NONLINEAR INTERACTION OF ELECTROMAGNETIC WAVES IN A PLASMA

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The nonlinear interaction of long transverse waves with transverse or longitudinal waves is considered by means of the nonlinear equation for evolution of field fluctuations in a plasma. The effect of a transverse field on the nonlinear relaxation of longitudinal waves is determined.

INTRODUCTION

THIS paper is devoted to the theory of nonlinear interaction of electromagnetic waves in a plasma. The theory is based on the equations of nonlinear electrodynamics, the statistical averaging of which makes it possible to obtain a nonlinear equation for the evolution of the electromagnetic field fluctuations. Principal attention is paid to effects connected with the transverse field.

The general equation for the change in oscillation energy was used in the past primarily for the analysis of the nonlinear interaction of electron Langmuir waves. Unlike in our earlier paper [1], in which only the Coulomb interaction of the plasma particles was taken into account, we determine here, first, the role of the formation of transverse waves by coalescence of longitudinal waves[1)] and, second, the conditions under which nonlinear interaction is determined by the intermediate transverse wave.

We then consider the interaction between long transverse and longitudinal Langmuir waves in a plasma. First, we disclose the conditions under which scattering of the oscillations by the ions predominates, and show that in this case the interaction of the oscillations with the ions is several orders of magnitude larger than the previously considered interaction with electrons [5-6]. Second, we determine the conditions under which the time of transformation of the oscillations is determined by the interaction with the electrons, an interaction characterized by the intermediate transverse wave. Third, we consider the effect of coalescence of a longitudinal and a long transverse wave to produce a transverse wave. Last, we study the induced scattering of long transverse waves and show that in this case an important role is played by the interaction with the intermediate transverse wave. No such interaction was previously observed, because the analysis was confined either to the theory of scattering by longitudinal plasma fluctuations or to short waves [7-9] [2)].

1. EVOLUTION OF ELECTROMAGNETIC FLUCTUATIONS

The kinetics of the interacting waves in a plasma should be based on the equations of nonlinear electrodynamics. The nonlinearity is due here to the nonlinear material equation. For a collision-free plasma the latter can be easily obtained by solving the kinetic equation in the self-consistent approximation, represented in the form of a series in powers of the field

$$
\begin{aligned}
f(\mathbf{p}, \mathbf{r}, t) &= f_0(\mathbf{p}, \mathbf{r}, t) \\
&+ \sum_{n=1}^{\infty} (-ie)^n \int d\omega d\mathbf{k} e^{-i\omega t+i\mathbf{kr}}\, d\omega_1 d\mathbf{k}_1 \dots d\omega_n d\mathbf{k}_n \\
&\times g_+(\omega, \mathbf{k}, \mathbf{v}) \Gamma_{j(1)} g_+^{(1)} \Gamma_{j(2)} \dots g_+^{(n-1)} \Gamma_{j(n)} f_0(\omega_n, \mathbf{k}_n, \mathbf{p}) \\
&\times E_{j(1)}(\omega - \omega_1, \mathbf{k} - \mathbf{k}_1) \dots E_{j(n)}(\omega_{n-1} - \omega_n, \mathbf{k}_{n-1} - \mathbf{k}_n).
\end{aligned} \tag{1.1}
$$

Here e—charge, **v**—velocity, **p**—particle momentum, $f_0(\mathbf{p}, \mathbf{r}, t)$—zeroth-approximation distribution function, and

$$
\mathbf{E}(\mathbf{r}, t) = \int d\omega d\mathbf{k} e^{-i\omega t+i\mathbf{kr}}\, \mathbf{E}(\omega, \mathbf{k});
$$

$$
f_0(\mathbf{p}, \mathbf{r}, t) = \int d\omega d\mathbf{k} e^{-i\omega t+i\mathbf{kr}}\, f_0(\omega, \mathbf{k}, \mathbf{p}), \tag{1.2}
$$

[1)]See also[2,4] concerning the coalescence (and decay) of waves.

[2)]Sholokhov[9] showed that the transverse fluctuations affect the scattering of transverse waves in a plasma with a beam.

$$g_+^{(n)} \equiv g_+(\omega_n, \mathbf{k}_n, \mathbf{v}) = \frac{1}{\omega_n - \mathbf{k}_n\mathbf{v} + i0}, \tag{1.3}$$

$$\Gamma_{j(n)} \equiv \Gamma_{j(n)}(\omega_{n-1} - \omega_n, \mathbf{k}_{n-1} - \mathbf{k}_n, \mathbf{p})$$
$$= \frac{1}{\omega_{n-1} - \omega_n + i0}[\delta_{lj(n)}(\omega_{n-1} - \omega_n - (\mathbf{k}_{n-1} - \mathbf{k}_n \cdot \mathbf{v}))$$
$$+ (\mathbf{k}_{n-1} - \mathbf{k}_n)_l v_{j(n)}] \frac{\partial}{\partial n}. \tag{1.4}$$

The Green's function of the free motion g_+ and the vertex part Γ_j have been written out for the case of a plasma without strong fields.

The solution (1.1) corresponds to the customarily employed approximation of adiabatically turning-on the field in an infinitely remote past. As follows from the papers of Klimontovich and one of the authors [10,11], such a solution does not make it possible to construct a complete statistical field theory. However, this solution is just sufficient for the description of the processes that are of the higher order in nonlinearity. For example, the solution (1.1) is sufficient to describe the induced scattering of waves by particles and, strictly speaking, is not sufficient for the description of simple scattering. We shall be interested in what follows only in the processes of higher order in nonlinearity. We therefore confine ourselves to the solution of (1.1), which corresponds to adiabatically turning-on the interaction in the infinitely remote past.

Substituting (1.1) into the definition of the current density

$$\mathbf{j} = \Sigma e \int d\mathbf{p} f \mathbf{v},$$

we obviously obtain immediately the sought nonlinear material equation, with the aid of which we can write the field equations in the form

$$\frac{c^2k^2}{\omega^2}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)E_j(\omega, \mathbf{k}) = \sum_{n=1}^{\infty} \int d\omega_1 \ldots d\mathbf{k}_n$$
$$\times \varepsilon_{ij(1)\ldots j(n)}(\omega, \mathbf{k}, \omega_1, \mathbf{k}_1, \ldots, \omega_n, \mathbf{k}_n)$$
$$\times E_{j(1)}(\omega - \omega_1, \mathbf{k} - \mathbf{k}_1) \ldots E_{j(n)}(\omega_{n-1} - \omega_n, \mathbf{k}_{n-1} - \mathbf{k}_n), \tag{1.5}$$

where it is assumed that the plasma in the state with f_0 contains no space charges or currents, and where we also use the notation

$$\varepsilon_{ij(1)\ldots j(n)}(\omega, \mathbf{k}, \omega_1, \mathbf{k}_1, \ldots, \omega_n, \mathbf{k}_n) = \delta_{n1}\delta_{ij}\delta(\omega_1)\delta\mathbf{k}_1)$$
$$- \sum 4\pi(-ie)^{n+1} \, d\mathbf{p}(v_i/\omega) g_+(\omega, \mathbf{k}, \mathbf{v}) \Gamma_{j(1)} g_+^{(1)} \Gamma_{j(2)}$$
$$\ldots g_+^{(n-1)} \Gamma_{j(n)} f_0(\omega_n, \mathbf{k}_n, \mathbf{p}). \tag{1.6}$$

The right side of (1.6) is a sum over all the particle species.

We are interested in the case when the dependence of f_0 on the time and on the coordinates can be neglected. In such a case

$$\varepsilon_{ij(1)\ldots j(n)}(\omega, \mathbf{k}, \omega_1, \mathbf{k}_1, \ldots, \omega_n, \mathbf{k}_n)$$
$$= \delta(\omega_n)\delta(\mathbf{k}_n)\varepsilon_{ij(1)\ldots j(n)}(\omega, \mathbf{k}, \omega_1, \mathbf{k}_1, \ldots, \omega_{n-1}, \mathbf{k}_{n-1}).$$

In particular

$$\varepsilon_{ij}(\omega, \mathbf{k}, \omega', \mathbf{k}') = \delta(\omega')\delta(\mathbf{k}')\varepsilon_{ij}(\omega, \mathbf{k}),$$

where $\epsilon_{ij}(\omega, \mathbf{k})$—complex dielectric tensor.

We must make one essential remark. Namely, the Green's functions $g_+(\omega, \mathbf{k}, \mathbf{v})$ in (1.1) and (1.6) become infinite, in accordance with (1.3), when the frequency and the wave vector are equal to zero. However, zero Green's-function arguments correspond to a homogeneous and static state of the medium, or, more generally speaking, to a slowly varying state. Assuming that the description of the slow variation of the plasma is given by the function f_0, we exclude from the integrals (1.1) and (1.5) the points that correspond simultaneously to zero values of the frequencies and the wave numbers which are arguments of the Green's functions. In other words, we shall assume that only ω_n and $\mathbf{k}_n$ can assume simultaneously zero values.

Assuming the nonlinearity to be slowly varying, we can readily obtain the spectra of the natural oscillations of the electromagnetic field in the plasma with the aid of the usual procedure of time-averaging of the following expression, which determines the time variation of the amplitudes of weakly-damped almost-monochromatic oscillations:

$$\frac{d}{dt}\{E_i^*(\omega, \mathbf{k})E_j(\omega, \mathbf{k})\}\left\{\frac{\partial(\omega\varepsilon_{ij}^H(\omega, \mathbf{k})}{\partial\omega} + \frac{c^2k^2}{\omega^2}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\right\}$$
$$+ \operatorname{div}\mathbf{S} = i\omega \sum_{n=1}^{\infty} \int d\omega_1, \ldots, d\mathbf{k}_{n-1}\varepsilon_{ij(1)\ldots j(n)}$$
$$\times (\omega, \mathbf{k}, \omega_1, \mathbf{k}_1, \ldots, \omega_{n-1}, \mathbf{k}_{n-1}) \, E_i^*(\omega, \mathbf{k})E_{j(1)}$$
$$\times (\omega - \omega_1, \mathbf{k} - \mathbf{k}_1) \ldots E_{j(n-1)}(\omega_{n-2} - \omega_{n-1}, \mathbf{k}_{n-2} - \mathbf{k}_{n-1})$$
$$\times E_{j(n)}(\omega_{n-1}, \mathbf{k}_{n-1}) + \text{c.c.} \tag{1.7}$$

Here $\epsilon_{ij}^H(\omega, \mathbf{k})$—Hermitian part of the dielectric tensor, and $\mathbf{S}$—field energy flux density vector in the medium [12,13]. Equation (1.7), which is the r equation of energy conservation, actually corresponds to the abbreviated field equations customarily used in nonlinear optics [14]. As is well known, such equations are useful when the phase relations between the different waves are fixed. For the case of random phases of interest to us it is necessary to make one more step, connected with the averaging of (1.7) over the phases or over the statistical ensemble.

We confine ourselves below to a description of processes for which it is sufficient to retain in (1.7) terms of fourth power in the field inclusive. In the statistical averaging of the products of the components of the electric field we take into account the fact that

$$\langle E_j^*(\omega', \mathbf{k}')E_i(\omega, \mathbf{k})\rangle = \delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')(E_iE_j)_{\omega,\mathbf{k}}.$$

If we neglect the correlation between the amplitudes of the different waves, which is equivalent to neglecting the nonlinear effects, the average of the product of the field amplitudes can be represented in the form of a sum of products of pair correlators. In the same approximation, the average of the product of three field amplitudes vanishes. However, an account of the first nonlinear correction, which can be readily obtained with the aid of (1.5), makes the average of the product of the three field amplitudes proportional to the product of two pair correlators. We then obtain from (1.7) after averaging

$$\frac{1}{\omega}\frac{d}{dt}(E_jE_i)_{\omega,\mathbf{k}}\left\{\frac{\partial(\omega\varepsilon_{ij}^H(\omega,\mathbf{k}))}{\partial\omega}+\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\right\}$$
$$+\operatorname{div}\left(\frac{\mathbf{S}}{\omega}\right)_{\omega,\mathbf{k}} = i\left[\varepsilon_{ij}(\omega,\mathbf{k})-\varepsilon_{ji}^*(\omega,\mathbf{k})\right](E_jE_i)_{\omega,\mathbf{k}}$$
$$-2\operatorname{Im}\Big\{(E_{j(1)}E_i)_{\omega,\mathbf{k}}\int d\omega' d\mathbf{k}'\,(E_{j(2)}E_{j(3)})_{\omega',\mathbf{k}'}$$
$$\times V_{ij(2)j(1)j(3)}(\omega,\mathbf{k},\omega',\mathbf{k}')\Big\}+\operatorname{Im}\int d\omega' d\mathbf{k}'\{A_{ir}^*(\omega,\mathbf{k})$$
$$\times S_i(\omega,\mathbf{k},\omega',\mathbf{k}')\,S^*_{rj(3)j(4)}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$\times(E_{j(2)}E_{j(4)})_{\omega',\mathbf{k}'}(E_{j(1)}E_{j(3)})_{\omega-\omega',\mathbf{k}-\mathbf{k}'}$$
$$+2S_{ij(1)j(2)}(\omega,\mathbf{k},\omega',\mathbf{k}')A_{j(1)r}(\omega-\omega',\mathbf{k}-\mathbf{k}')$$
$$\times S_{rj(4)j(3)}(\omega-\omega',\mathbf{k}-\mathbf{k}',\omega,\mathbf{k})(E_{j(2)}E_{j(4)})_{\omega',\mathbf{k}'}$$
$$\times(E_{j(3)}E_i)_{\omega,\mathbf{k}}\},\qquad(1.8)$$

$$A_{ij}(\omega,\mathbf{k})=\left[\varepsilon_{ij}(\omega,\mathbf{k})-\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\right]^{-1},\qquad(1.9)$$

$$V_{ij(2)j(1)j(3)}(\omega,\mathbf{k},\omega',\mathbf{k}')=\varepsilon_{ij(2)j(1)j(3)}(\omega,\mathbf{k},\omega+\omega',\mathbf{k}+\mathbf{k}',\omega',\mathbf{k}')$$
$$+\varepsilon_{ij(2)j(3)j(1)}(\omega,\mathbf{k},\omega+\omega',\mathbf{k}+\mathbf{k}',\omega,\mathbf{k}),\qquad(1.10)$$

$$S_{ij(1)j(2)}(\omega,\mathbf{k},\omega',\mathbf{k}')=\varepsilon_{ij(1)j(2)}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$+\varepsilon_{ij(2)j(1)}(\omega,\mathbf{k},\omega-\omega',\mathbf{k}-\mathbf{k}').\qquad(1.11)$$

When using Eq. (1.8), which describes the evolution of the electromagnetic field fluctuations, we must take into account the fact that the index i remembers the polarization of the oscillations. Then, for example, for an isotropic plasma, we obtain from (1.8) two equations that enable us to consider the time variation of the longitudinal and transverse field fluctuations.

We note here that Eq. (1.8) does not correspond to the equation obtained by Kadomtsev and Petviashvili [15,16] with the aid of the Wiener method; in our opinion this method not only fails to facilitate the construction of the vector field fluctuation theory, but does not permit a description of all the nonlinear interactions of equal order in nonlinearity. Our derivation of (1.8) is close in spirit to that used in [16] to describe stationary turbulence in a plasma, with account taken of Coulomb interaction only.

2. NONLINEAR INTERACTION BETWEEN LONGITUDINAL AND LONGITUDINAL WAVES

By way of a first application of the general equation (1.8), we consider longitudinal oscillations of an isotropic plasma. We can then write down the following equation for the nonlinear interaction of longitudinal waves:

$$\frac{d}{dt}(E_l^2)_{\omega,\mathbf{k}}\frac{\partial\varepsilon^{l'}(\omega,\mathbf{k})}{\partial\omega} = -2\varepsilon^{l''}(\omega,\mathbf{k})(E_l^2)_{\omega,\mathbf{k}}$$
$$-2(E_l^2)_{\omega,\mathbf{k}}\int d\omega' d\mathbf{k}'(E_l^2)_{\omega',\mathbf{k}'}$$
$$\times\operatorname{Im}V_{ijrs}(\omega,\mathbf{k},\omega',\mathbf{k}')\frac{k_ik_rk_j'k_s'}{k^2(k')^2}$$
$$+\int d\omega' d\mathbf{k}' d\omega'' d\mathbf{k}''\delta(\omega-\omega'-\omega'')$$
$$\times\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\Big\{2(E_l^2)_{\omega,\mathbf{k}}(E_l^2)_{\omega',\mathbf{k}'}$$
$$\times\operatorname{Im}\frac{1}{\varepsilon^l(\omega'',\mathbf{k}'')}\frac{k_ik_s''k_m'k_r''k_n'k_j}{(kk'k'')^2}S_{ism}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$\times S_{rnj}(\omega'',\mathbf{k}'',\omega,\mathbf{k})+\pi\operatorname{sign}\varepsilon^{l''}(\omega,\mathbf{k})\,\delta\left[\varepsilon^{l'}(\omega,\mathbf{k})\right]$$
$$\times\left|\frac{k_ik_j'k_r''}{kk'k''}S_{irj}(\omega,\mathbf{k},\omega',\mathbf{k}')\right|^2(E_l^2)_{\omega',\mathbf{k}'}(E_l^2)_{\omega'',\mathbf{k}''}\Big\}$$
$$+2\int d\omega' d\mathbf{k}' d\omega'' d\mathbf{k}''\delta(\omega-\omega'-\omega'')\,\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')$$
$$\times(E_l^2)_{\omega,\mathbf{k}}(E_l^2)_{\omega',\mathbf{k}'}\operatorname{Im}\Big\{\frac{\delta_{sr}-k_s''k_r''/k''^2}{\varepsilon^{tr}(\omega'',\mathbf{k}'')-c^2k''^2/\omega''^2}\frac{k_ik_m'k_n'k_j}{(kk')^2}$$
$$\times S_{ism}(\omega,\mathbf{k},\omega',\mathbf{k}')\,S_{rnj}(\omega'',\mathbf{k}'',\omega,\mathbf{k})\Big\}.\qquad(2.1)$$

Here ϵ^{tr} and ϵ^l are respectively the transverse and longitudinal dielectric constants, with $\epsilon^{l'}$ and $\epsilon^{l''}$ the real and imaginary parts. In deriving (2.1) from (1.8) we assumed that $(E_iE_j)_{\omega,\mathbf{k}} = (E_l^2)_{\omega,\mathbf{k}}\times k_ik_j/k^2$, meaning that only longitudinal oscillations of the electromagnetic field are considered.

Equation (2.1) without the last term of the right side is equivalent to Eq. (10) of [17] and Eq. (1.1) of [1]. For the time-independent oscillation intensity, Eq. (2.1) goes over into that used by Petviashvili [16,18] (see also [15]) only under conditions when the last term of the right side of (2.1) can be neglected. This difference is due to the fact that our equation describes the influence of the transverse field in a plasma on the longitudinal oscillations. In the present section we expose the role of this influence and determine the conditions under which the effects of the transverse field turn out to be essential for the nonlinear interaction of longitudinal oscillations.

For convenience we write first an expression for the rate of time variation of the energy of the electronic Langmuir longitudinal oscillations

$$W_l(\mathbf{k}) = (2\pi)^3 \int_0^\infty d\omega \frac{(E_l^2)_{\omega,\mathbf{k}}}{4\pi} \frac{\partial[\omega\varepsilon^{l'}(\omega, \mathbf{k})]}{\partial\omega}, \quad (2.2)$$

in which no account is taken of the last term of the right side of (2.1), and which was considered in detail in [1][3]. When the influence of the ions on the wave scattering is insignificant, as is the case when

$$(\omega - \omega')^2 = \frac{9}{4} v_{Te}^2 r_{De}^2 (k^2 - k'^2)^2 \gg v_{Ti}^2 (\mathbf{k} - \mathbf{k}')^2 \times \ln\left[\frac{e_i^2 M T_e^3}{e^2 m T_i^3}\right],$$

the principal role is assumed by scattering from the electrons and

$$\frac{dW_l(\mathbf{k})}{dt} = -\frac{3}{2(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_l(\mathbf{k})}{\varkappa T_e} r_{De}^6 \int d\mathbf{k}' W_l(\mathbf{k}') \times \left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2 \frac{k^2 - k'^2}{|\mathbf{k} - \mathbf{k}'|^3} [\mathbf{k}\mathbf{k}']^2. \quad (2.3)^*$$

In the case of the opposite inequality

$$\frac{9}{4} v_{Te}^2 r_{De}^2 (k^2 - k'^2)^2 \ll v_{Ti}^2 (\mathbf{k} - \mathbf{k}')^2 \ln\left[\frac{e_i^2 M T_e^3}{e^2 m T_i^3}\right]$$

the scattering by ions predominates and

$$\frac{dW_l(\mathbf{k})}{dt} = -\frac{3}{8(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_l(\mathbf{k})}{\varkappa T_e} \frac{v_{Te}}{v_{Ti}} r_{De}^4 \int d\mathbf{k}' W_l(\mathbf{k}') \times \left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2 \frac{k^2 - k'^2}{|\mathbf{k} - \mathbf{k}'|} \left[\frac{r_{Di}/r_{De}}{F + (r_{Di}/r_{De})^2}\right]^2 \times \left[1 + \frac{3}{4} r_{De}^2 (k^2 - k'^2)\right] \exp\left\{-\frac{9}{8} \frac{v_{Te}^2}{v_{Ti}^2} r_{De}^2 \frac{(k^2 - k'^2)^2}{(\mathbf{k} - \mathbf{k}')^2}\right\} \quad (2.4)$$

Here e, e_i, m, M, T_e, and T_i—charge, mass, and temperature of the electrons and ions, respectively, $\omega_{Le} = \sqrt{4\pi e^2 N/m}$—Langmuir frequency of the electrons, $v_{Te} = \sqrt{\kappa T_e/m}$ and $v_{Ti} = \sqrt{\kappa T_i/M}$—thermal velocities, $r_{De} = \sqrt{\kappa T_e/4\pi e^2 N}$ and $r_{Di} = \sqrt{\kappa T_i/4\pi |e| e_i N}$—Debye radii of the electrons and ions. Finally,

$$F = 0, \text{ if } 1 \gg v_{Ti}^2 (\mathbf{k} - \mathbf{k}')^2 / (\omega - \omega')^2 \gg \ln^{-1}[e_i^2 M T_e^3 / e^2 m T_i^3], \quad (2.5)$$

$$F = 1, \text{ if } v_{Ti}^2 (\mathbf{k} - \mathbf{k}')^2 \gg (\omega - \omega')^2. \quad (2.6)$$

The first effect of the nonlinear interaction of longitudinal waves, due to the last term of the right side of (2.1), is the coalescence of two longitudinal waves into a single transverse one. The corresponding contribution is due to the vanishing of the expression

$$\varepsilon^{tr}(\omega'', \mathbf{k}'') - c^2 k''^2 / \omega''^2. \quad (2.7)$$

The coalescence of two longitudinal waves with frequencies that are approximately equal to the electronic Langmuir frequency leads here to the occurrence of a transverse wave of frequency $2\omega_{Le}$ and wave vector of absolute magnitude $\sqrt{3}\omega_{Le}/c$. All the oscillation frequencies participating in such a process are large compared with the electron thermal velocity divided by the oscillation wavelength. We can therefore use the following approximate formula

$$S_{irj}(\omega, \mathbf{k}, \omega', \mathbf{k}') = \frac{i}{\omega\omega'\omega''} \frac{4\pi e^3 N}{m^2} \left\{\delta_{ir} \frac{k_j'}{\omega'} + \delta_{rj} \frac{k_i}{\omega} + \delta_{ij} \frac{k_r''}{\omega''}\right\} \quad (2.8)$$

As a result we get for the rate of decrease of the energy of the transverse oscillations, due to their coalescence and transformation into transverse oscillations,

$$\left[\frac{dW_l(\mathbf{k})}{dt}\right]_{\text{coal}} = -\frac{1}{12(2\pi)^2} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_l(\mathbf{k})}{\varkappa T_e} \frac{c^2}{v_{Te}^2} r_{De}^7 \int d\mathbf{k}' W_l(\mathbf{k}') \times \frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2} (k^2 - k'^2)^2 \delta\left[3 - \frac{c^2}{\omega_{Le}^2} (\mathbf{k} + \mathbf{k}')^2\right]. \quad (2.9)$$

From this, for example for an isotropic distribution of the longitudinal oscillations with wavelengths on the order of $c/\omega_{Le} \equiv \lambda_0$, it follows that the time within which the energy of the longitudinal oscillations is decreased at the expense of the formation of the transverse waves is given by the formula

$$\tau_{\text{coal}} \sim 10^3 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_l} \left(\frac{c}{v_{Te}}\right)^5. \quad (2.10)$$

A time of the same order characterizes the coalescence of the longitudinal oscillations in wave packets with larger wave vectors but with width $\sim \omega_{Le}/c$.

When the interaction between the ions is the principal factor, it is necessary to compare formula (2.10) with the time of the spectral energy

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3)The results of[1] were also partially used subsequently by Gailitis and Tsytovich.[5]

*$[\mathbf{k}\mathbf{k}'] = \mathbf{k} \times \mathbf{k}'$.

pumping due to the scattering by the ions. For wavelengths $\sim\lambda_0$ the order of magnitude of this time, as follows from (2.4), is

$$\tau \sim 10^2 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_l} \left(\frac{c}{v_{Te}}\right)^4 \frac{v_{Ti}}{v_{Te}} \tag{2.11}$$

and turns out to be smaller than the coalescence time (2.10). In other words, in this case, when the longitudinal oscillations relax, only a small fraction of their energy goes over as a result of coalescence into the transverse oscillations. The fraction of this energy is characterized by the ratio of the time of the spectral pumping, due to the scattering by the ions, to the coalescence time. This ratio, in accordance with (2.10) and (2.12), is equal to (v_{Ti}/c). We note that inasmuch as the intensity of the longitudinal oscillations can exceed greatly the intensity of the equilibrium noise, the transverse radiation resulting from the coalescence of the longitudinal waves can be quite large.

It must be pointed out, finally, that for wavelengths of the order of λ_0 the scattering by the ions greatly exceeds the scattering by the electrons under conditions when $(T_e/T_i)\kappa T_e \ll (m/M)mc^2$. In the opposite case, scattering by the ions is negligible. In particular, scattering by ions is neglible if $(T_e/T_i)\kappa T_e > 2\times 10^6$ deg, in a hydrogen plasma and $(T_e/T_i)\kappa T_e > 10^4$ deg in a mercury plasma. The spectral pumping is determined here by the scattering from the electrons and, as follows from (2.5), takes place for wavelengths $\sim\lambda_0$ within a time on the order of (see [1] for more details)

$$\tau \sim 10^2 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_l} \left(\frac{c}{v_{Te}}\right)^6. \tag{2.12}$$

From a comparison of (2.10) and (2.12) we see that under such conditions the pumping over the longitudinal-wave spectrum is insignificant.

In addition to the already considered coalescence effect, another nonlinear interaction of longitudinal waves arises, again as a result of the last term in the right side of (2.1). Such an interaction arises as a contribution from the region of values of ω'' and $\mathbf{k}''$ for which (2.7) can no longer vanish. We have in mind here the contribution made to the induced scattering of the longitudinal insulation by the interaction, which can be set in correspondence with the following picture: The lines of the incident and scattering longitudinal waves enter the two vertices of the triangle produced by the virtual-electron lines, while the line going out of the third vertex is that of the intermediate transverse wave, which is absorbed by the electron, thereby producing the scattering. We shall refer to such an interaction as "scattering via an intermediate wave" to distinguish it from the previously considered process [1], when the longitudinal wave was intermediate (for brevity we shall use the expression "Coulomb scattering" for this interaction).

The contribution made to the rate of change of the energy of the longitudinal oscillations at the expense of scattering via the transverse wave is

$$\delta\left[\frac{dW_l(\mathbf{k})}{dt}\right]_{\text{scatt}} = -\frac{1}{6(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_l(\mathbf{k})}{\varkappa T_e} r_{De}^2$$
$$\times \int d\mathbf{k}' W_l(\mathbf{k}') \frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2} \frac{|\mathbf{k}-\mathbf{k}'|}{(k^2-k'^2)} \left[1 - 4\frac{c^2 \mathbf{k}\mathbf{k}'}{\omega_{Le}^2}\right]$$
$$\times \left\{\frac{4}{9} \frac{c^2(\mathbf{k}-\mathbf{k}')^4}{v_{Te}^4 (k^2-k'^2)^2} + 2\frac{c^2}{v_{Te}^2} + \frac{\pi}{2} \frac{1}{r_{De}^2 (\mathbf{k}-\mathbf{k})^2}\right\}^{-1} \tag{2.13}$$

In deriving this relation we use for the tensor S the following approximate expression, obtained for an isotropic particle distribution under the conditions $\omega \gg \mathbf{k}\cdot\mathbf{v}$ and $\omega' \gg \mathbf{k}'\cdot\mathbf{v}$:

$$S_{irl}(\omega, \mathbf{k}, \omega', \mathbf{k}') = -\frac{i}{\omega\omega'} \frac{4\pi e^3 N}{m^2} \left\{ \frac{1}{\omega''} \left[\delta_{ri}\left(\frac{k_l}{\omega} - \frac{k_l'}{\omega'}\right) - \delta_{rl}\left(\frac{k_i}{\omega} - \frac{k_i'}{\omega'}\right)\right] + \frac{k_r''}{(k'')^4} \right.$$
$$\times \left[\frac{\mathbf{k}\mathbf{k}''}{\omega^2} k_l k_i'' + \frac{\mathbf{k}'\mathbf{k}''}{\omega'^2} k_i' k_l'' + \frac{\mathbf{k}\mathbf{k}'}{\omega\omega'} k_i'' k_l''\right]$$
$$-\left[\frac{k_l}{\omega}\left(\delta_{ri}' + \frac{k_r'' k_i''}{k''^2}\right) + \frac{k_i'}{\omega'}\left(\delta_{lr} - \frac{k_r'' k_l''}{k''^2}\right)\right]$$
$$\times \frac{1}{N} \int \frac{d\mathbf{p} f_0}{\omega'' + i0 - \mathbf{k}''\mathbf{v}} + \frac{k_r''}{(k'')^2}$$
$$\times \left[\delta_{il} + \frac{\omega''}{(k'')^2}\left(\frac{k_l k_i''}{\omega} + \frac{k_i' k_l''}{\omega'}\right) + \left(\frac{\omega''}{k''}\right)^2 \left(\frac{k_l k_i''}{\omega^2} \mathbf{k}\mathbf{k}'' \right.\right.$$
$$\left.\left. + \frac{\mathbf{k}'\mathbf{k}''}{\omega'^2} k_i' k_l'' + \frac{\mathbf{k}\mathbf{k}'}{\omega\omega'} k_i'' k_l''\right)\right] \frac{1}{N} \int \frac{d\mathbf{p}(\mathbf{k}''\partial f_0/\partial \mathbf{p})}{\omega'' + i0 - \mathbf{k}''\mathbf{v}}$$
$$-\left\{\frac{k''_r}{(k'')^2}\left[\frac{k_l}{\omega^2}\left(k_i - \frac{\mathbf{k}\mathbf{k}''}{k''^2} k_i''\right) + \frac{k_i'}{\omega'^2}\left(k_l' - \frac{\mathbf{k}'\mathbf{k}''}{k''^2} k_l''\right)\right.\right.$$
$$\left. + \frac{\mathbf{k}\mathbf{k}'}{\omega\omega'}\left(\delta_{il} - \frac{k_i'' k_l''}{k''^2}\right)\right]$$
$$+ \frac{k_l}{\omega^2 k''^2}\left[k_i''\left(k_r - \frac{\mathbf{k}\mathbf{k}''}{k''^2} k_r''\right) + \mathbf{k}\mathbf{k}'\left(\delta_{ir} - \frac{k_i'' k_r''}{k''^2}\right)\right]$$
$$+ \frac{k_i'}{\omega'^2 k''^2}\left[k_l''\left(k_r' - \frac{\mathbf{k}'\mathbf{k}''}{k''^2} k_r''\right) + \mathbf{k}'\mathbf{k}''\left(\delta_{lr} - \frac{k_l'' k_r''}{k''^2}\right)\right]$$
$$\left. + \frac{\mathbf{k}\mathbf{k}'}{\omega\omega' k''^2}\left[k_i''\left(\delta_{rl} - \frac{k_r'' k_l''}{k''^2}\right) + k_l''\left(\delta_{ir} - \frac{k_i'' k_r''}{k''^2}\right)\right]\right\}$$
$$\left. \times \frac{1}{N} \int d\mathbf{p} f_0 \frac{\mathbf{k}''\mathbf{v}}{\omega'' + i0 - \mathbf{k}''\mathbf{v}} \right\}, \tag{2.14}$$

and also the expression derivable from it

$$S_{ism}(\omega,\mathbf{k},\omega',\mathbf{k}')\,S_{rnj}(\omega'',\mathbf{k}'',\omega,\mathbf{k})\left(\delta_{sr}-\frac{k_s''k_r''}{k''^2}\right)$$
$$=\frac{1}{(\omega\omega'\omega'')^2}\left(\frac{4\pi Ne^3}{m^2}\right)^2\Big\{\delta_{is}k_m''-\delta_{sm}k_i''$$
$$-(k_m\delta_{is}+k_i'\delta_{sm})\frac{1}{N}\int d\mathbf{p}\,f_0\frac{\omega''}{\omega''+i0-\mathbf{k}''\mathbf{v}}\Big\}$$
$$\times\Big\{\delta_{jr}k_n''-\delta_{rn}k_j''-(k_n\delta_{jr}+k_j'\delta_{rn})\frac{1}{N}$$
$$\times\int d\mathbf{p}\,f_0\frac{\omega''}{\omega''+i0-\mathbf{k}''\mathbf{v}}\Big\}\left(\delta_{sr}-\frac{k_s''k_r''}{k''^2}\right)\qquad(2.15)$$

We note immediately that under the conditions when the wavelength of the longitudinal oscillations is small compared with λ_0, expression (2.13) leads to effects which are only small corrections to the effects resulting from the theory that takes into account only the longitudinal (Coulomb) interaction of the plasma particles. We shall therefore focus our attention on the opposite case. We assume that the following inequality is satisfied:

$$\omega_{Le}v_{Te}\Delta k\gg c^2k^2,\qquad(2.16)$$

where $\Delta k = |\mathbf{k}-\mathbf{k}'|$. Then the right side of (2.13) takes on the form

$$-\frac{2}{3(2\pi)^{7/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k}')}{\varkappa T_e}r_{De}^4\int d\mathbf{k}'W_l(\mathbf{k}')\frac{[\mathbf{kk}']^2}{(kk')^2}\frac{|\mathbf{k}-\mathbf{k}'|^3}{(k^2-k'^2)}.\qquad(2.17)$$

This expression corresponds to a situation wherein the oscillation energy is not transferred to the particles, and the relaxations of the oscillations constitute pumping of the longitudinal oscillations over the spectrum, from the short waves to the long ones.

The corresponding time of spectral pumping is

$$\tau\sim10^3\frac{Nr_{De}^3}{\omega_{Le}}\frac{\varkappa T_e}{W_l(k)}\frac{\Delta k}{r_{De}^4k^5}\qquad(2.18)$$

and is much shorter than the value obtained for the time connected with the induced scattering of the waves by the electrons when only the Coulomb interaction is taken into account. The latter, in turn, is shorter than the time of scattering by the waves if $v_{Ti}c^2\ll v^3_{Te}$ for wave numbers $k \gg (v_{Ti}/v_{Te})r_{De}^{-1}$. On the other hand, if the spectral pumping time connected with the scattering by the ions is shorter than the Coulomb time of scattering by the electrons, then (2.18) turns out to be smaller than the ion relaxation time if $\Delta k < 0.3\,k\sqrt{v_{Ti}/v_{Te}}$.

Finally, if the inequality $\omega_{Le}v_{Te}\Delta k\ll c^2k^2$, the opposite of (2.16), is satisfied, then the right side of (2.13) can be written in the form

$$-\frac{3}{8(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}r_{De}^2\frac{v_{Te}^4}{c^4}\int d\mathbf{k}'W_l(\mathbf{k}')$$
$$\times\frac{[\mathbf{kk}']^2}{(kk')^2}\frac{(k^2-k'^2)}{|\mathbf{k}-\mathbf{k}'|^3}.\qquad(2.19)$$

The spectral pumping time corresponding to this expression is equal to

$$\tau\sim10^2\frac{Nr_{De}^3}{\omega_{Le}}\frac{\varkappa T_e}{W_l(\mathbf{k})}\left(\frac{c}{v_{Te}}\right)^4\frac{1}{r_{De}^2k\Delta k}.\qquad(2.20)$$

This time is much shorter than the Coulomb time of scattering of the wave by the electrons. Therefore formula (2.19) describes spectral pumping of waves only when the scattering by the ions is small compared with the Coulomb scattering by the electrons, or when the time (2.20) is short compared with the time of induced scattering by the ions. The latter is satisfied if $c^2k^2 \ll 0.1\,\omega_{Le}^2(v_{Ti}v_{Te}/c^2)$. On the other hand, scattering by the ions is small compared with the Coulomb scattering by the electrons if $v_{Ti}c^2 \ll v_{Te}^3$.

In concluding this section we note that at first glance it may appear that the relativistic corrections to the Coulomb scattering could turn out to be of the same order of magnitude as the effects connected with the transverse intermediate waves. The analysis actually shows, however, that the relativistic additions to the Coulomb scattering are always corrections, whereas, as shown above, the scattering connected with the transverse intermediate wave may become the principal factor in the spectral distribution of the longitudinal oscillations. We note that this takes place not only in the case of interaction of longitudinal waves, but in all other cases considered below.

3. NONLINEAR INTERACTION OF LONGITUDINAL AND TRANSVERSE WAVES

This section is devoted to the nonlinear interaction between longitudinal and transverse waves. Here

$$(E_iE_j)_{\omega,\mathbf{k}}=\frac{k_ik_j}{k^2}(E_l^2)_{\omega,\mathbf{k}}+\frac{1}{2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\cdot(E_{tr}^2)_{\omega,\mathbf{k}}.\qquad(3.1)$$

Confining ourselves to the interaction between longitudinal and transverse waves, we obtain from (1.8) the following two equations:

$$\frac{d}{dt}(E_l^2)_{\omega,\mathbf{k}}\frac{\partial\varepsilon^{l'}(\omega,\mathbf{k})}{\partial\omega}=$$
$$-(E_l^2)_{\omega,\mathbf{k}}\int d\omega'd\mathbf{k}'(E_{tr}^2)_{\omega',\mathbf{k}'}\frac{k_ik_r}{k^2}\operatorname{Im}V_{ijrs}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$\times\left(\delta_{js}-\frac{k_j'k_s'}{k'^2}\right)+(E_l^2)_{\omega,\mathbf{k}}\operatorname{Im}\int d\omega'd\mathbf{k}'d\omega''d\mathbf{k}''$$
$$\times\delta(\omega-\omega'-\omega'')\,\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\frac{k_ik_j}{k^2}\left(\delta_{mn}-\frac{k_m'k_n'}{k'^2}\right)$$

$$\times (E_{tr}^2)_{\omega',\mathbf{k}'} S_{ism}(\omega, \mathbf{k}, \omega', \mathbf{k}') S_{rnj}(\omega'', \mathbf{k}'', \omega, \mathbf{k})$$
$$\times \left\{\frac{k_s''k_r''}{k''^2}\frac{1}{\varepsilon^l(\omega'', \mathbf{k}'')} + \frac{\delta_{sr} - k_s''k_r''/k''^2}{\varepsilon^{tr}(\omega'', \mathbf{k}'') - c^2k''^2/\omega''^2}\right\}$$
$$+ \frac{\pi}{2}\operatorname{sign}\varepsilon^{l''}(\omega, \mathbf{k})\,\delta[\varepsilon^{l'}(\omega, \mathbf{k})]\int d\omega' d\mathbf{k}' d\omega'' d\mathbf{k}''$$
$$\times\delta(\omega - \omega' - \omega'')\,\delta(\mathbf{k} - \mathbf{k}' - \mathbf{k}'')$$
$$\times\left\{\frac{k_ik_j'k_nk_m'}{(kk')^2}\left(\delta_{sr} - \frac{k_s''k_r''}{k''^2}\right)(E_l^2)_{\omega',\mathbf{k}'}(E_{tr}^2)_{\omega'',\mathbf{k}''}\right.$$
$$+ \frac{k_ik_r''k_nk_s''}{(kk'')^2}\left(\delta_{jm} - \frac{k_j'k_m'}{k'^2}\right)(E_l^2)_{\omega'',\mathbf{k}''}(E_{tr}^2)_{\omega',\mathbf{k}'}$$
$$+ \frac{1}{2}\frac{k_ik_n}{k^2}\left(\delta_{jm} - \frac{k_j'k_m'}{k'^2}\right)\left(\delta_{sr} - \frac{k_s''k_r''}{k''^2}\right)$$
$$\left.\times(E_{tr}^2)_{\omega',\mathbf{k}'}(E_{tr}^2)_{\omega'',\mathbf{k}''}\right\} S_{irj}(\omega, \mathbf{k}, \omega', \mathbf{k}') S_{nsm}^*(\omega, \mathbf{k}, \omega', \mathbf{k}'), \tag{3.2}$$

$$\frac{d}{dt}(E_{tr}^2)_{\omega,\mathbf{k}}\frac{1}{\omega}\left\{\frac{\partial[\omega\varepsilon^{tr}(\omega, \mathbf{k})]}{\partial\omega} + \frac{c^2k^2}{\omega^2}\right\}$$
$$= -2\varepsilon^{tr''}(\omega, \mathbf{k})(E_{tr}^2)_{\omega,\mathbf{k}} - (E_{tr}^2)_{\omega,\mathbf{k}}\operatorname{Im}\int d\omega' d\mathbf{k}'$$
$$\times V_{ijrs}(\omega, \mathbf{k}, \omega', \mathbf{k}')\left(\delta_{ir} - \frac{k_ik_r}{k^2}\right)\left(\frac{k_j'k_s'}{k'^2}\right)(E_l^2)_{\omega',\mathbf{k}'}$$
$$+ (E_{tr}^2)_{\omega,\mathbf{k}}\operatorname{Im}\int d\omega' d\mathbf{k}' d\omega'' d\mathbf{k}''$$
$$\times\delta(\omega - \omega' - \omega'')\,\delta(\mathbf{k} - \mathbf{k}' - \mathbf{k}'')\left\{\frac{k_s''k_r''}{k''^2}\frac{1}{\varepsilon^l(\omega'', \mathbf{k}'')}\right.$$
$$\left.+ \frac{\delta_{sr} - k_s''k_r''/k''^2}{\varepsilon^{tr}(\omega'', \mathbf{k}'') - c^2k''^2/\omega''^2}\right\}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)$$
$$\times S_{ism}(\omega, \mathbf{k}, \omega', \mathbf{k}') S_{rnj}(\omega'', k'', \omega, \mathbf{k})\frac{k_m'k_n'}{k'^2}(E_l^2)_{\omega',\mathbf{k}'}$$
$$+ \pi\operatorname{sign}\varepsilon^{tr''}(\omega, \mathbf{k})\,\delta\left[\varepsilon^{tr'}(\omega, \mathbf{k}) - \frac{c^2k^2}{\omega^2}\right]$$
$$\times\int d\omega' d\mathbf{k}' d\omega'' d\mathbf{k}''\delta(\omega - \omega' - \omega'')\,\delta(\mathbf{k} - \mathbf{k}' - \mathbf{k}'')$$
$$\times\left(\delta_{in} - \frac{k_ik_n}{k^2}\right) S_{irj}(\omega, \mathbf{k}, \omega', \mathbf{k}') S_{nsm}^*(\omega, \mathbf{k}, \omega', \mathbf{k}')$$
$$\times\left\{\frac{k_j'k_m'k_r''k_s''}{(k'k'')^2}(E_l^2)_{\omega',\mathbf{k}'}(E_l^2)_{\omega'',\mathbf{k}''} + \frac{1}{2}\frac{k_j'k_m'}{k'^2}\right.$$
$$\times\left(\delta_{sr} - \frac{k_s''k_r''}{k''^2}\right)(E_l^2)_{\omega',\mathbf{k}'}(E_{tr}^2)_{\omega'',\mathbf{k}''} + \frac{1}{2}\frac{k_r''k_s''}{k''^2}$$
$$\left.\times\left(\delta_{jm} - \frac{k_j'k_m'}{k'^2}\right)(E_l^2)_{\omega'',\mathbf{k}''}(E_{tr}^2)_{\omega',\mathbf{k}'}\right\}. \tag{3.3}$$

Under conditions where it is necessary to take into account the linear damping of the longitudinal waves, and also the nonlinear interaction of the longitudinal waves with the longitudinal waves, Eq. (3.2) is supplemented by the right side of (2.1). In the present section we are interested only in the nonlinear interaction of longitudinal waves with transverse ones. We therefore confine ourselves to a consideration of (3.2) and (3.3). We are interested in an interaction of waves having frequencies that differ little from the Langmuir frequency of electrons. For longitudinal electron oscillations this is always satisfied and corresponds to the fact that their wavelength is large compared with the electronic Debye radius. For transverse oscillations we assume that λ_0 is small compared with the length of the transverse waves. We start our analysis with the induced scattering of the waves under conditions when only the interaction with the ions is significant, and when it is also possible to neglect the contribution of the interaction that is characterized by the intermediate transverse oscillation. These conditions will be spelled out below.

The scattering by electrons only, without account of the interaction characterized by the intermediate transverse oscillations, is described by a tensor which has in our case of long-wave oscillations the form

$$\operatorname{Im}\left\{V_{ijsr}(\omega, \mathbf{k}, \omega', \mathbf{k}') - S_{inj}(\omega, \mathbf{k}, \omega', \mathbf{k}')\right.$$
$$\left.\times S_{mrs}(\omega'', \mathbf{k}'', \omega, \mathbf{k})\frac{k_n''k_m''}{k''^2}[\varepsilon^l(\omega'', \mathbf{k}'')]^{-1}\right\} \cong \frac{4\pi^2e^4\omega''}{m^2(\omega\omega')^3}$$
$$\times\left\{k_jk_r\left(\delta_{is} - \frac{k_i''k_s''}{k''^2}\right) + k_jk_s'\left(\delta_{ir}\right.\right.$$
$$\left.- \frac{k_i''k_r''}{k''^2}\right) + k_i'k_r\left(\delta_{js} - \frac{k_j''k_s''}{k''^2}\right)$$
$$\left.+ k_i'k_s'\left(\delta_{jr} - \frac{k_j''k_r''}{k''^2}\right)\right\}\int d\mathbf{p}\, f\delta(\omega'' - \mathbf{k}''\mathbf{v}). \tag{3.4}$$

We have used here formula (2.14). Formula (3.4) enables us to write down the following equations for the time variation of the energies of the longitudinal and transverse oscillations:

$$\frac{dW_l(\mathbf{k})}{dt} = -\frac{1}{16(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}r_{De}^4\int d\mathbf{k}' W_{tr}(\mathbf{k}')$$
$$\times\frac{\{3k^2r_{De}^2 - \lambda_0^2k'^2\}}{|\mathbf{k} - \mathbf{k}'|^3}\left\{\frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2}[k^2k'^2 - 4(\mathbf{k}\mathbf{k}')^2\right.$$
$$\left.+ 2k'^2(\mathbf{k}\mathbf{k}')] + 2\frac{(\mathbf{k}\mathbf{k}')^2}{k^2}(\mathbf{k} - \mathbf{k}')^2\right\}, \tag{3.5}$$

$$\frac{W_{tr}(\mathbf{k})}{dt} = -\frac{1}{16(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_{tr}(\mathbf{k})}{\varkappa T_e}r_{De}^4$$
$$\times\int d\mathbf{k}' W_l(\mathbf{k}')\frac{\{\lambda_0^2k^2 - 3k'^2r_{De}^2\}}{|\mathbf{k} - \mathbf{k}'|^3}\left\{\frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2}\right.$$
$$\left.\times[k^2k'^2 - 4(\mathbf{k}\mathbf{k}')^2 + 2k^2(\mathbf{k}\mathbf{k}')] + 2\frac{(\mathbf{k}\mathbf{k}')^2}{k'^2}(\mathbf{k} - \mathbf{k}')^2\right\}. \tag{3.6}$$

In deriving these formulas we have assumed that the particles have a Maxwellian distribution, and that the energy density of the transverse oscillations is determined by the relation

$$W_{tr}(\mathbf{k}) = (2\pi)^3\int_0^\infty d\omega\frac{(E_{tr}^2)_{\omega,\mathbf{k}}}{4\pi}\frac{\partial}{\partial\omega}\left\{\omega\left[\varepsilon^{tr'}(\omega, \mathbf{k}) - \frac{c^2k^2}{\omega^2}\right]\right\}. \tag{3.7}$$

In the particular case when the wavelengths of the transverse oscillations are much larger than those of the longitudinal waves, formulas (3.5) and (3.6) go over into those obtained by Gaĭlitis and Tsytovich [5] on the basis of the results of their paper [6]. We note that in accordance with (3.5) and (3.6) the oscillation energy is conserved and shifts from the high frequency oscillations to the low frequency ones. The fact that the scattering of the oscillations is elastic and that the energy is not transferred to the particles is manifest in the symmetry of the kernels of (3.5) and (3.6). We confine ourselves below to the elastic-scattering approximation. Therefore, bearing this symmetry in mind, we consider only the equation for the rate of change of the energy of longitudinal oscillations with time.[4)]

Equation (3.5) is no longer suitable if the following inequality takes place

$$(\omega-\omega')^2=\left[\frac{1}{2}\,\omega_{Le}(3k^2r_{De}^2-\lambda_0^2k'^2)\right]^2<v_{Ti}^2(\mathbf{k}-\mathbf{k}')^2 \times\ln\left[\frac{e_i^2MT_e^3}{e^2mT_i^3}\right]. \quad (3.8)$$

The decisive factor then becomes the scattering by the ions (see [1]). As a result we get in lieu of (3.5)

$$\frac{dW_l(\mathbf{k})}{dt}=-\frac{1}{16(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}\frac{v_{Te}}{v_{Ti}}r_{De}^2\int d\mathbf{k}'W_{tr}(\mathbf{k}') \times\frac{3k^2r_{De}^2-\lambda_0^2k'^2}{|\mathbf{k}-\mathbf{k}'|}\frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2}\left[\frac{r_{Di}/r_{De}}{F+(r_{Di}/r_{De})^2}\right]^2 \times\exp\left\{-\frac{1}{8}\frac{v_{Te}^2}{v_{Ti}^2}\frac{[3k^2r_{De}^2-\lambda_0^2k'^2]^2}{r_{De}^2(\mathbf{k}-\mathbf{k}')^2}\right\}, \quad (3.9)$$

where the function F is characterized by formulas (2.5) and (2.6). The time of spectral redistribution, described by (3.9) has an order of magnitude (in the case when the exponential in this equation can be set equal to unity)

$$\tau_i\sim10^3\left[\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l}{\varkappa T_e}\frac{v_{Te}}{v_{Ti}}r_{De}^2k^2(3k^2r_{De}^2-\lambda_0^2k'^2)\right]^{-1}. \quad (3.10)$$

The ratio of this time to the spectral redistribution time described by (3.5) is approximately equal to $(kr_{De})^2\,(v_{Ti}/v_{Te})$, which turns out to be smaller than $(v_{Ti}/v_{Te})^3\ln[e_i^2MT_e^3/e^2mT_i^3]$ when $kr_{De}\gtrsim\lambda_0k'$. Therefore, for wavelengths where scattering by the ions is decisive the time of spectral redistribution in an isothermal hydrogen plasma turns out to be four orders of magnitude smaller than the value previously obtained without account of scattering from the ions. For a mercury plasma τ_i turns out to be approximately eight orders of magnitude smaller[5)].

Scattering from electrons which is due to the interaction with an intermediate wave or, in other words, due to that term of (3.2) which contains expression (2.7) in the denominator, can under certain conditions exceed the scattering considered above in this section. The corresponding contribution to the rate of change of the energy of the longitudinal oscillations is of the form

$$\delta\left[\frac{dW_l(\mathbf{k})}{dt}\right]=-\frac{1}{16(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}r_{De}^4\int d\mathbf{k}'W_{tr}(\mathbf{k}') \times\frac{3r_{De}^2k^2-\lambda_0^2k'^2}{(kk')^2|\mathbf{k}-\mathbf{k}'|} \times\frac{k'^2(\mathbf{k}\cdot\mathbf{k}-\mathbf{k}')^2+(\mathbf{k}-\mathbf{k}')^2(\mathbf{k}\mathbf{k}')^2}{\lambda_0^4(\mathbf{k}-\mathbf{k}')^4+{}^1\!/_2(c/v_{Te})^2[1+\pi/4\lambda_0^2(\mathbf{k}-\mathbf{k}')^2][3k^2r_{De}^2-\lambda_0^2k'^2]^2}. \quad (3.11)$$

In the derivation of (3.11) we made use of (2.15).

It must be emphasized that interaction (3.11) is small compared with scattering by ions in the region where inequality (3.8) is satisfied. Therefore only in the region when the opposite inequality is satisfied can the interaction (3.11) be significant. Depending on the ratio between the lengths of the longitudinal and transverse waves, either the Coulomb scattering given by (3.5) or scattering by the transverse wave in accord with (3.11) will predominate. Thus, for example, when

$$(v_{Te}/c^2)\,\omega_{Le}\gg k\gg k'$$

and

$$(kr_{De})^2\gg(v_{Ti}/v_{Te})^2\ln[e_i^2MT_e^3/e^2mT_i^3]$$

the spectral redistribution is determined by the scattering via the transverse wave, and occurs within a time of the order of

$$\tau\sim10^3\frac{Nr_{De}^3}{\omega_{Le}}\frac{\varkappa T_e}{W_l}\frac{k^3}{(kr_{De})^4k'^3}. \quad (3.12)$$

In deriving the expressions for the interactions between the long transverse and longitudinal waves it was assumed that $\omega''=\omega-\omega'\ll\mathbf{k}''\cdot\mathbf{v}_e$. We can consider the opposite case, when $\omega\sim\omega'\gg\omega''\gg\mathbf{k}''\cdot\mathbf{v}_e$. Using (3.5) we obtain for Coulomb scattering of the waves

4)The difference between (3.5) and the corresponding formula (3.2) of Matsuura[19] is due to the latter neglecting the terms $\sim e^6$ or, in other words, the terms containing the tensor S in our formula (3.2).

5)In deriving formula (3.9) we did not take into account the interaction with the ions, characterized by an intermediate transverse wave. The corresponding contribution may turn out to be appreciable only if the plasma is very highly isothermal, when $(T_e/T_i)\gg(M/m)$.

$$-\frac{1}{16(2\pi)^{5/2}}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}r_{De}^4\int d\mathbf{k}'W_{tr}(\mathbf{k}')(\lambda_0^2k'^2-3k^2r_{De}^2)$$

$$\times\exp\left\{-\frac{r_{De}^2}{8|\mathbf{k}-\mathbf{k}'|^2}\left(\frac{k'^2c^2}{v_{Te}^2}-3k^2\right)^2\right\}\frac{Q(\mathbf{k},\mathbf{k}')}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^3}, \quad (3.13)$$

$$Q(\mathbf{k},\mathbf{k}') = [\mathbf{kk}']^2(k^2k'^2-4(\mathbf{kk}')^2+2k'^2(\mathbf{kk}')) + 2(\mathbf{kk}')^2k'^2(\mathbf{k}-\mathbf{k}')^2. \quad (3.14)$$

The scattering of a transverse wave by a longitudinal one via an intermediate transverse wave leads to different expressions, depending on the ratio of the longitudinal wavelength to λ_0. If the longitudinal wavelength exceeds λ_0, then we must put in (3.13)

$$Q(\mathbf{k},\mathbf{k}') = k'^2(\mathbf{k}-\mathbf{k}')^2(\mathbf{k}\cdot\mathbf{k}-\mathbf{k}')^2 + (\mathbf{k}-\mathbf{k}')^4(\mathbf{kk}')^2 - k'^2(\mathbf{k}-\mathbf{k}')^2(\mathbf{kk}')^2 - ([\mathbf{kk}']^2+(\mathbf{kk}')\mathbf{k}'\cdot\mathbf{k}-\mathbf{k}'))^2. \quad (3.15)$$

For the opposite case, when the longitudinal wave is shorter than λ_0, the corresponding contribution to the change in the transverse-oscillation energy is obtained from (3.13) by using the expression

$$Q(\mathbf{k},\mathbf{k}') = -\frac{1}{2}\left(\frac{v_{Te}}{c}\right)^2 r_{De}^{-2}\left[k^4\frac{(\mathbf{k}'\cdot\mathbf{k}-\mathbf{k}')^2}{(\mathbf{k}-\mathbf{k}')^2}+k^2k'^2(\mathbf{kk}') + k^2(\mathbf{kk}')(\mathbf{k}'\cdot\mathbf{k}-\mathbf{k}')\right]. \quad (3.16)$$

The role of the ions is always negligibly small in our case, for the ion term contains an exponentially small factor with an exponent that is $(v_{Te}/v_{Ti})^2$ times larger than the argument of the exponent in (3.13).

Comparison of the time of variation of the intensity of the transverse oscillations due to the interaction with the longitudinal shows that the principal role is always played by the interaction via the intermediate transverse wave, except for the case $kc/v_{Te} \gg k' \gg \lambda_0^{-1}$, when interaction via the longitudinal wave can be of the same order of magnitude.

In concluding this section we must stop to consider one more possible process of nonlinear interaction of longitudinal and transverse waves. Namely, the long-wave oscillations which we are considering (both transverse and longitudinal) can coalesce into transverse waves with frequency $2\omega_{Le}$ and wave vector $\sqrt{3}\omega_{Le}/c$[6]. The contribution of such a process to the rate of decrease of longitudinal oscillation energy is of the form

$$\left[\frac{dW_l(\mathbf{k})}{dt}\right]_{\text{coal}} = -\frac{1}{8(2\pi)^2}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_l(\mathbf{k})}{\varkappa T_e}\delta\left(3-\frac{c^2k^2}{\omega_{Le}^2}\right)r_{De}^5 \times\int d\mathbf{k}'W_{tr}(\mathbf{k}')\left(k^2+\frac{(\mathbf{kk}')^2}{k'^2}\right). \quad (3.17)$$

Analogously, for the rate of decrease of energy of the transverse oscillations we obtain

$$\left[\frac{dW_{tr}(\mathbf{k})}{dt}\right]_{\text{coal}} = -\frac{1}{2(2\pi)^2}\frac{\omega_{Le}}{Nr_{De}^3}\frac{W_{tr}(\mathbf{k})}{\varkappa T_e}r_{De}^5\int d\mathbf{k}'W_l(\mathbf{k}') \times\delta\left(3-\frac{c^2k'^2}{\omega_{Le}^2}\right)\left(k'^2+\frac{(\mathbf{kk}')^2}{k^2}\right). \quad (3.18)$$

Inasmuch as the wavelength of the transverse oscillation participating in the coalescence is much larger than λ_0, we can state with a corresponding degree of accuracy that the wavelength of the longitudinal oscillations in (3.17) and (3.18) is equal to $\lambda_0/\sqrt{3}$.

4. NONLINEAR INTERACTION OF LONG WAVE TRANSVERSE OSCILLATIONS

For the nonlinear interaction of transverse waves we obtain from (1.8) the equation

$$\frac{d}{dt}(E_{tr}^2)_{\omega,\mathbf{k}}\frac{\partial}{\partial\omega}\left[\varepsilon'_{tr}(\omega,\mathbf{k})-\frac{c^2k^2}{\omega^2}\right] =$$
$$-(E_{tr}^2)_{\omega,\mathbf{k}}\frac{1}{2}\int d\omega'\,d\mathbf{k}'\,(E_{tr}^2)_{\omega',\mathbf{k}'} \times\operatorname{Im}V_{ijrs}(\omega,\mathbf{k},\omega',\mathbf{k}')\left(\delta_{ir}-\frac{k_ik_r}{k^2}\right)\left(\delta_{js}-\frac{k_j'k_s'}{k'^2}\right)$$
$$+(E_{tr}^2)_{\omega,\mathbf{k}}\frac{1}{2}\operatorname{Im}\int d\omega'\,d\mathbf{k}'\,d\omega''\,d\mathbf{k}''\delta(\omega-\omega'-\omega'') \times\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\left\{\frac{k_s''k_r''}{k''^2}\frac{1}{\varepsilon^l(\omega'',\mathbf{k}'')}+\frac{\delta_{sr}-k_s''k_r''/k''^2}{\varepsilon^{tr}(\omega'',\mathbf{k}'')-c^2k''^2/\omega''^2}\right\}$$
$$\times\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)S_{ism}(\omega,\mathbf{k},\omega',\mathbf{k}')S_{rnj}(\omega'',\mathbf{k}'',\omega,\mathbf{k}) \times\left(\delta_{mn}-\frac{k_m'k_n'}{k'^2}\right)(E_{tr}^2)_{\omega',\mathbf{k}'}+\frac{\pi}{4}\operatorname{sign}\varepsilon^{tr''}(\omega,\mathbf{k})$$
$$\times\delta\left[\varepsilon^{tr'}(\omega,\mathbf{k})-\frac{c^2k^2}{\omega^2}\right]\int d\omega'\,d\mathbf{k}'\,d\omega''\,d\mathbf{k}''\delta(\omega-\omega'-\omega'') \times\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')(\delta_{in}-k_ik_n/k^2)S_{irj}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$\times\overset{*}{S}_{nsm}(\omega,\mathbf{k},\omega',\mathbf{k}')(\delta_{jm}-k_j'k_m'/k'^2)(\delta_{sr}-k_s''k_r''/k''^2) \times(E_{tr}^2)_{\omega',\mathbf{k}'}(E_{tr}^2)_{\omega'',\mathbf{k}''}. \quad (4.1)$$

We confine ourselves below to long-wave transverse oscillations, the frequency of which is close to the Langmuir electron frequency. For such oscillations, coalescence and decay are impossible. Therefore the nonlinear interactions are determined by the induced scattering of transverse waves. Inasmuch as the theory of scattering of transverse waves in a plasma is developed, we

[6]The coalescence of short transverse waves ($\omega \approx ck$) with longitudinal waves was considered by Kovrizhnykh and Tsytovich.[20]

could make use of the results obtained there. However, the case of interest to us, that of long waves, was never considered before. In addition, it must be noted that Akhiezer et al. [7] and Rosenbluth and Rostoker [8] considered the scattering of transverse waves within the framework of a theory that takes into account only longitudinal plasma fluctuations. In the language of our equation (4.1), this means that we have disregarded the term that contains expression (2.7) in the denominator. Actually, however, account of precisely these terms when using formula (2.15) for the case of small Δk

$$1 \gg \frac{\omega''}{k'' v_{Te}} \equiv \frac{c^2(k^2 - k'^2)}{2\omega_{Le} v_{Te} |\mathbf{k} - \mathbf{k}'|} \approx \frac{c}{v_{Te}} \frac{\Delta k c}{\omega_{Le}}$$

yields

$$\frac{dW_{tr}(\mathbf{k})}{dt} = -\frac{1}{4(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_{tr}(\mathbf{k})}{\varkappa T_e} r_{De}^4 \left(\frac{\omega_{Le}}{c}\right)^2 \int d\mathbf{k}' W_{tr}(\mathbf{k}')$$
$$\times \frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2} \frac{k^2 + k'^2 - \mathbf{k}\mathbf{k}'}{|\mathbf{k} - \mathbf{k}'| (k^2 - k'^2)} \left\{ 4 \frac{(\mathbf{k} - \mathbf{k}')^4}{(k^2 - k'^2)^2} + \frac{\pi}{2} \frac{c^2}{v_{Te}^2} \frac{\omega_{Le}^2}{c^2} \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} \right\}^{-1}. \qquad (4.2)$$

When $(\Delta/k) \ll k r_{De}$, the characteristic time of the spectral redistribution is of the order of

$$\tau \sim 10^3 \left(\frac{c}{v_{Te}}\right)^2 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_{tr}} \frac{1}{r_{De}^2 k \Delta k}. \qquad (4.3)$$

In the opposite case $(\Delta k/k) \gg k r_{De} \ll 1$ we have

$$\tau \sim 10^3 \left(\frac{c}{v_{Te}}\right)^2 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_{tr}} \frac{\Delta k}{r_{De}^4 k^5}. \qquad (4.4)$$

We note that the characteristic time resulting from the remaining terms of (4.1), without account of scattering by the ions, is of the order of magnitude

$$\tau \sim 10^3 \frac{N r_{De}^3}{\omega_{Le}} \frac{\varkappa T_e}{W_{tr}} \frac{v_{Te}^2}{c^2} \frac{1}{r_{De}^6 k^5 \Delta k},$$

which greatly exceeds the time (4.3) or (4.4).

When the following inequality is satisfied

$$(\omega - \omega')^2 \equiv [c^2(k^2 - k'^2) / 2\omega_{Le}]^2 \lesssim v_{Ti}^2 (\mathbf{k} - \mathbf{k}')^2 \times \ln(e_i^2 M T_e^3 / e^2 m T_i^3), \qquad (4.5)$$

a sharp increase takes place in the contribution of the scattering of waves by ions, which becomes essential for the nonlinear interaction of transverse waves. We then obtain

$$\frac{dW_{tr}(\mathbf{k})}{dt} = -\frac{1}{32(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_{tr}(\mathbf{k})}{\varkappa T_e} \frac{c^2}{v_{Te} v_{Ti}}$$
$$\times r_{De}^4 \int d\mathbf{k}' W_{tr}(\mathbf{k}') \frac{k^2 - k'^2}{|\mathbf{k} - \mathbf{k}'|} \left[1 + \frac{(\mathbf{k}\mathbf{k}')^2}{k^2 k'^2}\right]$$
$$\times \left[\frac{r_{Di}/r_{De}}{F + (r_{Di}/r_{De})^2}\right]^2 \exp\left\{-\frac{c^4 (k^2 - k'^2)^2}{8 v_{Ti}^2 \omega_{Le}^2 (\mathbf{k} - \mathbf{k}')^2}\right\} \qquad (4.6)$$

where the function F is determined by (2.5) and (2.6). The characteristic time of the spectral redistribution is then equal to

$$\tau_i \sim 10^3 \frac{N r_{De}^3}{\omega_{Le}} \frac{v_{Te} v_{Ti}}{c^2} \frac{\varkappa T_e}{W_{tr}} \frac{1}{r_{De}^4 k^3 \Delta k}. \qquad (4.7)$$

So far we assumed that in the nonlinear interaction of long transverse waves $\omega'' = \omega - \omega' \ll \mathbf{k} - \mathbf{k}' \cdot \mathbf{v}_e$. As in Sec. 3, let us consider the opposite case $\omega \sim \omega' \gg \omega'' \gg \mathbf{k} - \mathbf{k}' \cdot \mathbf{v}_e$. The scattering of waves by ions is in this case always negligibly small, inasmuch as the times characterizing the scattering contain an exponential with a large argument. As to the scattering by the electrons, the corresponding expressions can be obtained with the aid of (3.4) and (2.14):

$$\delta \left[\frac{dW_{tr}(\mathbf{k})}{dt}\right] = -\frac{1}{16(2\pi)^{5/2}} \frac{\omega_{Le}}{N r_{De}^3} \frac{W_{tr}(\mathbf{k})}{\varkappa T_e}$$
$$\times r_{De}^6 \left(\frac{c}{v_{Te}}\right)^2 \int d\mathbf{k}' W_{tr}(\mathbf{k}') \frac{[\mathbf{k}\mathbf{k}']^2}{(kk')^2} \frac{(k^2 - k'^2)}{|\mathbf{k} - \mathbf{k}'|}$$
$$\times (k^2 + k'^2 - \mathbf{k}\mathbf{k}') \left[1 + 8 \left(\frac{v_{Te}}{c}\right)^4 \frac{(\mathbf{k} - \mathbf{k}')^2}{r_{De}^2 (k^2 - k'^2)^2}\right]$$
$$\times \exp\left\{-\frac{1}{8} \left(\frac{c}{v_{Te}}\right)^4 \frac{(k^2 - k'^2)^2 r_{De}^2}{(\mathbf{k} - \mathbf{k}')^2}\right\}. \qquad (4.8)$$

The second term in the square brackets of (4.8) is due to scattering via a transverse wave, and is always negligibly small compared with unity. Therefore in the case in question the principal role is played by scattering from electrons, with account of the Coulomb interaction between particles only.

In conclusion attention must be paid to the following circumstance. Eqs. (2.1), (3.2), and (4.1) contain terms whose sign depends on the sign of the imaginary part of the dielectric constant. If the distribution function of the particles in the plasma is such that the sign of the imaginary part of ϵ^l or ϵ^{tr} is negative, then a new nonlinear kinetic instability is possible. At sufficiently large oscillation amplitudes this instability can turn out to be more significant than the linear instability which also takes place in this case.

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