(\mathbf{e}, \mathbf{e}')\, d\sigma / d\Omega &= T_{lm}(\mathbf{e}', \mathbf{e})\, d\sigma / d\Omega. \end{aligned} \tag{12}$$

Comparing relations (9a), (9b), and (11) we find that the quantities $P_m\, d\sigma/d\Omega$ and $T_{nl}\, d\sigma/d\Omega$ are determined by the same combinations of the scalar amplitudes; the same property is possessed also by the pairs $P_l\, d\sigma/d\Omega$, $T_{mn}\, d\sigma/d\Omega$ and $P_n\, d\sigma/d\Omega$, $T_{ml}\, d\sigma/d\Omega$. Therefore the information contained in the off-diagonal tensor component is equivalent to the information that can be extracted by studying the polarization of the recoil nucleons. This equivalence occurs when simultaneous use is made of polarized initial photons with subsequent registration of polarized photons. In the opposite case, the pairs indicated above will contain mutually-supplementary information.

4. The polarization of the initial and final photons enters into all the expressions via the quantities $(\mathbf{e}\cdot\mathbf{m})$ and $(\mathbf{e}\cdot\mathbf{n})$ which, depending on the photon polarization, are of the form

$$(\mathbf{em}) = \cos\varphi, \qquad (\mathbf{en}) = \sin\varphi\cos\theta/2$$

(for linear polarization) and

$$(\mathbf{e}^{\pm}\mathbf{m}) = -i/\sqrt{2}, \qquad (\mathbf{e}^{\pm}\mathbf{n}) = \mp 1/\sqrt{2}$$

(for circular polarization), where φ —angle between the scattering plane and the plane made up by the polarization and photon-momentum vectors. The rules for averaging over the photon polarizations take the following form:

$$|\overline{(\mathbf{em})}|^2 = 1, \qquad |\overline{(\mathbf{en})}|^2 = 1, \qquad \overline{(\mathbf{en})(\mathbf{e}^*\mathbf{m})} = 0. \tag{13}$$

Thus, the combinations $E_3E'_{1,2}$ are purely imaginary if the initial nucleon is circularly polarized, and are real in the case of linear polarization of the initial photon; the same holds true also for the quantities $E_{1,2}E'_3$, depending on the polarizations of the final photons. Therefore the number of combinations of scalar amplitudes which can be determined by measuring the corresponding quantity is reduced if, for example, only linearly polarized initial and final photons are used. If the initial and final polarizations of the photons are of the same type, then the quantities $E_3E'_3$ are real; if one of the polarizations is linear and the other is circular, then such quantities are pure imaginary.

5. Let us examine now what measurements must be made to determine the 11 quantities necessary to reconstitute the amplitude of the scattering of γ quanta by nucleons.

If we use linearly-polarized photons and an unpolarized target, and if we are not interested in the polarization of the scattered photon, then the measurements of the differential cross section can be used to determine two combinations of scalar amplitudes:

$$|f_1|^2 + |f_3|^2 + \frac{1}{2}(1+\cos\theta)(|f_5|^2 + |f_6|^2),$$
$$|f_5|^2 + |f_6|^2 + \frac{1}{2}(1+\cos\theta)(|f_2|^2 + |f_4|^2). \tag{14a}$$

Measurements of the polarization of the recoil nucleon in the direction of $\mathbf{m}$ yields two quantities:

$$\mathrm{Im}\, f_1f_3^*, \qquad \mathrm{Im}\, f_2f_4^*. \tag{14b}$$

Measurements of the polarization of the recoil nucleon in the direction of $\mathbf{n}$ yields one quantity:

$$\mathrm{Im}(f_1f_5^* - f_3f_6^*) + \frac{1}{2}(1+\cos\theta)\,\mathrm{Im}(f_2f_5^* + f_4f_6^*). \tag{14c}$$

Measurement of the polarization of the recoil nucleon in the direction of $\mathbf{l}$ —also one quantity

$$\mathrm{Im}(f_1f_6^* + f_3f_5^*) - \frac{1}{2}(1+\cos\theta)\,\mathrm{Im}(f_2f_6^* - f_4f_5^*). \tag{14d}$$

Altogether six quantities can be determined on the basis of these measurements. If we include furthermore a measurement of the azimuthal asymmetry in the polarization of the initial photons, it becomes possible to determine four additional quantities, i.e., one more measurement is necessary for the complete experiment. To this end we can make use of the scattering of circularly polarized photons, and then the measurements of P_n and P_l, and also of A_n and A_l, make it possible to determine one additional quantity each. Thus, measurement of the differential cross section, of the recoil-nucleon polarization, and of the asymmetry with initial polarized photons makes it possible to determine 14 combinations of amplitudes, from which 11 independent combinations can be chosen in many ways, but in this case we must measure the recoil-nucleon polarization and the asymmetry for polarized target nucleons.

If we do not use circularly polarized initial photons, we can obtain sufficient additional information, over and above the information obtained from the scattering of linearly polarized photons with subsequent averaging over the polarizations of the scattered photon, by studying the dependence of the recoil-nucleon polarization or the asymmetry of scattered-photon polarization.

In the absence of a polarized target, we can determine the necessary 11 combinations of the amplitudes by measuring the differential cross section and the polarization of the recoil nucleons as a function of the polarizations of the initial and final photons, and we can confine ourselves to only linearly polarized initial photons.

Thus, there are enough possibilities for choosing the necessary experiments to determine the amplitude for the scattering of photons by nucleons.

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