KINETIC THEORY OF PROPAGATION OF STRONG ELECTROMAGNETIC WAVES IN SEMICONDUCTORS AND IN A PLASMA

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Submitted to JETP editor February 27, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 1322-1341 (October, 1964)

The effect of heating of electrons in a plasma and in semiconductors by an electromagnetic field is investigated, along with the effect on the field propagation of the nonlinearities associated with this heating. The character of the damping of the field is studied both for the nonresonant case, when the nonlinearity of the Maxwell equations is small, and also for magnetoplasma and cyclotron resonances, when there is strong nonlinearity. The dependence of the surface impedance on the amplitude and frequency of the incident electromagnetic field and on the external constant magnetic field is also determined.

THE nonlinear dependence of the electric current on the electric field in the electron gas of a plasma and in semiconductors is evident even for relatively low electric field intensity.[1-3] This fact is connected with the large value of the mean free path of the electron in these media and the slow transfer of the energy from the electron to the scattering centers.

The effect of the nonlinearity on the propagation of electromagnetic waves in the electron gas is of considerable interest, both theoretically and experimentally. However, to our knowledge, only the paper of Gurevich[4] has been devoted to this particular problem. In this paper, the analysis is carried out with the help of an elementary theory applicable to the ionosphere, while the effect of the magnetic field has not been taken into account in any practical sense. In the work mentioned, several interesting results were obtained which show the sharp difference between nonlinear and linear propagation.

However, it is necessary to remark that the elementary theory is applicable fundamentally for scattering of current carriers by neutral molecules. Even for scattering by charged ions, the application of the elementary theory can lead to significant errors.[4] This is even more true for semiconductors, in which different types of scattering centers exist, and the simultaneous scattering by several of them can be substantial. The elementary theory does not in principle allow us to calculate the distribution function of the current carriers, which is essential for a number of problems. Thus, the range of problems that can be solved by the elementary theory is very limited.

There is great interest in the effect of a constant magnetic field on the propagation of strong electromagnetic waves[1)] both in the nonresonant and in the resonant regions. For nonlinear media, significant interest attaches to the interaction of the harmonics. From what has been pointed out above, it follows that an analysis by means of the elementary theory, as carried out by Gurevich,[4] does not take into account a number of important factors.

In the present paper, a kinetic theory is constructed for the propagation of strong electromagnetic fields in an electron gas, in semiconductors and in a plasma located in an external magnetic field. In contrast with the work of Gurevich,[4] where it is applied to the ionosphere, and where it was assumed that the electron gas filled a layer with smoothly changing properties, we shall assume that the current carriers fill a half-space. This corresponds to the experimental conditions for semiconductors and for plasma under laboratory conditions.

1. KINETIC EQUATION. DIELECTRIC PERMITTIVITY TENSOR

The distribution function of electrons in a semiconductor and electron-ion plasma in an electric field of arbitrary magnitude and in an external constant magnetic field Φ can be written in the form[1,2]

$$\Phi(\mathbf{p}, \mathbf{r}, t) = f(\varepsilon, \mathbf{r}, t) + \chi(\varepsilon, \mathbf{r}, t)\mathbf{p}/p, \qquad (1.1)$$

where $\mathbf{p}$ is the momentum (in the case of a semiconductor, the quasi-momentum) of the carrier, $\mathbf{r}$ is its coordinate, t the time, $\epsilon = p^2/2m$ the energy of the carrier, m the effective mass of the carrier.

[1)]Those electric fields will be called strong for which there is a nonlinear dependence of the current on the field.

The functions $f(\epsilon, r, t)$ and $\chi(\epsilon, r, t)$ satisfy the following equations

$$\frac{\partial f}{\partial t}+\frac{1}{n(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left(n(\varepsilon)\left\{\frac{ep}{3m}\chi\mathbf{E}-A(\varepsilon)\left[\frac{\partial f}{\partial\varepsilon}+\frac{f}{T}\right]\right\}\right)=0, \tag{1.2}$$

$$\frac{\partial\chi}{\partial t}+\frac{ep}{m}\mathbf{E}\frac{\partial f}{\partial\varepsilon}-\omega_H[\mathbf{h},\chi]+\frac{\chi}{\tau(\varepsilon)}=0. \qquad (1.3)^*$$

Here the following notation has been introduced: $n(\epsilon) = 4\pi p^2 dp/d\epsilon$ is the density of states in energy space, $\mathbf{E}(\mathbf{r}, t)$ is the electric field, $A(\epsilon)$ the diffusion coefficient in energy space, T the equilibrium temperature in energy units, $\omega_H = |e|H/mc$, H being the constant magnetic field and $\mathbf{h}$ the local magnetic field, and $\tau(\epsilon)$ is the relaxation time. If the carrier interacts with several scattering objects simultaneously, then $A(\epsilon) = \Sigma A_i(\epsilon)$, $\tau^{-1}(\epsilon) = \Sigma\tau_i^{-1}(\epsilon)$; the summation is carried out over all forms of scattering.[2)]

In the present work, effects associated with spatial dispersion are not taken into account, i.e., space derivatives are omitted from (1.2) and (1.3). The conditions under which this neglect is possible were discussed by Ginzburg and Gurevich.[1,2,4] Further simplifications of the system (1.2), (1.3) can be obtained under definite assumptions regarding the time dependence of the electric field.

Most interest attaches to the case in which the electric field is a periodic function of time. We expand the field in a Fourier series:

$$\mathbf{E}(\mathbf{r},t)=\sum_{q=-\infty}^{\infty}{}' \mathbf{E}^{(q)}(\mathbf{r})e^{iq\omega t}. \tag{1.4}$$

Since $\mathbf{E}(\mathbf{r}, t)$ is real, it follows that $\mathbf{E}^{(q)} = \mathbf{E}^{(-q)*}$. The prime on the sum indicates that the term with $q = 0$ has been omitted in the sum of (1.4). We shall seek also the solution of the system (1.2), (1.3) in the form of a Fourier series:

$$f(\varepsilon,\mathbf{r},t)=\sum_{q=-\infty}^{\infty}f^{(q)}(\varepsilon,\mathbf{r})e^{iq\omega t},$$

$$\chi(\varepsilon,\mathbf{r},t)=\sum_{q=-\infty}^{\infty}\chi^{(q)}(\varepsilon,\mathbf{r})e^{iq\omega t}, \tag{1.5}$$

where

$$f^{(q)}=f^{(-q)*},\qquad \chi^{(q)}=\chi^{(-q)*}.$$

Eliminating $\chi^{(q)}$ from (1.2) with the help of (1.3), and equating the coefficients of each of the factors of $e^{iq\omega t}$ to zero, we get the following set of equations for the determination of $f^{(q)}$:

$$iq\omega f^{(q)}-\frac{1}{n(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left\{n(\varepsilon)\left[\frac{2e^2\varepsilon}{3m}\sum_{q_1=-\infty}^{\infty}{}'\sum_{q_2=-\infty}^{\infty}B_{ik}[(q_1+q_2)\omega]\right.\right.$$

$$\times E_k^{(q_1)}E_i^{(q-q_1-q_2)}\partial f_0^{(q_2)}/\partial\varepsilon$$

$$\left.\left.+A(\varepsilon)\left(\partial f^{(q)}/\partial\varepsilon+f^{(q)}/T\right)\right]\right\}=0. \tag{1.6}$$

As has been shown in a number of papers (see, for example,[1,2]),

$$\frac{1}{n(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left\{n(\varepsilon)A(\varepsilon)\left(\frac{\partial f^{(q)}}{\partial\varepsilon}+\frac{f^{(q)}}{T}\right)\right\}\sim\frac{f^{(q)}}{\tau_e},$$

where τ_e is the relaxation time for the energy associated with the relaxation time for the momentum τ by the relation $\tau_e \sim \epsilon^2\tau/(\Delta\epsilon)^2$. Here $\Delta\epsilon$ is the energy transferred by the current carrier in a single collision. The system (1.2), (1.3) holds if $\Delta\epsilon/\epsilon \ll 1$ so that $\tau_e \gg \tau$. If it is assumed that $\omega\tau_e \gg 1$, then it follows from (1.6) that $f^{(q)}/f^{(0)} \ll (q\omega\tau_e)^{-1} \ll 1$ for $q \neq 0$.

Solving Eq. (1.6) by the method of successive approximations in $(\omega\tau_e)^{-1}$, we get the following equation for $f^{(0)}$ after several transformations:

$$df^{(0)}/d\varepsilon+f^{(0)}/T(\varepsilon)=0, \tag{1.7}$$

where

$$T(\varepsilon)=T\left(1+\frac{4e^2\varepsilon}{3mA(\varepsilon)}\sum_{q=1}^{\infty}B_{ik}^{(q)}E_i^{(q)}E_k^{(q)*}\right),$$

$$B_{ik}^{(q)}=\frac{\nu}{(\omega_H^2-\omega_q^2+\nu^2)^2+4\omega_q^2\nu^2}\left\{(\omega_H^2+\omega_q^2+\nu^2)\delta_{ik}\right.$$

$$\left.+\frac{\omega_H^2}{\omega^2+\nu^2}(\omega_H^2-3\omega_q^2+\nu^2)h_ih_k-2i\omega_q\omega_Hh_le_{lik}\right\},$$

$$\omega_q=q\omega,\qquad \nu=\tau^{-1}, \tag{1.8}$$

e_{lik} is a completely antisymmetric unit tensor of third rank. Equation (1.7) should be normalized.

The solution of Eq. (1.7) has the form (the index 0 in $f^{(0)}$ will be omitted in what follows)

$$f(\varepsilon)=CN\exp\left\{-\int_0^{\varepsilon}\frac{d\varepsilon}{T(\varepsilon)}\right\},$$

$$C=\int_0^{\infty}d\varepsilon\, n(\varepsilon)\exp\left\{-\int_0^{\varepsilon}\frac{d\varepsilon}{T(\varepsilon)}\right\}, \tag{1.9}$$

N is the carrier density.

It follows from (1.9) that $f(\epsilon)$ does not depend on time. A detailed discussion of this fact for a monochromatic wave and its physical interpretation are given in the works of Ginzburg and Gurevich.[1,2] We note that

$$\sum_{q=1}^{\infty}B_{ik}^{(q)}E_i^{(q)}E_k^{(q)*}$$

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*$[\mathbf{h}, \chi] = \mathbf{h} \times \chi$

2)In [3,5,6] $A(\epsilon)$ and $\tau(\epsilon)$ are expressed in terms of the scattering probability.

is always larger than zero.

Since the kinetic equation for χ [Eq. (1.3)] has the same form as for weak fields, the dielectric tensor is given by the usual expressions (see, for example,[1]), where, however, f is determined by Eq. (1.9).

In the set of coordinates (1)—(3) in which the magnetic field is directed along the axis 3, we get the following relations for the components of the permittivity tensor:

$$\varepsilon_{11}^{(q)} = 1 + \frac{32\sqrt{2}\pi^2 m^{1/2} e^2}{3\omega_q} \times \int_0^\infty \frac{d\varepsilon}{T(\varepsilon)} \frac{\omega_q - i\nu(\varepsilon)}{\omega_H^2 - (\omega_q - i\nu(\varepsilon))^2} \varepsilon^{3/2} f(\varepsilon),$$

$$\varepsilon_{12}^{(q)} = \frac{32\sqrt{2}\pi^2 i m^{1/2} e^2 \omega_H}{3\omega_q} \int_0^\infty \frac{d\varepsilon}{T(\varepsilon)} \frac{1}{\omega_H^2 - (\omega_q - i\nu(\varepsilon))^2} \varepsilon^{3/2} f(\varepsilon),$$

$$\varepsilon_{33}^{(q)} = 1 - \frac{32\sqrt{2}\pi^2 m^{1/2} e^2}{3\omega_q} \int_0^\infty \frac{d\varepsilon}{T(\varepsilon)} \frac{1}{\omega_q - i\nu(\varepsilon)} \varepsilon^{3/2} f(\varepsilon). \quad (1.10)$$

2. PROPAGATION OF STRONG ELECTROMAGNETIC WAVES OF HIGH FREQUENCY FAR AWAY FROM RESONANCE (GENERAL THEORY)

We shall consider the field to be high frequency if the following inequality is satisfied: $\omega_q \tau_e \gg 1$. Furthermore, it is also assumed in Secs. 2 and 3 that all frequencies ω_q differ appreciably from the frequencies of magnetoplasma resonance $\omega_{1,2}$ and the Larmor frequency ω_H, i.e., that the following inequalities are satisfied:

$$|\omega_H - \omega_q|\tau \gg 1, \qquad |\omega_{1,2} - \omega_q|\tau \gg 1.$$

We now introduce the set of coordinates (x, y, z) in which the magnetic field lies in the yz plane. We restrict ourselves to one dimensional problems, i.e., we assume that the field and the distribution function depend only on a single coordinate, for example, on z. The Maxwell equations in the set of coordinates (x, y, z) for the one dimensional case have the following form

$$\frac{d^2 E_x^{(q)}}{dz^2} + \frac{\omega_q^2}{c^2}[A_q E_x^{(q)} + iB_q E_y^{(q)}] = 0,$$

$$\frac{d^2 E_y^{(q)}}{dz^2} + \frac{\omega_q^2}{c^2}[-iB_q E_x^{(q)} + G_q E_y^{(q)}] = 0,$$

$$E_z^{(q)} = -\frac{\sin\vartheta}{\sin^2\vartheta\,\varepsilon_{11}^{(q)} + \cos^2\vartheta\,\varepsilon_{33}^{(q)}}\{\varepsilon_{12}^{(q)} E_x^{(q)} + \cos\vartheta(\varepsilon_{33}^{(q)} - \varepsilon_{11}^{(q)})E_y^{(q)}\}; \quad (2.1)$$

$$A_q = \varepsilon_{11}^{(q)} + \frac{\sin^2\vartheta\,\varepsilon_{12}^{(q)2}}{\sin^2\vartheta\,\varepsilon_{11}^{(q)} + \cos^2\vartheta\,\varepsilon_{33}^{(q)}}$$

$$iB_q = \frac{\cos\vartheta\,\varepsilon_{12}^{(q)}\varepsilon_{33}^{(q)}}{\sin^2\vartheta\,\varepsilon_{11}^{(q)} + \cos^2\vartheta\,\varepsilon_{33}^{(q)}},$$

$$G_q = \frac{\varepsilon_{11}^{(q)}\varepsilon_{33}^{(q)}}{\sin^2\vartheta\,\varepsilon_{11}^{(q)} + \cos^2\vartheta\,\varepsilon_{33}^{(q)}}. \quad (2.1')$$

ϑ is the angle between the z axis and the magnetic field.

The set of equations (2.1) is identical in form with the analogous set for the case of weak fields. There is an essential difference, however, in that A_q, B_q, and G_q depend on all the Fourier components $E^{(q)}$ and, consequently, the propagation of strong electric fields is described by an infinite set of nonlinear differential equations.

We note one peculiarity of the propagation of strong electromagnetic waves for $\omega\tau_e \gg 1$. As is seen from Eqs. (1.10), the dielectric tensor does not depend on the time. This circumstance leads to the result that the frequency spectrum of the propagating field does not change. Thus, if a strong electromagnetic wave containing the two harmonics with frequencies ω_1 and ω_2 is incident on a half-space filled with plasma, then the spectrum of the wave which propagates in the half-space will also consist only of these two frequencies. Combination and multiple frequencies appear in the higher approximations in $(\omega\tau_e)^{-1}$. On the other hand, the damping of the field of the q-th harmonic, and, under the definite conditions, its phase also will be determined by the Fourier amplitudes of all the remaining harmonics.

If damping can be neglected (if $\nu = 0$), then, since ω_H does not depend on the energy in the case of a quadratic dispersion law, all the integrals in (1.10) can be computed. In this case the nondissipative part of the dielectric tensor has the same form as in linear theory. If at the same time the electron concentration does not depend on the field, then for any strength of electric field in the medium, plane monochromatic waves will be propagated with the same index as for linear theory. One can show that in the case of a nonquadratic dispersion law, under the neglect of damping, monochromatic plane waves are also propagated in a semiconductor and the principle of superposition is satisfied, although the nondissipative part of the dielectric tensor depends on the amplitude of the field in this case.

After neglecting quantities of the order of ν/ω_q, $\nu/|\omega_H - \omega_q|$ in $B_{ik}^{(q)}$ in comparison with unity, the equation for $T(\epsilon)$ in (1.8) is described in the following way:

$$T(\varepsilon, v) = T\left(1 + \frac{\nu(\varepsilon)\varepsilon}{A(\varepsilon)} u_0^2 v^2\right),$$

$$v^2 = \frac{u^2}{u_0^2}, \quad u^2 = \sum_{q=1}^{\infty} \gamma_{ik}^{(q)} E_i^{(q)} E_k^{(q)*},$$

$$\gamma_{ik}^{(q)} = \frac{4e^2}{3m} (\omega_H^2 - \omega_q^2)^{-2} \Big\{ (\omega_H^2 + \omega_q^2) \delta_{ik} + \frac{\omega_H^2}{\omega_q^2} (\omega_H^2 - 3\omega_q^2) h_i h_k - 2i\omega_H \omega_q h_l e_{lik} \Big\}, \qquad (2.2)$$

where u_0 is the value of u at the point z = 0.

Substituting (2.2) in Eqs. (1.10) and expanding the integrands in these formulas in powers of $\nu/|\omega_H - \omega_q|$, we get

$$\varepsilon_{11}^{(q)} = 1 - \frac{\omega_0^2}{\omega_H^2 - \omega_q^2} - i \frac{(\omega_H^2 + \omega_q^2)\omega_0^2 \nu_0^2}{\omega_q (\omega_H^2 - \omega_q^2)^2} \varphi(v^2),$$

$$\varepsilon_{12}^{(q)} = i \frac{\omega_0^2 \omega_H}{(\omega_H^2 - \omega_q^2)\omega_q} + \frac{2\omega_0^2 \omega_H \nu_0}{(\omega_H^2 - \omega_q^2)^2 \omega_q} \varphi(v^2),$$

$$\varepsilon_{33}^{(q)} = 1 - \frac{\omega_0^2}{\omega_q^2} - i \frac{\omega_0^2 \nu_0}{\omega_q^3} \varphi(v^2). \qquad (2.3)$$

Using (2.3) and the definition of A_q, B_q, G_q [see (2.1)], we also find expressions for these quantities:

$$A_q = A_q' + iA_q''\varphi(v^2), \qquad B_q = B_q' + iB_q''\varphi(v^2),$$
$$G_q = G_q' + iG_q''\varphi(v^2). \qquad (2.4)$$

In Eqs. (2.3) and (2.4), the following designations have been introduced: the prime and the double prime denote the real and imaginary parts of the corresponding quantities which have the same form as in linear theory, while $\varphi(v^2)$ is defined by the formula

$$\varphi(v^2) = \frac{CT}{\nu_0} \int_0^{\infty} \frac{d\varepsilon}{T(\varepsilon, v)} \varepsilon^{3/2} \nu(\varepsilon) \exp\Big\{ -\int_0^{\varepsilon} \frac{d\varepsilon}{T(\varepsilon, v)} \Big\},$$

$$\nu_0 = C_0 \int_0^{\infty} d\varepsilon \cdot \varepsilon^{3/2} \nu(\varepsilon) e^{-\varepsilon/T}; \qquad (2.5)$$

$\omega_0^2 = 4\pi e^2 N/m$, C_0 is the normalized equilibrium constant of the Maxwell function.

If the electron concentration does not depend on the field, and in what follows we shall assume this to be the case, then the entire dependence of the dielectric permittivity on the electric field is determined by the functions $\varphi(v^2)$. We note that the second components in Eqs. (2.3) and (2.4), which are proportional to $\varphi(v^2)$, determine the dissipation field and are small in comparison with the first components.

Solving the set (2.1) by the WKB method (for the applicability of this method to the present case, see[1]), we get

$$E_i^{(q)} = E_{i0}^{(q)} \exp\Big\{ -\frac{\omega_q}{c} \Big[in_q z + \varkappa_q \int_0^z \varphi(v^2)\, dz \Big] \Big\}, \qquad (2.6)$$

where n_q is the index of refraction of the ordinary or extraordinary wave, κ_q is the damping of the ordinary or extraordinary wave in linear theory.[1] In Secs. 2 and 3, we do not need to know their implicit form. We note that under the approximations we have made, $\kappa \ll n_q$.

The Maxwell equations (2.1) give the following connection between the different components of the field:

$$E_y^{(q)} = \frac{i(A_q' - n_q^2)}{B_q'} E_x^{(q)} = K_q E_x^{(q)},$$

$$E_z^{(q)} = -\frac{\sin\vartheta}{\sin^2\vartheta\, \varepsilon_{11}^{(q)} + \cos^2\vartheta\, \varepsilon_{33}^{(q)}} \times \{ \varepsilon_{12}^{(q)\prime} + \cos\vartheta (\varepsilon_{33}^{(q)\prime} - \varepsilon_{11}^{(q)\prime}) K_q \} E_x^{(q)} = L_q E_x^{(q)}. \qquad (2.7)$$

It is evident that K_q and L_q do not depend on the electric field and have the same form as in the linear theory.

The infinite set of nonlinear integral equations (2.6) can be solved in the following way. Substituting $E_i^{(q)}$ from (2.6) into the expression for v^2, we get

$$v^2 = u_0^{-2} \sum_{q=1}^{\infty} \gamma_{ik}^{(q)} E_{i0}^{(q)} E_{k0}^{(q)*} \exp\Big\{ -\frac{2\omega_q}{c} \varkappa_q \int_0^z \varphi(v^2)\, dz \Big\}.$$

We make a change in variable, setting

$$w = \exp\Big\{ -2\frac{\omega}{c} \int_0^z \varphi(v^2)\, dz \Big\}.$$

We then have the following expression for v^2:

$$v^2(w) = u_0^{-2} \sum_{q=1}^{\infty} \gamma_{ik}^{(q)} E_{i0}^{(q)} E_k^{(q)*} w^{q\varkappa_q}. \qquad (2.8)$$

We introduce the function $\xi(w) = \varphi(v^2(w))$. It is easy to see that $\varphi(v^2)$ can also be expressed in terms of w in the following form:

$$\varphi(v^2) = -\frac{c}{2\omega} \frac{1}{w} \frac{dw}{dz}.$$

Equating these two expressions for $\varphi(v^2)$, we finally obtain the following differential equation for the determination of w:

$$-\frac{c}{2\omega} \frac{1}{w} \frac{dw}{dz} = \xi(w). \qquad (2.9)$$

The solution of this equation, under the obvious additional condition w(0) = 1, has the form

$$\int_w^1 \frac{dw}{w\xi(w)} = 2\frac{\omega}{c} z, \qquad E_i^{(q)} = E_{i0}^{(q)} \exp\Big(-\frac{i\omega_q}{c} n_q z \Big) w^{q\varkappa_q/2}. \qquad (2.10)$$

Equations (2.10) in principle completely solve the stated problem—the first of them gives w(z) in implicit form, while the second expresses any of the Fourier components of the electric field in terms of w(z).

Let us consider the limiting cases of small and large z. For small z, the function w is close to unity, and $w\xi(w)$ in the first equation of (2.10) can be removed from under the integral sign at the point w = 1. In this limiting case, Eqs. (2.10) take the following form:

$$w = 1 - \frac{2\omega}{c}\xi(1)z,$$

$$E_i^{(q)} = E_{i0}^{(q)}\exp\left\{-i\frac{\omega_q}{c}n_q z\right\}\left(1 - \frac{\omega_q}{c}\varkappa_q\xi(1)z\right). \quad (2.11)$$

Now let z be large. If the right hand side of the first of Eqs. (2.10) approaches infinity, then the left hand side must approach infinity also. In view of the fact that $\xi(w)$ nowhere vanishes, the left hand side of the first of Eqs. (2.10) can approach infinity only as $w \to 0$, because then the integral in this expression diverges logarithmically.

We now determine the asymptotic value of w for large z. We note that $\xi(0) = 1$. This follows from (2.5) and (2.8). Actually, for w = 0, the function $v^2(w) = 0$ and, consequently, the distribution function (2.7) goes over into the Maxwell equilibrium distribution function, as a consequence of which $\xi(w) = \varphi(v^2(w))$ goes to unity. Taking this circumstance into account, we rewrite the integral on the left hand side of (2.10):

$$\int_w^1 \frac{dw}{w\xi(w)} = \int_w^1 \frac{dw}{w}(\xi^{-1}(w) - 1) + \int_w^1 \frac{dw}{w}$$

$$= \int_w^1 dw\frac{1-\xi(w)}{w\xi(w)} - \ln w.$$

The integral on the right hand side of this identity converges, because $1 - \xi(w) \to 0$ as $w \to 0$. Therefore, the upper limit in this integral can be set equal to zero for large z (small w). Thus, the following equality holds for the determination of w at large z:

$$\int_0^1 dw\frac{1-\xi(w)}{w\xi(w)} - \ln w = \frac{2\omega}{c}z,$$

$$w = \exp\left\{\int_0 dw\frac{1-\xi(w)}{w\xi(w)} - \frac{2\omega}{c}z\right\},$$

whence

$$E_i^{(q)} = E_{i0}^{(q)}\exp\left[\frac{1}{2}q\varkappa_q\int_0^1\frac{1-\xi(w)}{w\xi(w)}dw - \frac{\omega_q}{c}(in_q + \varkappa_q)z\right]. \quad (2.12)$$

We now compute the change of the reflection coefficient and the index of refraction as the result of the nonlinear dependence of the current on the field. We limit ourselves to the case of normal incidence of the electromagnetic wave from the vacuum on the plane separating the vacuum from the plasma semiconductor.[3] For definiteness, we consider the incidence and refraction of the x component; the y and z components of the electric field can be obtained by means of (2.7). On the plane of separation z = 0, the following boundary conditions must be satisfied:

$$E_x^{(q)}(0) = E_x^{(q)}(-0), \quad \frac{\partial E_x^{(q)}(0)}{\partial z} = \frac{\partial E_x^{(q)}(-0)}{\partial z}. \quad (2.13)$$

The last condition in (2.13) follows from the continuity of the magnetic field.

We obtain the reflection coefficient and the index of refraction in the following way. The field incident from the vacuum will be sought in the form[4]

$$E_z^{(q)} = \mathcal{E}^{(q)}\left[\exp\left(-i\frac{\omega_q}{c}z\right) + P_q\exp\left(i\frac{\omega_q}{c}z\right)\right], \quad (2.14)$$

where P_q is the reflection coefficient and $\mathcal{E}^{(q)}$ is the amplitude of the field in the vacuum. The index of refraction R_q is defined by the relation

$$R_q = E^{(q)}(-0)\ /\ \mathcal{E}^{(q)}.$$

Substituting the expression for the electric field in the semiconductor from (2.10), and the field in the vacuum from (2.14) in (2.13), we find expressions for P_q and R_q:

$$P_q = P_{q0} + \delta P_q, \quad R_q = R_{q0} + \delta R_q,$$

$$P_{q0} = \frac{1-n_q}{1+n_q}, \quad R_{q0} = \frac{2}{1+n_q},$$

$$\delta P_q = \delta R_q = \frac{2i\varkappa_q}{(1+n_q)^2}\xi(1). \quad (2.15)$$

Here P_{q_0} and R_{q_0} are the reflection coefficient and the index of refraction in the absence of dissipation, while δP_q and δR_q are the corrections to them associated with the presence of damping in the medium. In a weak electric field, $\xi(1) = 1$ and $\delta R_q = \delta R_{q_0} = 2i\kappa_q/(1 + n_q)^2$, where δR_{q_0} is the corresponding value of linear theory. It is interesting that the ratio $\delta R_q/\delta R_{q_0}$ does not depend on the number of the harmonic q.

The field $E_{i0}^{(q)}$ entering into $\xi(1)$ can be ex-

[3]The generalization to the case of oblique incidence is trivial (see [4]).

[4]The index x is omitted everywhere below where it does not lead to ambiguities.

pressed in terms of the amplitude of the incident wave by means of the relation $E_0^{(q)} = R_{q_0} \mathcal{E}^{(q)}$ and the relations (2.7). Making the corresponding substitutions, we find the following expression for the value of u_0^2 entering into $\xi(1)$:

$$u_0^2 = \sum_{q=1}^{\infty} M_q \,|\, \mathcal{E}^{(q)} |^2, \quad M_q = \frac{16e^2}{3m(1+n_q)^2(\omega_H^2-\omega_q^2)^2}$$
$$\times \Big[(\omega_H^2+\omega_q^2)(1+|K_q|^2+|L_q|^2)$$
$$+\frac{\omega_H^2}{\omega_q^2}(\omega_H^2-3\omega_q^2)\,|\,L_q\cos\vartheta+K_q\sin\vartheta\,|^2$$
$$+4\omega_q\omega_H \operatorname{Im}(L_q\sin\vartheta-K_q\cos\vartheta)\Big]. \qquad (2.16)$$

We note that equations of the type (2.15) do not solve the boundary problem for an arbitrary form of the field incident from the vacuum on the separation boundary.

In the linear theory, the field inside the magnetoactive plasma consists of two normal waves, the superposition of which is also a solution. By means of this superposition, one can satisfy the boundary conditions (2.13) for arbitrary polarization of the field incident from a vacuum.

Under conditions of nonlinear propagation, the principle of superposition does not hold, and the theory constructed in the present paper describes the possibility of excitation of only one of the normal waves. In order that the "ordinary" or "extraordinary" waves be excited in the specimen, the x and y components of the incident field must be connected with the first of the relations (2.7), which imposes additional requirements on the polarization of the wave incident from the vacuum.

3. PROPAGATION OF STRONG ELECTROMAGNETIC WAVES OF HIGH FREQUENCY FAR FROM RESONANCE (LIMITING AND PARTICULAR CASES)

In the preceding section, formulas were obtained which describe the distribution of the field in an electron gas, the change in the coefficient of reflection and the index of refraction, etc. The results were expressed there in the form of quadratures. For subsequent investigation, it is necessary to make some additional assumptions.

To evaluate the integrals in Eq. (2.5) it is necessary to know the dependence of $A(\epsilon)$ and $\tau(\epsilon)$ on energy in explicit form. This dependence has been repeatedly computed for various types of scattering centers, and it has been shown that $A(\epsilon)$ and $\tau(\epsilon)$ are determined by the following formulas:

$$A(\varepsilon) = A_0(T)(\varepsilon/T)^r, \quad \tau(\varepsilon) = \tau_0(T)(\varepsilon/T)^l. \quad (3.1)$$

The values of r and l for various forms of scattering are given in the Table. The bars in the first column of the last three rows indicate that energy exchange does not take place in scattering by the impurities. Generally speaking, it is possible that the transfer of energy by the carrier is realized by one type of scattering, and that of momentum by another type, so that r from one row can correspond to l from another.

The integral in (2.5) can be computed in terms of elementary functions in two cases:

1) the external field can be regarded as very strong, i.e.,

$$u^2\nu(\varepsilon)\varepsilon/A(\varepsilon) \gg 1;$$

2) the fields are arbitrary, but r and l are connected by the relation $r + l = 1$; this relation is satisfied in scattering by acoustic, piezoacoustic, and optical phonons.

Let us first consider case one. Neglecting small quantities in the integrand expressions of (2.5), we obtain the following expression for $\xi(w)$:

$$\xi(w) = \psi_1(r,l)\left(\frac{T}{\varepsilon_0}\right)^l [v(w)]^{-2l/(r+l)},$$
$$\psi_1(r,l) = \Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3-2l}{2(r+l)}\right) \Big/ \Gamma\left(\frac{3-2l}{2}\right)\Gamma\left(\frac{3}{2(r+l)}\right),$$
$$\frac{\varepsilon_0}{T} = \left(\frac{Tu_0^2(r+l)}{A_0\tau_0}\right)^{1/(r+l)}. \qquad (3.2)$$

The distribution function in a strong electric field has the form

$$f(\varepsilon) = \frac{r+l}{4\pi\sqrt{2}}\Gamma^{-1}\left(\frac{3}{2(2+l)}\right) m^{-3/2}\varepsilon_0^{-3/2}v^{-3}$$
$$\times\exp\left\{-\left(\frac{\varepsilon}{\varepsilon_0 v^2}\right)^{1/(r+l)}\right\}. \qquad (3.3)$$

It follows from (3.3) that the distribution function is normalizable if $r + l > 0$. It has been shown previously[6] that the distribution function is normalizable for arbitrary statistical electric and magnetic fields if the inequalities

Form of Scattering	r	l	Form of Scattering	r	l
Acoustic oscillations	3/2	−1/2	Neutral impurities	—	0
Optical oscillations	1	0	Charged impurities	—	3/2
Piezoacoustic oscillations	1/2	1/2	Dipole impurities	—	1/2

$$r+l>0, \quad r-l>0 \tag{3.4}$$

are satisfied or, which amounts to the same thing, $r > 0$, $r > l > -r$. These conditions are carried over without change to variable electric fields. In what follows they will be assumed to be satisfied. Starting out from Eq. (3.2), one can immediately write down the expressions for the change in the coefficient of reflection and the index of refraction:

$$\delta R_q = \frac{2i\varkappa_q}{(1+n_q)^2}\psi_1(r,l)\left(\frac{A_0\tau_0}{Tu_0^2(r+l)}\right)^{l/(r+l)}. \tag{3.5}$$

In order to carry through the integration in the first of Eqs. (2.10) to conclusion, we make another assumption, namely, we assume that one of the harmonics in the incident wave, for example, the first, has an amplitude that is significantly larger than the rest; then only the first term in the infinite sum (2.8) remains, as the result of which the following formulas are obtained:

$$\xi(w) = \psi_1(r,l)\left(\frac{T}{\varepsilon_0}\right)^l w^{-l\varkappa_1/(r+l)},$$

$$\frac{\varepsilon_0}{T} = \left(\frac{TM_1|\mathcal{E}^{(1)}|^2(r+l)}{A_0\tau_0}\right)^{1/(r+l)}. \tag{3.2'}$$

Carrying out the integration in (2.10), we immediately get

$$w = \left[1 - 2\frac{\omega}{c}\frac{l}{(r+l)}\psi_1(r,l)\varkappa_1\left(\frac{T}{\varepsilon_0}\right)^l z\right]^{(r+l)/\varkappa_1 l}.$$

$$E^{(q)} = R_{0q}\mathcal{E}^{(q)}\left[1 - \frac{2\omega}{c}\frac{l}{r+l}\psi_1(r,l)\varkappa_1\left(\frac{T}{\varepsilon_0}\right)^l z\right]^{q(r+l)\varkappa_q/2l\varkappa_1}$$

$$\times \exp\left(-\frac{i\omega_q}{c}n_q z\right). \tag{3.6}$$

The average energy of the current carrier is also of interest:

$$\bar{\varepsilon} = \Gamma\left(\frac{5}{2(r+l)}\right)\Gamma^{-1}\left(\frac{3}{2(r+l)}\right)$$

$$\times\varepsilon_0\left[1 - 2\frac{\omega}{c}\frac{l}{r+l}\psi_1(r,l)\varkappa_1\left(\frac{T}{\varepsilon_0}\right)^l z\right]^{1/l}. \tag{3.7}$$

It is interesting to note that when $l = 0$ the linear relation between the current and the field is preserved regardless of the dependence on the field strength, and the expression for the field (3.6) goes over into the ordinary formula of the theory of the linear skin effect with an exponentially damped field. For other values of l, the field is damped according to a law that is slower than exponential.

We now proceed to the analysis of the derived expressions. We first investigate the dependence of the effects under investigation on the amplitude of the external electric field $\mathcal{E}^{(q)}$. It is seen from (3.1) that $\epsilon_0/T \sim |\mathcal{E}^{(q)}|^{2/(r+l)}$, i.e., ϵ_0/T increases with the increase in the electric field because $r + l > 0$. The value of l can be both larger and smaller than 0. The electric field and the average energy fall off with increase in z both for $l > 0$ and for $l < 0$; however the character of the fall-off is different for each of the two cases. The characteristic dimension over which the field decreases significantly is

$$\delta \sim \frac{c}{\omega\varkappa_1}\left(\frac{\varepsilon_0}{T}\right)^l \sim \delta_0\left(\frac{\varepsilon_0}{T}\right)^l,$$

where the damping length in linear theory is denoted by $\delta_0 = c/\omega\kappa_1$. Equations (3.2)—(3.7) were obtained under assumptions equivalent to the assumption $\bar{\epsilon}/T \gg 1$, and consequently, $\epsilon_0/T \gg 1$.

If $l > 0$, then $\delta > \delta_0$, i.e., the characteristic damping distance in the theory of a strong field for this case is much larger than in the theory of a weak field. Here all the electrons whose energy is much larger than T (the "hot" electrons) are concentrated in a layer whose thickness does not exceed δ in order of magnitude. This is seen directly from Eq. (3.7). Actually, ϵ goes to zero, for the case $l > 0$, for $z = (r + l)\delta/2l\psi_1(r, l)$. The factor in front of δ is of the order of unity. Further, the criterion for the applicability of the theory is the condition $\bar{\epsilon}/T \gg 1$. Thus the theory is inapplicable for $z \sim \delta$.

The thickness δ_1 of the layer in which the "hot" electrons are concentrated can be estimated roughly by assuming ϵ to be of the order of T: $\delta_1 \sim [1 - (T/\epsilon_0)^l]\delta$. In the estimate, the numerical factors of the order of unity have been omitted. We note that $\delta_1 \gg \delta_0$. For $l < 0$ and $\delta \ll \delta_0$, similar estimates show that the "hot electrons" can occupy many layers of the thickness δ, but $\delta_1 < \delta_0$ because $\bar{\epsilon} \sim T$ for $\delta \sim \delta_0$. The dependence of the penetration depth of the field and the change in the coefficient of reflection and index of refraction on $\mathcal{E}^{(q)}$ is easily established:

$$\delta \sim (\delta R_q)^{-1} \sim |\mathcal{E}^{(q)}|^{2l/(r+l)}.$$

If the condition $r + l = 1$ is satisfied, then the distribution function (2.7) becomes Maxwellian with effective temperature

$$\Theta(w) = T\left[1 + \frac{u_0^2 T}{A_0\tau_0}v^2(w)\right]. \tag{3.8}$$

Simple integration leads to the following expression for $\xi(w)$:

$$\xi(w) = (T/\Theta(w))^l, \tag{3.9}$$

whence we have for δR_q:

$$\delta R_q = \frac{2i\varkappa_q}{(1+n_q)^2}\left[1+\frac{u_0{}^2 T}{A_0\tau_0}\right]^{-l}. \tag{3.10}$$

In order to carry out the integration in Eqs. (2.10), we again assume that the amplitude of the first harmonic is much larger than the amplitude of the remaining harmonics. We further note that for all real processes of scattering, l is a half integer (see the table) ($l = \mu - 1/2$ where μ is an integer). The only exception to this rule, $l = 0$, is not of interest because in this case Ohm's law holds. With account of these circumstances, we have

$$2\frac{\omega}{c}\varkappa_1 z = \frac{\mu}{|\mu|}\sum_k \frac{2}{2k-1}\left(\frac{\Theta'(x)}{T}\right)^{k-1/2}\Bigg|_w^1 + \ln\frac{(\Theta'(x)/T)^{1/2}-1}{(\Theta'(x)/T)^{1/2}+1}\Bigg|_w^1,$$

$$\frac{\Theta'(x)}{T} = 1 + \frac{TM_1\,|\mathcal{E}^{(1)}|^2}{A_0\tau_0}x^{\varkappa_1},$$

$$\Theta'(x)\big|_a^b = \Theta'(b) - \Theta'(a). \tag{3.11}$$

Here the summation over k is carried out in the limits from 1 to μ for $\mu > 0$ and from 0 to $-(\mu+1)$ for $\mu < 0$.

Equation (3.11) gives w(z) in implicit form. We consider the limiting case of large z:

$$z\varkappa_1\omega / c \gg 1, \quad w \ll 1, \quad \Theta'(w)/T \approx 1.$$

We write down the expressions for the fields:

$$E^{(q)}(z) = R_{q0}\mathcal{E}^{(q)}\left(\frac{4}{\Theta'(1)/T-1}\right)^{q\varkappa_q/2\varkappa_1} \times\left[\frac{(\Theta'(1)/T)^{1/2}-1}{(\Theta'(1)/T)^{1/2}+1}\right]^{q\varkappa_q/2\varkappa_1} \times\exp\left\{\frac{q\varkappa_q}{\varkappa_1}\left(\frac{\mu}{|\mu|}\sum_k\frac{1}{2k-1}\left[\frac{\Theta'(1)}{T}-1\right]\right) - \frac{\omega_q}{c}(in_q+\varkappa_q)z\right\}. \tag{3.12}$$

In the case of a strong electric field $\Theta'(1)/T \gg 1$ for the ratio of the field within the body of the specimen to its value in the linear theory we have the following formula:

$$\frac{E^{(q)}}{E_\lambda{}^{(q)}} = \left(\frac{4T}{\Theta'(1)}\right)^{q\varkappa_q/2\varkappa_1}\exp\left\{\frac{q\varkappa_q}{2l\varkappa_1}\left(\frac{\Theta'(1)}{T}\right)^l\right\} \gg 1 \quad \text{for}\ \mu > 0,$$

$$\frac{E^{(q)}}{E_\lambda{}^{(q)}} = \left(\frac{4T}{\Theta'(1)}\right)^{q\varkappa_q/2\varkappa_1}\exp\left\{\frac{q\varkappa_q}{\varkappa_1}\sum_{k=0}^{-(\mu+1)}\frac{2}{2k-1}\right\} \ll 1 \quad \text{for}\ \mu < 0. \tag{3.13}$$

For $l = -1/2$, the fields are determined by the expression

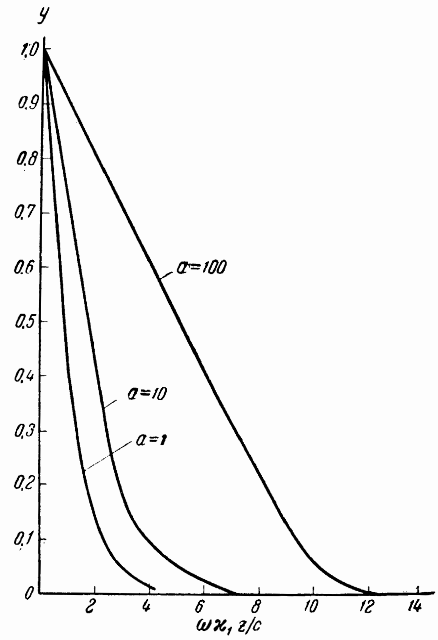

FIG. 1

$$E^{(q)} = R_{q0}\mathcal{E}^{(q)}\left(\frac{4}{\Theta'(1)/T-1}\right)^{q\varkappa_q/2\varkappa_1}\left[\frac{(\Theta'(1)/T)^{1/2}-1}{(\Theta'(1)/T)^{1/2}+1}\right]^{q\varkappa_q/2\varkappa_1} \times\left[1-\frac{(\Theta'(1)/T)^{1/2}-1}{(\Theta'(1)/T^{1/2}+1}\exp\left(-\frac{2\omega}{c}\varkappa_1 z\right)\right]^{-1} \times\exp\left[-\frac{\omega_q}{c}z(in_q+\varkappa_q)z\right]. \tag{3.14}$$

In order to obtain a representation of the behavior of the field in the medium for arbitrary fields incident from a vacuum, the graphical dependence of $y(z) = E_1(z)/E_{10}$ on $\omega\kappa_1 z/c$ was constructed for the values $a = (\Theta'(1) - T)/T$ equal to 100, 10, 1 for $l = \pm 1/2$ for $q = 1$. It is seen on the graphs that for $l = 1/2$ (Fig. 1) the penetration depth for $a > 1$ is much larger than in the linear theory while for $l = -1/2$ (Fig. 2) it is much less.

We note that the expressions for the fields obtained with the help of the kinetic theory for $l = -1/2$, $\omega_H = 0$, $q = 1$ are qualitatively identical with the results of elementary theory (compare, for example, Eq. (3.13) with Eq. (16) of the work of Gurevich[3]). It has already been shown there that upon passage of a strong electromagnetic field through a layer of completely ionized plasma, the penetration depth can be appreciably greater than in the linear theory, which is also in agreement with the results of a kinetic consideration.

The dependence on the frequency and on the magnetic field for the effects under consideration is determined by the value of M_q. The general case can only be studied graphically. Here we

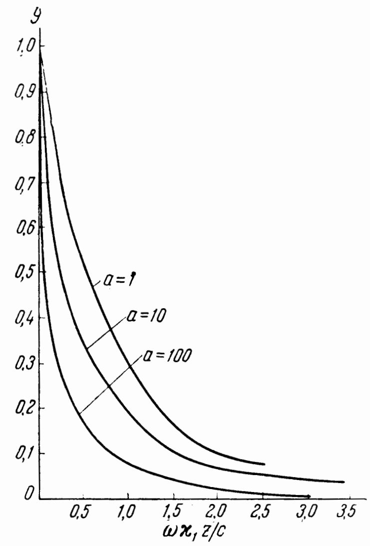

FIG. 2

limit ourselves to the longitudinal propagation of strong electric fields, q = 1, and we shall also consider that ω, $\omega_H \ll \omega_0$ and $\omega_H > \omega$. Under these assumptions, the relative change in the penetration depth is proportional to $((\omega_H - \omega)/\omega)^{l/(r+l)}$. We shall not linger here on the case $\omega\tau \ll 1$. We note only that for longitudinal propagation, the results obtained are valid only for the single limitation $|\omega - \omega_H|\tau \ll 1$ and, consequently, are applicable also for $\omega\tau \ll 1$ if $\omega_H\tau \gg 1$.

4. NONLINEAR RESONANCE EFFECTS

It is seen from Eqs. (2.1) that the coefficients A_q, B_q, and G_q in Eqs. (2.1) increase strongly for certain values of the frequency of the electromagnetic field, while their imaginary part remains much larger than the real. This leads to a sharp increase in the dissipation of the electromagnetic field. This phenomenon is known as resonance and has been studied thoroughly in linear theory. In the present section, we shall consider nonlinear resonance effects.

We assume that a monochromatic wave is propagated in the medium with resonance frequency ω. We first consider the case in which the inequalities $|\omega - \omega_H|\tau \gg 1$ are satisfied. It follows from Eqs. (2.1) that the resonance increase of A_q, B_q, and G_q takes place if the real part of the denominator of these quantities vanishes:

$$\sin^2\vartheta\varepsilon_{11}' + \cos^2\vartheta\varepsilon_{33}' = 0. \tag{4.1}$$

Substituting the real parts ϵ_{11}' and ϵ_{33}' from Eqs. (2.3) in (4.1), we get an expression for the resonance frequencies:

$$\omega_{1,2}^2 = {}^1\!/_2(\omega_0^2 + \omega_H^2) \pm [{}^1\!/_4(\omega_0^2 + \omega_H^2)^2 - \omega_0^2\omega_H^2\cos^2\vartheta]^{1/2}. \tag{4.2}$$

This is the ordinary expression for the frequencies of magnetoplasma resonance.[1] Since we have assumed $\omega \neq \omega_H$, then longitudinal propagation of electromagnetic waves ($\vartheta = 0$) drops out of consideration. This case will be investigated separately below.

We write down the set of Maxwell equations for the resonance electromagnetic field. It follows from the third equation of (2.1) that E_z is ω/ν times larger than E_x or E_y. Keeping the principal term in ω/ν in u^2 in Eq. (2.2), and considering the field to be strong, we get the expression (3.2) for $\varphi(v^2)$, in which it is necessary to set

$$v^2 = \frac{|E_z|^2}{|E_{0z}|^2}, \qquad \frac{\varepsilon_0}{T} = \left(\frac{4(r+l)\,|E_{0z}|^2 e^2 T}{3A_0\tau_0 m W(\omega)}\right)^{1/(r+l)},$$

$$W(\omega) = \frac{\omega^2(\omega^2 - \omega_H^2)}{\omega^4 + \omega_H^4\cos^2\vartheta + \omega^2\omega_H^2(1 - 3\cos^2\vartheta)}. \tag{4.3}$$

With account of (2.4) and (4.1), the Maxwell equations are written as follows:

$$\frac{d^2E_x}{dz^2} - i\frac{\omega^2}{c^2}[aE_x + ibE_y]\,\psi_1^{-1}(r, l)\left(\frac{\varepsilon_0}{T}\right)^l v^{2l/(r+l)} = 0,$$

$$\frac{d^2E_y}{dz^2} - i\frac{\omega^2}{c^2}[-ibE_x + gE_y]\,\psi_1^{-1}(r, l)\left(\frac{\varepsilon_0}{T}\right)^l v^{2l/(r+l)} = 0,$$

$$E_z = \frac{i\sin\vartheta}{\sin^2\vartheta\varepsilon_{11}'' + \cos^2\vartheta\varepsilon_{33}''}\{i\varepsilon_{12}''E_x$$

$$+ \cos\vartheta(\varepsilon_{33}' - \varepsilon_{11}')E_y\}\,\psi_1^{-1}(r, l)\left(\frac{\varepsilon_0}{T}\right)^l v^{2l/(r+l)}, \tag{4.4}$$

where

$$a = -\frac{\sin^2\vartheta\varepsilon_{12}''^2}{\sin^2\vartheta\varepsilon_{11}'' + \cos^2\vartheta\varepsilon_{33}''}$$

$$b = \frac{\cos\vartheta\varepsilon_{12}''\varepsilon_{33}'}{\sin^2\vartheta\varepsilon_{11}'' + \cos^2\vartheta\varepsilon_{33}''}, \tag{4.5}$$

$$g = \frac{\varepsilon_{11}'\varepsilon_{33}'}{\sin^2\vartheta\varepsilon_{11}'' + \cos^2\vartheta\varepsilon_{33}''}.$$

We note that in the resonance case the nonlinearity of the Maxwell equations is nowhere small.

It is convenient to solve the nonlinear system (4.4) if one transforms to normal waves. We replace E_x and E_y in (4.4) by the linear combination $F_{1,2} = E_x - k_{1,2}E_y$. If we define $k_{1,2}$ by the formulas

$$k_{1,2} = \frac{i}{2b}\left[\frac{a-g}{2} \pm \left(\frac{(a-g)^2}{4} + b^2\right)^{1/2}\right], \tag{4.6}$$

then the set (4.4) reduces to the following form:

$$\frac{d^2F_{1,2}}{dz^2} - i\frac{\omega^2}{c^2}n_{1,2}^2\psi_1^{-1}(r, l)\left(\frac{\varepsilon_0}{T}\right)^l v^{2l/(r+l)}F_{1,2} = 0,$$

$$E_z = (\gamma_1F_1 + \gamma_2F_2)\,\psi_1^{-1}(r, l)\left(\frac{\varepsilon_0}{T}\right)^l v^{2l/(r+l)}, \tag{4.7}$$

where

$$n_{1,2}^2 = \frac{1}{2}(a+g) \pm \left[\frac{1}{4}(a-g)^2 + b^2\right]^{1/2},$$

$$\gamma_{1,2} = \pm\frac{\sin\vartheta b\,[i\varepsilon_{12}k_{1,2} + \cos\vartheta(\varepsilon_{33}' - \varepsilon_{11}')]}{(\sin^2\vartheta\varepsilon_{11}'' + \cos^2\vartheta\varepsilon_{33}'')\,[(a-g)^2 + 4b^2]^{1/2}}. \quad (4.8)$$

It follows from Eq. (4.8) that $n_{1,2}^2$, $k_{1,2}$ and $\gamma_{1,2}$ do not depend on the electric field.

Equations (4.7) go over to the equation of linear theory for $l = 0$. It follows from this fact that the $n_{1,2}$ are indices of refraction in the linear theory if the field in the linear theory is sought in the form $\exp(i^{3/2}\omega n_{1,2}z/c)$.

It was shown above that it is impossible in the nonlinear theory to consider simultaneous propagation of both normal waves because their superposition is not a solution. For example, let us investigate the propagation of the first wave by setting $F_2 = 0$. It follows from $F_2 = 0$ that

$$E_x / E_y = k_2. \quad (4.9)$$

Thus the $k_{2,1}$ are polarization coefficients for the first and second waves, respectively. It is seen from (4.8) that the polarization coefficients in the nonlinear theory are described by the same relations as in the linear.

In the set of equations (4.7) we put $F_2 = 0$ and using the second of the equations of the system, eliminate v from the first equation. Finally, we have for the determination of F_1 the formula[5)]

$$\frac{d^2F}{dz^2} - i\frac{\omega^2}{c^2}n^2\psi_1^{-1}(r,l)\left(\frac{\varepsilon_0}{T}\right)^l\left|\frac{F}{F_0'}\right|^{2l/(r-l)}F = 0, \quad (4.10)$$

where $F_0' = F(-0)$. We shall seek a solution of (4.10) in the form

$$F / F_0' = (1 + \omega\varkappa z / c)^{-(\alpha+i\beta)}. \quad (4.11)$$

Substituting (4.11) in (4.10), we get

$$\varkappa^2(\alpha + i\beta)(\alpha + i\beta + 1)(1 + \omega\varkappa z / c)^{-2}$$
$$- in^2\psi_1^{-1}(r,\ l)(\varepsilon_0 / T)^l(1 + \omega\varkappa z / c)^{-\alpha 2l/(r-l)} = 0.$$

To satisfy this equation, it is necessary that the following conditions hold:

$$\alpha\frac{l}{(r-l)} = 1,$$

$$\varkappa^2(\alpha + i\beta)(\alpha + i\beta + 1) = in^2\psi_1^{-1}(r,l)(\varepsilon_0/T)^l. \quad (4.12)$$

By equating the real and imaginary parts in the second of Eq. (4.12) separately, we get three equations for the determination of the three unknowns α, β, and κ, whose solutions have the following form:

$$\alpha = \frac{r-l}{l}, \quad \beta = \frac{1}{l}[r(r-l)]^{1/2};$$

$$\varkappa = \frac{l(\varepsilon_0/T)^l n}{[(2r-l)(r(r-l))^{1/2}]^{1/2}\,\psi_1(r,l)}. \quad (4.13)$$

We note that the set (4.12) is in a definite sense equivalent to the dispersion equation of linear theory.

By virtue of the inequalities (3.4), β is an imaginary quantity; the sign in front of the square roots in Eq. (4.13) is so chosen that κ is larger than zero.

The value of E_z is easily determined from the second equation of the set (4.7):

$$E_z / E_{0z} = (1 + \omega\varkappa z / c)^{\alpha-2-i\beta}, \quad (4.14)$$

whence it is seen that E_z falls off with increase of z more rapidly than F, that is, the larger the value of z, the closer the wave is to transverse:

We proceed to the solution of the boundary problem. Let the wave

$$F(z) = F_0(e^{-i\omega z/c} + Pe^{i\omega z/c}), \quad (4.15)$$

be incident from the vacuum ($z < 0$) on the half-space $z > 0$, which is filled with semiconductor or plasma; here P is the reflection coefficient. F(z) satisfies on the interface $z = 0$ the following conditions by virtue of (2.11):

$$F(0) = F(-0), \qquad \partial F(0) / \partial z = \partial F(-0) / \partial z. \quad (4.16)$$

We must still determine the reflection coefficient P and the index of refraction $R = F(-0)/F_0$, which is connected with the impedance ζ by the relation $R = 2\zeta$.

It is necessary to keep in mind that E_{0z}, which enters into ϵ_0, must be expressed in terms of F_0. This can be done if one sets $z = 0$ in the second of the equations of the set (4.7) and assumes that

$$F(-0) = 2\zeta F_0.$$

Substituting F from (4.11) and (4.15) in the boundary condition (4.16), we get a set of two equations for the determination of P and ζ; this set is easily solved if we assume that $|\zeta| \ll 1$. The smallness of $|\zeta|$ follows from the fact that the penetration depth in the resonance case is small in comparison with the wavelength in the vacuum, while $|\zeta|$ is of the order of their ratio. Finally, we get

$$P = 1 + 2\zeta, \quad (4.17)$$

$$F(z) = 2\zeta F_0\left(1 + \frac{l}{[(2r-l)(r-l)]^{1/2}}\frac{\omega z}{c|\zeta|}\right)^{-(r-l)/l - i[r(r-l)]^{1/2}/l}, \quad (4.18)$$

5)The index 1 will be omitted below.

$$E_z(z) = \frac{2i|\gamma|\zeta F_0}{|\zeta|^2 n^2}\left[\frac{r}{r-l}\right]^{1/2} \times\left(1+\frac{l}{[(2r-l)(2-l)]^{1/2}}\frac{\omega z}{c|\zeta|}\right)^{-(r+l)/l-i[r(r-l)]^{1/2}/l}, \quad (4.19)$$

$$\zeta = 2^{-l/r}[\psi_1(r,l)]^{(r+l)/2r}\left(\frac{r}{r-l}\right)^{(r-l)/4r}|\gamma|^{-l/r}n^{-(r-l)/r} \times\left[\frac{4(r+l)e^2T|F_0|^2}{3mW(\omega)A_0\tau_0}\right]^{-l/2r}\exp\left\{i\operatorname{arctg}\left[\frac{r-l}{r}\right]^{1/2}\right\}. \quad (4.20)^*$$

We write out the equation for the mean energy:

$$\varepsilon/T = 2^{2/r}\left[\Gamma\left(\frac{5}{2(r+l)}\right)\Big/\Gamma\left(\frac{3}{2(r+l)}\right)\right]\left(\frac{r}{r-l}\right)^{1/2r} \times[\psi_1(r,l)]^{-1/r}|\gamma|^{2/r}n^{-2/r}\left[\frac{4(r+l)e^2T|F_0|^2}{3mW(\omega)A_0\tau_0}\right]^{1/r} \times\left(1+\frac{l}{[(2r-l)(r-l)]^{1/2}}\frac{\omega z}{c|\zeta|}\right)^{-2/l}. \quad (4.21)$$

These results are suitable both for the first and the second waves.

We shall discuss the results at the end of this section, but now we shall consider the case of a transverse propagation of the wave ($\vartheta = \pi/2$). For transverse propagation, the wave with index 2 is nonresonant; therefore, we limit ourselves to the consideration of a wave with index 1. The expression for the impedance for a transverse propagation has the following form:

$$\zeta = \frac{\sqrt{2}}{2^{l/r}}\left(\frac{r}{r-l}\right)^{(r-l)/4r}\left[\Gamma\left(\frac{5-2l}{2}\right)\Big/\Gamma\left(\frac{5}{2}\right)\right]^{(r+l)/2r} \times[\psi_1(r,l)]^{(r+l)/2r}\left(1+\frac{\omega_0^2}{2\omega_H^2}\right)^{1/2}\left(\frac{\omega_0}{\omega_H}\right)^{l/2r} \times\left(1+\frac{\omega_0^2}{\omega_H^2}\right)^{(r-l)/r}\left[\frac{4(r+l)e^2T|F_0|^2}{3mA_0\tau_0}\right]^{-l/2r} \times\exp\left\{i\operatorname{arc\,tg}\left[\frac{r-l}{r}\right]^{1/2}\right\}. \quad (4.22)$$

For $\bar{\varepsilon}/T$, the following equation is obtained:

$$\bar{\varepsilon}/T = 2^{2/r}\left[\Gamma\left(\frac{5}{2(r+l)}\right)\Big/\Gamma\left(\frac{3}{2(r+l)}\right)\right] \times\left[\Gamma\left(\frac{5}{2}\right)\Big/\Gamma\left(\frac{5-2l}{2}\right)\right]^{1/r}\left(\frac{r}{r-l}\right)^{2/r} \times[\psi_1(r,l)]^{-1/r}\left(1+\frac{\omega_H^2}{\omega_0^2}\right)^{1/2r}\left[\frac{4(r+l)e^2T|F_0|^2}{3mA_0\omega_0}\right]^{1/r} \times\left[1+\frac{l}{[(2r-l)(r-l)]^{1/2}}\frac{\omega z}{c|\zeta|}\right]^{-2/l} \quad (4.23)$$

*arctg = $\tan^{-1}$

We proceed to the case of a purely longitudinal propagation $\vartheta = 0$. The following expression is obtained from (1.9) for the symmetric part of the propagation function:

$$f(\varepsilon) = C\exp\left\{-\frac{3mA_0\varepsilon^{r-l}}{4(r-l)e^2T\tau_0|E_0|^2v^2}\right\};$$

$$|E_0|^2 = |E_{0x}|^2+|E_{0y}|^2, \quad v^2 = \frac{|E_x|^2+|E_y|^2}{|E_{0x}|^2+|E_{0y}|^2}. \quad (4.24)$$

For longitudinal propagation, it is also convenient to introduce the normal $F_{1,2} = E_x \pm iE_y$. The normal wave F_1 is resonant (the index 1 for the resonant harmonic is omitted in what follows). Setting $\vartheta = 0$ and $\omega = \omega_H$ in (2.1) we obtain the following relation for F:

$$\frac{d^2F}{dz^2}+\frac{\omega_H^2}{c^2}\left(1-i\psi_2(r,l)\left(\frac{\varepsilon_0'}{T}\right)^l\eta^2\left|\frac{F}{F_0}\right|^{2l/(r-l)}\right)F = 0,$$

$$\psi_2(r,l) = \Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3+2l}{2(r-l)}\right)\Big/\Gamma\left(\frac{3+2l}{2}\right)\Gamma\left(\frac{3}{2(r-l)}\right),$$

$$\eta^2 = \frac{\omega_0^2\tau_0}{\omega_H}\Gamma\left(\frac{5+2l}{2}\right)\Big/\Gamma\left(\frac{5}{2}\right),$$

$$\frac{\varepsilon_0'}{T} = \left(\frac{2(r-l)e^2T\tau_0|F_0|^2}{3mA_0}\right)^{1/(r-l)}. \quad (4.25)$$

The resonance is most sharply pronounced if one can neglect unity in the Eq. (4.25). Physically, this corresponds to a smallness of the displacement current in comparison with the conduction current. After neglect of unity, Eq. (4.25) is identical, apart from sign with Eq. (4.10), in which connection the field is determined by Eq. (4.18) with impedance ζ which has the following form:

$$\zeta = \left(\frac{r}{r-l}\right)^{(r-l)/4r}\left[\Gamma\left(\frac{5}{2}\right)\Big/\Gamma\left(\frac{5+2l}{2}\right)\right]^{(r-l)/2r} \times[\psi_2(r,l)]^{-(r-l)/2r}\left(\frac{\omega_0}{\omega_H}\right)^{l/2r}\left(\frac{\omega_H}{\omega_0^2\tau_0}\right)^{1/2} \times\left(\frac{2(r-l)e^2T|F_0|^2}{3mA_0\omega_0}\right)^{-l/2r}\exp\left\{i\operatorname{arc\,tg}\left[\frac{r-l}{r}\right]^{1/2}\right\}. \quad (4.26)$$

For the average value of the energy, we have

$$\frac{\bar{\varepsilon}}{T} = \left[\Gamma\left(\frac{5}{2(r-l)}\right)\Big/\Gamma\left(\frac{3}{2(r-l)}\right)\right]\left[\Gamma\left(\frac{5}{2}\right)\Big/\Gamma\left(\frac{5+2l}{2}\right)\right]^{1/r} \times[\psi_2(r,l)]^{-1/r}\left(\frac{\omega_H}{\omega}\right)^{1/r}\left(\frac{2(r-l)e^2T|F_0|^2}{3mA_0\omega_0}\right)^{1/r} \times\left[1+\frac{l}{[(2r-l)(r-l)]^{1/2}}\frac{\omega z}{c|\zeta|}\right]^{-2/l}. \quad (4.27)$$

It can be shown that for $\omega_H = 0$ and $\omega\tau \ll 1$, the same formulas hold for the field and impedance if one introduces the following transformation in them: $F \to E_x$, $|F_0|^2 = 2|E_{x0}|^2(1 + |K|^2)$, where $K = E_y/E_x$ is the coefficient of polarization.

We proceed to a discussion of the results. In finding the distribution function in the kinetic equation, spatial derivatives have been neglected. For this neglect to be valid the condition $\delta \gg L(\overline{\Delta\epsilon}/\epsilon)^{-1}$ must be satisfied (see[1]), where δ is the distance over which the field changes appreciably and L is the mean free path. As is seen from the formula for the resonance harmonic, in the case of a strong nonlinearity of (4.21), $\delta \sim c|\zeta|/\omega$. There then follows the inequality for the impedance $|\zeta| \gg \omega L(\overline{\Delta\epsilon}/\epsilon)/c$, which limits the region of applicability of the theory. On the other hand, it follows from the condition of resonance that $|\zeta| \ll 1$. Thus the values of the impedance are limited by the conditions of applicability of the theory both above and below.

We have in fact made use above of the condition $\epsilon(0)/T \gg 1$. This in turn imposes the following condition on the incident field (see (4.23)):

$$|F_0| \gg (mA_0\omega_0/e^2T)^{1/2}.$$

We consider the penetration of the field inside the specimen. The penetration depth $\delta \sim \delta(\overline{\epsilon}(0)/T)^{l/2}$, where δ_0 is the penetration depth in linear theory. For $l < 0$, we have $\delta \ll \delta_0$, for $l > 0$ we have $\delta \gg \delta_0$. This situation is the reverse of that which is obtained in the high frequency nonresonant case (Sec. 3). Similarly to what was done in the previous section, one can show that for $l > 0$, the "hot electrons" are contained in the layer of thickness $\delta_1 \gg \delta_0$; for $l < 0$, one gets $\delta_1 \ll \delta_0$. The penetration depth is $\delta \sim |\zeta|^{-1} \sim |F_0|^{l/r}$.

It is of interest to compare the dependence of the field on the coordinates in the linear theory, the nonlinear theory in the nonresonant case and in the nonlinear theory for the case of resonance. In the linear theory, the field is exponentially damped with increasing z, while the phase is a linear function of the coordinate so that the phase velocity does not depend on the time. In nonlinear theory, in the absence of resonance, the phase also depends linearly on the coordinate but the field falls off according to a power law. In the presence of resonance in the nonlinear theory the field falls off with increase in the coordinate according to a power law and the phase φ varies like

$$\varphi = \omega t - \frac{(r-l)^{1/2}}{l}\ln\left(1 + \frac{\omega z}{c|\zeta|}\right), \qquad (4.27b)$$

so that the phase velocity is

$$v_\Phi = \frac{dz}{dt} = \frac{l}{(r-l)^{1/2}}c|\zeta|\exp\left\{\frac{l}{(r-l)^{1/2}}\omega t\right\}.$$

The author thanks M. I. Kaganov and M. Ya. Azbel' for discussions.

[1] V. L. Ginzburg, Raspostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960.

[2] V. L. Ginzburg and A. V. Gurevich, UFN 70, 201, 393 (1960), Soviet Phys. Uspekhi 3, 115, 175 (1960).

[3] B. I. Davydov, JETP 7, 1069 (1937).

[4] A. V. Gurevich, Radiotekhn. i. élektron. 3, 704 (1956).

[5] V. P. Shabanskiĭ, JETP 27, 142 (1954).

[6] F. G. Bass, FMM (Physics of Metals and Metallography) 6, 961 (1958).

Translated by R. T. Beyer
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