THE PRESSURE OF AN INTENSE PLANE WAVE ON A FREE CHARGE AND ON A CHARGE IN A MAGNETIC FIELD

V. S. VORONIN and A. A. KOLOMENSKIĬ

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor April 23, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 1528-1535 (October, 1964)

A complete solution is obtained for the problem of the motion of a charged particle in the field of a plane electromagnetic wave of arbitrary form, the radiation-reaction force being taken into account. For the case in which the plane wave consists of a set of monochromatic waves with arbitrary frequencies, polarizations, and intensities a general formula is derived for the pressure of the wave on the particle; the well known Thomson formula is a special case of this expression. A general formula is found for the pressure of a monochromatic plane wave on a charge moving in a magnetic field. The limit is found which in principle is imposed on the mechanism of autoresonance acceleration by the radiation reaction.

1. INTRODUCTION

THE motion of a charge in the field of a plane electromagnetic wave has been treated in a number of papers (see, e.g.,[1,2]). In these papers, however, the effect of the radiation reaction on the charge was taken into account only for particular special cases, for example for a wave of small intensity, for which the effective scattering cross section is described by the Thomson formula. Meanwhile, owing to the recent development and application of extremely powerful beams of microwaves and of light, a further development of the theory of the interaction of such beams with charged particles may be not only of general physical interest, but also of practical interest.

In the present paper we obtain the solution of the problem of the motion of a free charged particle in the field of a plane wave of large intensity which is made up of a set of monochromatic waves of arbitrary polarizations, and that of the problem of the motion of a charge in the field of a monochromatic plane wave of arbitrary polarization together with a constant uniform magnetic field, with the radiation reaction force taken into account. We find general formulas expressing the radiation pressure on a free charge and on a charge moving in a magnetic field. These formulas are generalizations of the expression for the Thomson light pressure. Furthermore, in this work no restrictions of any kind are imposed on the intensity or wavelength of the electromagnetic wave, and the results can be used for calculations in various topics where one encounters the interaction of waves with charges in a magnetic field (acceleration of particles, amplification and generation of waves, interaction of radio waves with particles in the earth's magnetic field or in interstellar magnetic fields, and so on).

This problem is also of interest in connection with the recently investigated phenomenon of autoresonance between an electromagnetic wave and a particle moving in a magnetic field (see[3]). In the case of autoresonance the radiation reaction force also plays a definite role, since in principle it detunes the exact resonance and limits the time during which it acts. In this paper we obtain estimates of the limitations on the effectiveness of the autoresonant reaction between a wave and a particle in a magnetic field which appear when radiation reaction is taken into account.

If we introduce the notations

$$\tau = ct, \qquad \gamma = 1/\sqrt{1-\dot{\mathbf{r}}^2},$$
$$\mathbf{e} = q\mathbf{E}/mc^2, \qquad \mathbf{h} = q\mathbf{H}/mc^2, \; g = 2q^2/3mc^2, \tag{1}$$

then we can write the equations of motion of a charge q in the field **E**, **H**, with the radiation reaction force included, in the following way (cf.[1]):

$$\frac{d}{d\tau}(\gamma\dot{\mathbf{r}}) = \mathbf{e} + [\dot{\mathbf{r}}\mathbf{h}] + \mathbf{f}, \tag{2*}$$

where

$$\mathbf{f} = g\Big\{\gamma\left(\frac{\partial}{\partial\tau} + \dot{\mathbf{r}}\nabla\right)\mathbf{e} + \gamma\Big[\dot{\mathbf{r}}\left(\frac{\partial}{\partial\tau} + \dot{\mathbf{r}}\nabla\right)\mathbf{h}\Big] + [(\mathbf{e} + [\dot{\mathbf{r}}\mathbf{h}])\,\mathbf{h}] + \mathbf{e}\,(\mathbf{e}\dot{\mathbf{r}}) - \gamma^2\dot{\mathbf{r}}\,((\mathbf{e} + [\dot{\mathbf{r}}\mathbf{h}])^2 - (\mathbf{e}\dot{\mathbf{r}})^2)\Big\} \tag{3}$$

*[r h] = r × h.

is the radiation reaction force. Here γ is the relativistic factor, and $3g/2$ is the classical radius of the electron.

For an arbitrary plane wave **E**, **H** travelling in the direction of the unit vector **n** (alone the z axis) and a constant magnetic field H_0 directed along **n** we have

$$\mathbf{e}=\mathbf{e}(\xi), \quad \xi=\tau-z, \quad (\mathbf{en})=0, \quad \mathbf{h}=[\mathbf{ne}]+\mathbf{n}h_0. \tag{4}$$

Let us substitute (4) in (2) and (3) and change from the variables **r**, τ to new variables J, $\boldsymbol{\rho}$, ξ by using the relations

$$\gamma(1-\dot{z})=J, \quad \dot{\mathbf{r}}=\dot{\mathbf{r}}_\perp+\mathbf{n}\dot{z},$$
$$\boldsymbol{\rho}=\frac{\dot{\mathbf{r}}_\perp}{1-\dot{z}}, \quad \frac{d}{d\tau}=\frac{J}{\gamma}\frac{d}{d\xi}, \tag{5}$$

where $\mathbf{r}_\perp$ is the component of the radius vector of the particle that lies in a plane perpendicular to the vector **n**.

In the variables J, ρ, ξ the equations (2) take a form which is simpler and more convenient for analysis:

$$\frac{d\boldsymbol{\rho}}{d\xi}=\frac{\mathbf{e}}{J}+\frac{h_0}{J}[\boldsymbol{\rho}\mathbf{n}]+g\left\{\frac{d\mathbf{e}}{d\xi}-\frac{h_0}{J}[\mathbf{ne}]-\frac{h_0^2}{J}\boldsymbol{\rho}\right\}, \tag{6a}$$

$$\frac{d}{d\xi}\left(\frac{1}{J}\right)=g\,(h_0\boldsymbol{\rho}+[\mathbf{ne}])^2, \tag{6b}$$

and the original quantities are expressed in terms of the new variables by the formulas

$$\gamma\dot{z}=\frac{J}{2}\rho^2+\frac{1-J^2}{2J},$$
$$\gamma=\frac{J}{2}\rho^2+\frac{1+J^2}{2J}, \quad \tau=\int\frac{\gamma}{J}\,d\xi. \tag{7}$$

2. THE MOTION OF A FREE CHARGE AND THE RADIATION REACTION IN THE FIELD OF AN INTENSE PLANE WAVE

We at first assume that there is no constant magnetic field ($h_0 = 0$). Then the equations (6) take the simple form

$$\frac{d\boldsymbol{\rho}}{d\xi}=\frac{\mathbf{e}}{J}+g\frac{d\mathbf{e}}{d\xi}, \quad \frac{d}{d\xi}\left(\frac{1}{J}\right)=ge^2 \tag{8}$$

and can be integrated immediately:

$$\frac{1}{J}=\frac{1}{J_0}+g\int e^2d\xi,$$
$$\boldsymbol{\rho}=\boldsymbol{\rho}_0+\frac{1}{J_0}\int\mathbf{e}d\xi+g\mathbf{e}+g\int\mathbf{e}\left(\int e^2d\xi\right)d\xi, \tag{9}$$

where J_0 = const, $\boldsymbol{\rho}_0$ = const.

Substituting (9) in (7), we can find any characteristics of the motion. For example, dropping higher powers of the small quantity g, we get as the expression for the longitudinal momentum

$$2p_z=2\gamma\dot{z}=J_0\left(\boldsymbol{\rho}_0+\frac{1}{J_0}\int\mathbf{e}d\xi\right)^2+\frac{1-J_0^2}{J_0}$$
$$+g\left\{(1+J_0^2)\int e^2d\xi+2J_0\left(\boldsymbol{\rho}_0+\frac{1}{J_0}\int\mathbf{e}d\xi\right)\right.$$
$$\left.\times\left[\mathbf{e}+\int\mathbf{e}\left(\int e^2d\xi\right)d\xi\right]-J_0^2\left(\boldsymbol{\rho}_0+\frac{1}{J_0}\int\mathbf{e}d\xi\right)^2\int e^2d\xi\right\}. \tag{10}$$

In the case of a free charge we can solve the problem for the general case in which the wave $\mathbf{e}(\xi)$ is a superposition of monochromatic waves with wave numbers k_i and arbitrary elliptical polarizations:

$$\mathbf{e}(\xi)=\begin{cases} e_y(\xi)=\sum_i(e_{i2}-e_{i1})\sin(k_i\xi+\varphi_i) \\ e_x(\xi)=\sum_i(e_{i1}+e_{i2})\cos(k_i\xi+\varphi_i)\end{cases} \tag{11}$$

Substituting (11) in (9), we can verify that if we neglect the radiation reaction force (i.e., set g = 0) the particle executes a periodic motion, and the average result of the action of the wave is zero. In this case the quantity J [see (5)] is an integral of the motion $J \equiv J_0$.

It must be emphasized that the wave pressure force F_{press}, defined as the average value of the longitudinal force acting on the particle, is not equal to the average value of the longitudinal component of the radiation reaction force (3), i.e., $F_{press} \neq \langle f_z\rangle$. This can be seen intuitively, for example, in the case of a monochromatic wave with circular polarization, if we choose a coordinate system in which the particle describes a circle and is at rest on the average. In this case the longitudinal component of the radiation reaction force is zero, as is quite clear, since as the particle moves in a circle the radiation does not carry away any longitudinal momentum. At the same time the pressure force of the light is of course different from zero, since F_{press} includes besides $\langle f_z\rangle$ the average of the Lorentz force component $ec^{-1}[\Delta\dot{\mathbf{r}}\mathbf{H}]_z$ associated with $\Delta\dot{\mathbf{r}}$, the increment of the velocity of the particle which occurs under the effect of the radiation reaction. Therefore we shall define the pressure force F_{press} as the average value of the quantity $d(\gamma\dot{z})/d\tau$ in the actual motion of the particle:

$$F_{\text{press}}=\left\langle\frac{d}{d\tau}(\gamma\dot{z})\right\rangle_\tau\equiv\int_0^T\frac{d}{d\tau}(\gamma\dot{z})\,d\tau\bigg/\int_0^T d\tau$$
$$=\lim\left(\gamma\dot{z}\Big|_0^\xi\bigg/\int_0^\xi\frac{\gamma}{J}d\xi\right). \tag{12}$$

In calculating (12) we use the expressions (7) and (10), and keep in (10) only the terms that contain the factor ξ, dropping all oscillatory terms. When we carry out the calculations, we get

$$F_{\text{press}} = g\sum_i (e_{i1}^2 + e_{i2}^2)\frac{1 + J_0^2 - J_0^2\rho_0^2 + \Sigma}{1 + \rho_0^2 + J_0^{-2} + J_0^{-2}\Sigma}; \quad (13)$$

$$\langle \dot{z} \rangle = 1 - \frac{2}{1 + \rho_0^2 + J_0^{-2} + J_0^{-2}\Sigma}, \quad \langle \dot{\mathbf{r}}_\perp \rangle = \boldsymbol{\rho}_0(1 - \langle \dot{z} \rangle), \quad (14)$$

where we have used the notation

$$\Sigma = \sum_i \frac{e_{i1}^2 + e_{i2}^2}{k_i^2}.$$

In the general case the formula (13) contains two parameters, ρ_0 and J_0. For convenience we can replace them by more intuitive quantities, which we choose to be $\langle \dot{z} \rangle$ and the half sum of the maximum and minimum values of the energy γ_0:

$$\gamma_0 = \frac{\gamma_{\max} + \gamma_{\min}}{2} = \frac{1}{2}\left[J_0\rho_0^2 + \frac{1 + J_0^2}{J_0}\Sigma\right]. \quad (15)$$

We get

$$F_{\text{press}} = g\sum_i (e_{i1}^2 + e_{i2}^2)[1 - \langle \dot{z} \rangle] \times [1 + \Sigma - \gamma_0^2\langle \dot{z} \rangle(1 - \langle \dot{z} \rangle)]. \quad (16)$$

The formulas (13) and (16) take a simpler form if we choose a coordinate system in which the particle moves on the average only along the z axis, which means that we must set the parameter ρ_0 equal to zero. In this case we can eliminate the second parameter J_0 from the expressions (12) and (13) and get

$$F_{\text{press}} = g\sum_i (e_{i1}^2 + e_{i2}^2)[1 + \Sigma]\frac{1 - \langle \dot{z} \rangle}{1 + \langle \dot{z} \rangle} \quad (\langle \dot{\mathbf{r}}_\perp \rangle = 0). \quad (17)$$

The quantity $\Sigma(e_{i1}^2 + e_{i2}^2)$ is the average energy density in the wave, so that for $\langle \dot{z} \rangle = 0$ Eq. (17) goes over into an expression which we can call a generalized Thomson formula. The additional factor $1 + \Sigma$ by which (17) differs from the ordinary Thomson formula is usually very close to unity. In principle, however, it can become significant for waves of very great intensity, and also for very long waves. In these cases the average kinetic energy of the particle under the action of the wave becomes relativistic already in a time of the order of the period of the wave. In fact, we can rewrite the formula (15) in the following form:

$$\gamma_0^2 = (1 + \Sigma) / (1 - \langle \dot{z} \rangle^2 - \langle \dot{\mathbf{r}}_\perp \rangle^2), \quad (18)$$

from which it can be seen that $\gamma_0^2 \geq 1 + \Sigma$.

Thus it has been shown that the pressure force is proportional to the mean energy density of the wave also in the case in which the wave is a superposition of plane waves with arbitrary frequencies and polarizations. At very high intensities, when the oscillations of the energy of the particle under the action of the wave come to be of the order of the rest energy, the magnitude of the pressure can be considerably larger than that indicated by the Thomson formula.

Besides this, we point out that, as can be seen from (7), the pressure force in the relativistic case ($|\dot{\mathbf{r}}| \approx \dot{z} = 1$) has a decided dependence on the directions of motion of the particle and the wave. If these directions are the same, F_{press} is diminished by a factor of approximately $4\gamma^2$, and if the directions are opposite, F_{press} is increased by this same factor.

3. THE PRESSURE OF AN ELECTROMAGNETIC WAVE ON A CHARGE IN A MAGNETIC FIELD[1)]

Now let $h_0 \neq 0$, and let $\mathbf{e}(\xi)$ be a monochromatic plane wave with elliptical polarization:

$$\mathbf{e} = \begin{cases} e_x = (e_1 + e_2)\cos\psi \\ e_y = (e_2 - e_1)\sin\psi, \qquad \psi = k\xi, \\ e_z = 0 \end{cases} \quad (19)$$

where ψ is the phase of the wave and k is the wave vector.

It is easy to see that the case $h_0/Jk = a = \pm 1$ corresponds to resonance acceleration of the particle by one of the circular components which can be separated out from the assumed elliptically polarized wave. The direction of rotation in this circular component is the same as the direction of revolution of the particle in the magnetic field H_0. The resonance acceleration, which in the absence of radiative effects ($g = 0$) can continue for an indefinitely long time, is the previously mentioned autoresonance effect investigated in[3]. We shall assume at first that the conditions of the motion are such that there is no resonance, i.e., $a \neq \pm 1$. Then, if we neglect radiation, we find that the particle executes a periodic motion, and on the average is moving uniformly in a straight line. To take the radiation reaction into account we have only to solve the equations (6) in first order with respect to the extremely small quantity g. To do this we set

$$\boldsymbol{\rho} = \boldsymbol{\rho}_0 + g\boldsymbol{\rho}_{1\perp}, \qquad 1/J = 1/J_0 + gj_1 \quad (20)$$

and get for $\boldsymbol{\rho}_0$, $\boldsymbol{\rho}_1$, j_1 the solutions

1)The problem of the pressure of a wave on a charge in the presence of a magnetic field has also been treated in a paper by Faĭnberg and Kurilko [4] in the approximation of weak nonlinearity. Effects associated with autoresonance were not taken into consideration in that paper.

$$\rho_0 = \Lambda(a\psi)\,\rho_{00} + \Lambda(a\psi)\int \Lambda(-a\psi)\,\frac{\mathbf{e}}{J_0}\frac{d\psi}{k}, \qquad (21)$$

$$\rho_1 = \Lambda(a\psi)\int \Lambda(-a\psi)\left\{j_1\mathbf{e} + k\frac{d\mathbf{e}}{d\psi} + h_0 j_1 \Lambda\left(\frac{\pi}{2}\right)\rho_0 + \frac{h_0}{J_0}\Lambda\left(\frac{\pi}{2}\right)\mathbf{e} - \frac{h_0^2}{J_0}\rho_0\right\}\frac{d\psi}{k}, \qquad (22)$$

$$j_1 = \int (h_0\rho_0 + [\mathbf{he}])^2\,\frac{d\psi}{k}, \qquad (23)$$

where J_0 = const, ρ_{00} = const, and $\Lambda(\varphi)$ is the operator for rotation by the angle φ, with the matrix

$$\Lambda(\varphi) = \begin{vmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{vmatrix}.$$

Equations (21)—(23) contain the law of motion of the particle, with radiation reaction taken into account. To determine the pressure force of the wave on the particle, F_{press}, we must take the time average of the derivative of the longitudinal momentum in the actual motion. Our aim is to include not only the radiation reaction force which appears directly in the equations of motion, but also the increment of velocity owing to the action of this force. Since the independent variable in the equations is the phase ψ, the mean value corresponding to F_{press} is found in the following way:

$$F_{\text{press}} = \left\langle \frac{d}{d\tau}(\gamma\dot z)\right\rangle = \int_0^T \frac{d}{d\tau}(\gamma\dot z)\,d\tau \Big/ \int_0^T d\tau = \int^{\psi} \frac{d}{d\psi}(\gamma\dot z)\,d\psi \Big/ \int^{\psi} \frac{\gamma}{J}\frac{d\psi}{k} = \left\langle \frac{d}{d\psi}(\gamma\dot z)\right\rangle_{\psi} \Big/ \left\langle \frac{\gamma}{Jk}\right\rangle_{\psi} \qquad (24)$$

and analogously

$$\langle \dot z\rangle_{\tau} = \left\langle \frac{\gamma\dot z}{J}\right\rangle_{\psi} \Big/ \left\langle \frac{\gamma}{J}\right\rangle_{\psi}, \qquad (25)$$

where $\langle\ \rangle_{\tau}$ and $\langle\ \rangle_{\varphi}$ denote averages with respect to τ and with respect to φ.

In order to make use of the solutions (21)—(23), we use (20) to transform (6) to the form

$$k\frac{d}{d\psi}(\gamma\dot z) = (\rho_0\mathbf{e}) + g\left\{\frac{1+J_0^2}{2}e^2 + \frac{J_0^2-1}{2}h_0^2\rho_0^2 + J_0^2 h_0(\rho_0[\mathbf{ne}]) + J_0\left(\rho_0 k\frac{d\mathbf{e}}{d\psi}\right) - \frac{J_0^2}{2}\rho_0^2(h_0\rho_0 + [\mathbf{ne}])^2 + (\rho_1\mathbf{e})\right\}. \qquad (26)$$

Substituting the expressions (21)—(23) in (26) and averaging, we get after a number of transformations the desired formula for the radiation pressure of a wave on a particle in a magnetic field:

$$F_{\text{press}} = gh_0^2\langle\dot z\rangle\left[1 + \frac{1}{(1-a)^2}\left(\frac{e_1}{k}\right)^2 + \frac{1}{(1+a)^2}\left(\frac{e_2}{k}\right)^2 - J_0^2\frac{1+\langle\dot z\rangle}{1-\langle\dot z\rangle}\right] + g[1-\langle\dot z\rangle]\left[\left(\frac{e_1}{1-a}\right)^2 + \left(\frac{e_2}{1+a}\right)^2\right]$$
$$\times\left[1 + \frac{1+a}{(1-a)^3}\left(\frac{e_1}{k}\right)^2 + \frac{1-a}{(1+a)^3}\left(\frac{e_2}{k}\right)^2 - J_0^2\frac{\langle\dot z\rangle}{1-\langle\dot z\rangle}\right] + g[1-\langle\dot z\rangle]\left[\frac{ae_1^2}{(1-a)^3} - \frac{ae_2^2}{(1+a)^3}\right]\left[1 - J_0^2\frac{1+\langle\dot z\rangle}{1-\langle\dot z\rangle}\right], \qquad (27)$$

where the parameter ρ_{00} has now been expressed in terms of $\langle\dot z\rangle$, the mean longitudinal velocity of the particle.

A characteristic feature of this expression is the presence of the coefficient g, which shows that the radiation pressure force is relatively extremely small, and also the presence of resonance denominators $1 \pm a$. The quantity $|a| \sim 1/J$ increases monotonically during the motion, since according to (6b) the quantity J decreases monotonically owing to emission of radiation. Therefore we can distinguish two physically different cases, according to whether the motion begins before or after resonance sets in. If the initial conditions are such that $|a| < 1$, then subsequently the particle will inevitably pass through the resonance region, in which, owing to the part of the force which is linear in the field, there is a great increase in the particle's energy. If, on the other hand, $|a| > 1$ at the beginning of the motion, then the particle will never pass through resonance and will be subject only to the action of radiation pressure.

If there is no applied constant magnetic field, or if this field is so small that $|a| \ll 1$, then, as we find from (14), the pressure force is given by the formula

$$F_{\text{press}} = g[1-\langle\dot z\rangle](e_1^2+e_2^2)\times\left[1 + \left(\frac{e_1}{k}\right)^2 + \left(\frac{e_2}{k}\right)^2 - J_0^2\frac{\langle\dot z\rangle}{1-\langle\dot z\rangle}\right], \qquad (28)$$

which is a special case of the formula (16).

Since we have defined F_{press} by Eq. (24), there has appeared in (27) the term

$$F_{h_0} = gh_0^2\langle\dot z\rangle\left[1 - J_0^2\frac{1+\langle\dot z\rangle}{1-\langle\dot z\rangle}\right], \qquad (29)$$

which is independent of the field of the wave and corresponds to the radiative energy loss of the particle in its motion in the external magnetic field H_0. It can be seen that F_{h_0} is always directed opposite to the motion of the particle. If the magnetic field H_0 is so large that $|a| \gg 1$, this term becomes the decisive one in the expression for F_{press}.

Let us now turn to the case of resonance, $|a| \approx 1$. If we neglect the radiation reaction, then, setting $J = h_0/k$ in Eq. (6a) and neglecting the last, "radiative," term in this equation, we get the result known from[3],

$$\rho \approx \frac{e_1}{h_0}\psi, \qquad \gamma \approx \frac{e_1^2}{2kh_0}\psi^2, \tag{30}$$

which gives the way ρ and γ increase with increase of ψ. The effect of the radiation reaction is that the increase of ρ and γ according to (30) cannot continue indefinitely. Owing to the effects of radiation there will be a disturbance, albeit a very slow one, of the resonance condition $|a| = 1$, because there is a change of the phase $\Delta\Phi$ of the particle relative to the wave.

Let us estimate the size of the maximum energy γ_{max} that the electron can acquire before the resonance gets out of tune. Substituting (30) in (6a) and making the justified assumption $h_{0\rho} \gg e$, we get an expression for $\Delta\Phi$

$$\Delta\Phi = \Delta\int \frac{h_0}{J}\frac{d\psi}{k} \approx gh_0\left(\frac{e_1}{k}\right)^2\frac{\psi^4}{12}. \tag{31}$$

It is obvious that the resonance will have got out of tune at phases $\psi \approx \psi_{max}$ for which $\Delta\Phi(\psi_{max}) \approx 1$. Then we have

$$\rho^2_{max} \approx (e_1 k / h_0^2)(12 / gh_0)^{1/2}, \tag{32}$$

and the value we get for the maximum energy the particle acquires in the resonance region is

$$\gamma_{max} \approx (e_1 / h_0)(3 / gh_0)^{1/2} \tag{33}$$

or

$$\gamma_{max} \approx 0.5 \cdot 10^3 E / H_0^{3/2} \tag{34}$$

(where E is in kV/cm and H_0 is in G).

It can be verified that the assumptions we have made and the estimate obtained are valid under the conditions

$$g^2h_0^2(12 / gh_0)^{1/2} \ll e / k \ll (12 / gh_0)^{1/2}, \tag{35}$$

which are practically always satisfied.

It is not hard to see that at values of the parameters for which the autoresonance method of acceleration is feasible the limit that the radiation reaction in principle imposes on the acceleration mechanism is an extremely high one.

---

[1] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Fizmatgiz, 1962.

[2] W. Heitler, The Quantum Theory of Radiation, Oxford, 1954 (third edition).

[3] A. A. Kolomenskiĭ and A. N. Lebedev, DAN SSSR **145**, 1259 (1962), Soviet Phys. Doklady **7**, 754 (1963); JETP **44**, 261 (1963), Soviet Phys. JETP **17**, 179 (1963).

[4] Ya. B. Faĭnberg and V. I. Kurilko, ZhTF **29**, 939 (1959), Soviet Phys. Tech. Phys. **4**, 855 (1960).

Translated by W. H. Furry
212