# THE ANGULAR DISTRIBUTION OF INTENSITY OF CERENKOV RADIATION FROM EXTENSIVE COSMIC-RAY AIR SHOWERS

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The angular distribution of intensity is calculated for the Cerenkov radiation produced in the terrestrial atmosphere by extensive air showers of cosmic rays. Calculations are made for showers arriving from the zenith and for conditions of observation at sea level and at an altitude of 3860 m above sea level. Photographic observation of the shape of the flash of light against the celestial sphere, as obtained in [2,3] is evidently in satisfactory agreement with the calculations.

## 1. INTRODUCTION

IN the registration of extensive air showers (EAS) by means of Cerenkov counters, [1,2] a knowledge of the angular distribution of the Cerenkov radiation is important primarily from the methodological point of view (choice of the angle subtended by the Cerenkov counters to obtain optimal signal-to-noise ratio, estimates of the accuracy of the angular coordinates of high-energy primary particles, and so on). Besides this, the angular distribution of the light from showers is already itself the object of physical investigation, [3] and therefore it is important to ascertain what kind of information about a shower can be obtained from such data. The present calculation has been made for this purpose, and is based on the following ideas.

Cerenkov radiation is mainly caused by the electronic component, which makes up the bulk of the charged particles in a shower. Owing to multiple Coulomb scattering by the nuclei of atoms in the air, electrons of energy E at a depth p have a Gaussian distribution of distances r from the axis of the shower, and a Gaussian distribution of angles relative to a mean angle $\vartheta$, which depends on r. The dispersions of the transverse and angular distributions depend on E. The energy spectrum of the electrons is an equilibrium one and does not depend on the degree of development of the shower in depth. For the case of primary photons the variation of the electrons with height is taken to be that given by the electromagnetic cascade theory, [4] and for the case of primary protons, that given by the calculations of Nikol'skiĭ and Pomanskiĭ. [5] The light emitted by the electrons is at the angle $\vartheta_{\text{Cer}}$ with the direction of their motion. Neither the scattering of the light by density inhomogeneities in the air nor absorption of the light is taken into account.

## 2. STATEMENT OF PROBLEM AND METHOD OF CALCULATION

The purpose of the calculation is to determine the number I of light quanta in the frequency range from $\lambda_1$ to $\lambda_2$ that fall on unit area of the earth's surface at distance R from the axis of the shower, and in the direction from any given point of the celestial sphere.

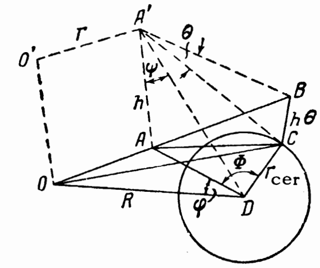


FIG. 1

Let us turn to Fig. 1. Here O is the trace of the axis of the shower on the earth's surface, D is the point of observation, and A′ is an arbitrary point which is at height h over the level of observation and is characterized by the angular coordinates $\psi$ (the zenith angle) and $\varphi$ (the azimuthal angle). We agree to measure the azimuthal angle from the direction from the point of observation D to the trace O of the axis of the shower on the earth's surface. The figure OBCD lies in the plane of the drawing, and OO′A′B in the perpendicular plane. We shall determine for the neighborhood of the point D the density of the luminous flux that reaches the earth's surface from the volume element dv with its center at A′. Let us consider the electrons with energy E that are in this volume element. Since the Cerenkov light is concentrated on the surface of a cone whose axis is the direction of motion of the electron at the instant of emission, light reaches D only from those electrons with the given energy E whose directions of motion in the element dv will, when prolonged to intersect with the earth's surface, fall on the circle of radius $r_{\text{Cer}}(h)$ and center at D, where $r_{\text{Cer}}(h)$ is the radius of the cone of light at the earth's surface.[1)]

We calculate the density of the axes of cones of light around some point C chosen arbitrarily on this circle. For this we must know the density of the electrons at A′ and their angular distribution. As in [6], we assume that at A′ the electrons of a given energy have a Gaussian angular distribution around the direction A′B making an angle $\vartheta(r)$ with the axis of the shower, and a Gaussian distribution of their distances from the axis of the shower. We denote the angle between the direction of motion of an electron at A′ and the direction A′B by $\theta$. Under these conditions the density $\kappa$ of axes of cones of light at the point C will be given by

$$\varkappa = N(E_0, p)F(E)C_1 \exp\left[-r^2/r_0^2\right]C_2 \exp\left[-\theta^2/\theta_0^2\right], \quad (1)$$

where $N(E_0, p)$ is the total number of electrons in the shower with the primary energy $E_0$ at the depth $p(h)$, and $C_1$ and $C_2$ are constants to be determined from the normalization conditions

$$\int_0^\infty C_1 \exp\left[-r^2/r_0^2\right]\cdot 2\pi r\,dr = 1,$$

$$\int_0^\pi C_2 \exp\left[-\theta^2/\theta_0^2\right]\cdot 2\pi \sin\vartheta\, d\vartheta = 1. \quad (2)$$

We assume the differential energy spectrum $F(E)$ of the electrons independent of h. As in [6], we take

$$F(E)\,dE = 0.75(1+\varepsilon)^{-2}d\varepsilon, \quad \varepsilon = 2.3E/\beta, \quad \beta = 72\,\text{MeV};$$
$$r_0^2(E, p) = 0.545(1+\varepsilon)^{-2}(2.3E_s/\beta)^2 t_0^2/\rho^2;$$

$$\theta_0^2 = 0.545(1+\varepsilon)^{-2}(2.3E_s/\beta)^2; \quad \bar{\vartheta}(r) = r/(t_0/\rho);$$
$$E_s = 21\ \text{MeV}; \quad t_0 = 34.2\ \text{g/cm}^2 \quad (3)$$

[E is expressed in MeV; $\rho$ is the density of the air at depth $p(h)$].

The quantity $\kappa$ must obviously depend on the position of the point C on the circle. A change of the position of the point C leads to a change of the required angle $\theta$. For fixed values of h, R, $\varphi$, $\psi$, and $\Phi$ the angle $\theta$ must satisfy the following condition

$$h^2\theta^2 = r^2 h\rho(1 + h\rho/t_0)/t_0 - h\rho \times\left[R^2 + r_{\text{cer}}^2 - 2Rr_{\text{cer}}\cos(\varphi+\Phi)\right]/t_0 + (1 + h\rho/t_0)(h^2\psi^2 + r_{\text{cer}}^2 - 2h\psi r_{\text{cer}}\cos\Phi), \quad (4)$$

and the quantity $r^2$ which appears in (1) and (4) is given by

$$r^2 = h^2\psi^2 + R^2 - 2Rh\psi\cos\varphi. \quad (5)$$

These two conditions are found from an analysis of the geometrical relations shown in Fig. 1. We thus have

$$\varkappa = \varkappa(E_0, p, E, R, \psi, \varphi, \Phi).$$

If $a(E,p)$ is the number of light quanta with wavelengths from $\lambda_1$ to $\lambda_2$ emitted by an electron of energy E in unit length of its path, the radiant flux density falling on the earth's surface at the distance R from the axis of the shower and in the direction $\varphi$ and $\psi$ is given by

$$I(E_0, R, \varphi, \psi) = \int_0^{p_0}\int_{E_{\text{thr}}}^{E_0}\int_0^{2\pi} \frac{a(E,p)}{2\pi r_{\text{cer}}} \times \chi(E_0, p, E, R, \psi, \varphi, \Phi)\, r_{\text{cer}}\, d\Phi\, dE\, dp. \quad (6)$$

The quantity $a(E,p)$ in (6) depends on the range of wavelengths chosen. In the interval 3000–6000 A this function is

$$a(E, p) = 354(1 - E^2_{\text{thr}}/E^2),$$

where $E_{\text{thr}}$ is the threshold energy for the production of Cerenkov radiation in air, and E is the kinetic energy of the radiating particle. To simplify the problem we shall regard the quantity $A(E,p)$ as a constant, equal to 354 quanta per g/cm$^3$, and to keep the total amount of light the same we shall replace the lower limit of the integration over energies in (6) by an $E_{\text{eff}}(p)$ which we determine from the condition

$$\int_{E_{\text{thr}}}^{E_0} a(p, E)\,F(E)\,dE = \int_{E_{\text{eff}}}^{E_0} 354F(E)\,dE. \quad (7)$$

Substituting in (6) the function $\kappa$ from (1) and using the relations (2)–(5) and (7), we get

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1)Accordingly, the assumption here is that the trace of the cone of light on the earth's surface is a circle. This is in general true only for the case $\psi = 0$. If $\psi \neq 0$, the trace of the cone of light on the earth's surface will be an ellipse. For small angles $\psi$, however, the ellipticity will be small, and neglect of this fact will not lead to any important error. Therefore we shall assume throughout that the trace of the cone of light on the earth's surface is a circle whose radius is $r_{\text{Cer}}(h) = h\vartheta_{\text{Cer}} = 0.671h\rho^{1/2}$, where $\rho$ is the density of the air at altitude h, expressed in g cm$^{-3}$.

$$I(E_0, R, \varphi, \psi) = 611 \int_0^{p_0} \rho^2(p) N(E_0, p) \int_{z_{\rm eff}}^{z_0} z^2 \exp(-\Lambda z^2)$$
$$\times \int_0^{2\pi} \exp\{-z^2[\mu\cos(\varphi+\Phi) - q\cos\Phi]\} d\Phi\, dz\, dp, \quad (8)$$

where

$$z = 1+\varepsilon, \quad q = 16(0.342 + 10^3 h\rho)\psi\sqrt{\rho}, \quad \mu = 16R\rho\sqrt{\rho},$$
$$\Lambda = [(1.193\cdot 10^{-2}\rho/h) + 69.8\rho^2]$$
$$\times (R^2 + 10^6 h^2\psi^2 - 2\cdot 10^3 Rh\psi\cos\varphi)$$
$$+ 4.082(\psi^2 + 0.45\rho) + 1.193\cdot 10^{-2}(10^6 h^2\psi^2 - R^2)\rho/h. \quad (9)$$

In these formulas (8) and (9) the density $\rho$ is expressed in g/cm$^{-3}$, R in meters, h in kilometers, and $I(E_0, R, \varphi, \psi)$ as number of quanta per m$^2$ sterad.

The connection between the height h above the level of observation, the air density $\rho(h)$, and the pressure $p(h)$ depends on the temperature distribution in the atmosphere. As in [6], we have assumed the following temperature distribution:

$$T(h) = 288°\text{ K} - bh \text{ for } 0 \leqslant h \leqslant 11 \text{ km};$$
$$T = 218°\text{ K} \quad \text{for } 11 \leqslant h \leqslant 30 \text{ km}, \quad (10)$$

where b = 6.35 deg/km.

## 3. RESULTS OF THE CALCULATIONS

The results presented below were obtained by means of an electronic computer and, as has already been noted, are valid only for showers arriving from directions close to the zenith. The numerical quantities are computed for two levels of observation; for sea level and for an altitude of 3860 m above sea level. For both levels of observation the angular distributions of the intensity of Cerenkov light were calculated both for showers from primary protons and for showers from high-energy photons.

Figure 2 represents graphically a section of the angular distribution against zenith angle for the azimuthal angle $\varphi = 0$ (for $R \neq 0$ the azimuth $\varphi = 0$ is in the direction from the detector to the axis of the shower) and distances R = 0, 100, and 400 m. Figure 2 is constructed from the data for showers from primary protons with energies $E_{0p} = 4.5 \times 10^6$ BeV and $E_{0p} = 1.5 \times 10^3$ BeV. From Fig. 2 it follows, first of all, that the angle at which the maximum intensity of light is observed increases with increase of the distance along the earth's surface from the detector to the axis of the shower. In showers from protons with $E_{0p} = 4.5 \times 10^6$ BeV the maximum intensity of the light for R = 0 coincides with the direction of arrival of the primary particle ($\psi = 0$). For R = 100 m the maximum of the light intensity already makes an angle $\approx 1°$ with the direction of the primary particle, and for R = 400 m this angle is $\approx 3°$. In showers with smaller initial energy the deflection of the maximum of the light from the direction of arrival of the primary particle is somewhat smaller.

A further idea of the nature of the angular distribution of the intensity of Cerenkov light is given by Fig. 3, which is constructed for a shower from a primary photon with initial energy $4.5 \times 10^6$ BeV as observed from sea level. This figure shows the positions of the expected lines of equal intensity of the light for distances R of 0 (diagram a), 100 m (diagram b), and 400 m (diagram c). On curves 1, 2, and 3 in each diagram the intensity of the light is

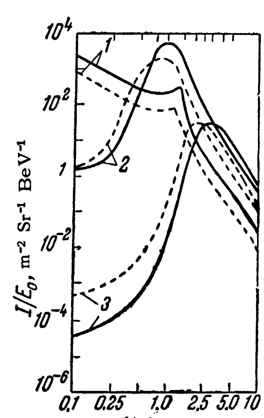


FIG. 2. Section of the angular distribution of the intensity of Cerenkov light against zenith angle $\psi$, for the value $\varphi = 0$ of the azimuthal angle (sea level). The solid curves correspond to a primary proton with $E_{0p} = 4.5 \times 10^6$ BeV, and the dashed curves to a primary proton with energy $1.5 \times 10^3$ BeV; curves 1, 2, and 3 correspond to the respective distances 0, 100, and 400 m from the axis.

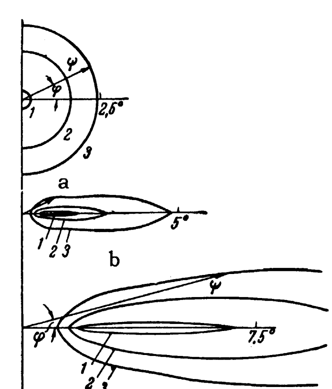


FIG. 3. Positions of lines of equal intensity on the celestial sphere in the flash from a shower produced by a primary proton with initial energy $E_{0p} = 4.5 \times 10^6$ BeV and observed at sea level. Curves a, b, c correspond to cases in which the axis of the shower passed at respective distances of 0, 100, and 400 m from the detector of the light.

$10^{-1}$, $10^{-2}$, and $10^{-3}$ of the maximum intensity of the light at that distance from the axis of the shower. The maximum value of the intensity of the light at distances 100 and 400 m can be found from Fig. 2 (for R = 0 the value is $I_{max} = 6.63 \times 10^3 \, m^{-2} sr^{-1} BeV^{-1}$). To find the total intensity per $m^2$sr one must multiply the numbers found from the diagrams by the primary energy in BeV.

Figure 3 is drawn in a polar coordinate system and reproduces the shape of the flash of light as it can be obtained by photographing showers by means of electron-optical converters. In this figure the zenith angle is represented by the length of the radius vector; the polar axis is directed along the axis of symmetry. The table presents results of numerical computations of the total amount of light contained inside the lines of equal intensity drawn in this figure. It can be seen from the table that for R equal to 100 and 400 meters almost all of the light is concentrated inside the line $I = 10^{-3} I_{max}(R)$. About half of all the light is inside the line $I = 10^{-1} I_{max}(R)$.

| | $R = 100$ m | $R = 400$ m |
|---|---|---|
| Total intensity of light* according to[6] | $4.8 \cdot 10^5$ | $5.75 \cdot 10^4$ |
| $I = 0.1 \ I_{max}(R)$ | $1.82 \cdot 10^5$ (38%) | $2.18 \cdot 10^4$ (38%) |
| $I = 0.01 \ I_{max}(R)$ | $3.9 \cdot 10^5$ (81,2%) | $5.4 \cdot 10^4$ (94%) |
| $I = 0.001 \ I_{max}(R)$ | $4.8 \cdot 10^5$ (100%) | $5.72 \cdot 10^4$ (100%) |

*All intensities here are given in quanta per $m^2$.

It is interesting to compare the results shown in Fig. 3 with the photograph of the light flash of a shower published in a paper by Hill and Porter.[3] In Fig. 4 the outline of the spot of light on the celestial sphere as constructed on the basis of the photograph given in [3] is shown as a dashed curve. The solid curve is the calculated contour line $I = 10^{-3} I_{max}$ for a shower from a primary proton with $E_0 = 4.5 \times 10^6$ BeV and the case in which the axis of the shower passed at a distance of 100 m from the detector (at sea level). It is qualitatively clear that the calculation gives the correct shape of the spot of light, but of course one cannot draw more concrete conclusions about the agreement with experiment on the basis of Fig. 4.

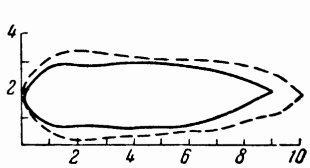

FIG. 4. Comparison between calculation and experiment. The dashed curve is experimental,[3] the solid one calculated.

It can be seen from Fig. 3 that the shape of the spot of light depends strongly on the distance of the detector from the axis of the shower. An analysis of the corresponding data for showers from protons of smaller energies shows that the curves shown in Fig. 3 depend so weakly on the primary energy[2)] that one can practically neglect the dependence. Since for a known distance of the detector from the axis of the shower the total light intensity within the spot of light is characteristic of the primary energy, from the point of view of methods an analysis of the shape of the spot of light and the total intensity can serve as a source of information both about the total energy of the shower and about the distance from the detector at which the axis of the shower passed. As will be seen from what follows, difficulties can arise only when there are large fluctuations in the position of the maxima of showers.

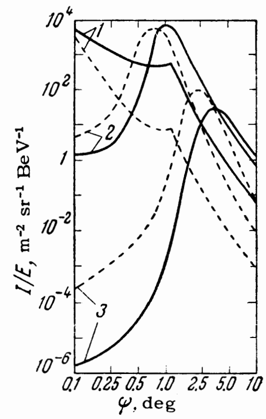

FIG. 5. Section of the angular distribution of intensity of Cerenkov light against zenith angle, for azimuthal angle $\varphi = 0$. The solid curves correspond to a primary photon with energy $E_0\gamma = 5 \times 10^6$ BeV, and the dashed curves to a primary photon with $E_0\gamma = 10^3$ BeV. Curves 1, 2, and 3 correspond to distances 0, 100, and 400 m from the axis of the shower (sea level).

Figure 5 shows sections of the angular distributions of intensity of Cerenkov light to be expected for showers from high-energy $\gamma$ rays. From Figs. 2 and 5 and from a comparison of the curves shown in Fig. 3 with the corresponding curves for showers from $\gamma$ rays we can assert that in the region of very high energies ($E_0 \approx 10^6$ BeV) the shape of the spot of light is almost independent of the nature of the primary particle, and consequently in this energy region the shape of the spot of light cannot be used to distinguish between showers from primary protons and from

2)For R = 100 m the correctness of this assertion follows from an analysis of Fig. 6 (see below).

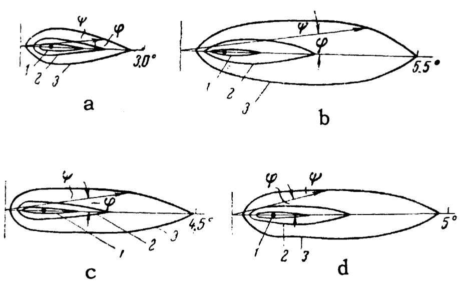


FIG. 6. Contours of equal intensity in light flashes from showers from primary protons and primary photons of various energies, for sea level and R = 100 m from the axis. The curves 1, 2, 3 correspond to intensity values $10^{-1}\, I_{max}(100)$, $10^{-2}\, I_{max}(100)$, and $10^{-3}\, I_{max}(100)$. Diagrams a and b correspond to primary photons of energies $10^3$ and $5 \times 10^6$ BeV, and diagrams c and d to primary protons of energies $1.5 \times 10^3$ and $4.5 \times 10^6$ BeV.

primary photons. For lower energies of the primary particles ($E_0 \approx 10^{12}$ eV) the situation is somewhat better (Fig. 6). Here the shape of the line $I = 10^{-3} I_{max}$ in showers from photons differs appreciably from that of the corresponding line in showers from protons. This difference, however, is entirely due to the difference in the shapes of the cascade curves. If we allow for the fact that owing to fluctuations the cascade curves for proton showers can differ decidedly from the average curve, the difference in the shape of the light spots which we have mentioned can also be insufficient for a reliable distinction between showers produced by photons and those produced by protons. Figures 7 and 8, which are analogous to Figs. 2 and 3, give an idea of the angular distribution of the light in showers from primary protons when the observation is at altitude 3860 meters above sea level. A comparison of Figs. 3 and 8 shows that on mountains the spot of light from a shower from a proton

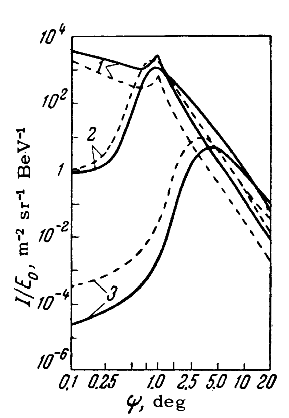


FIG. 7. Section of angular distribution of the intensity of Cerenkov light against zenith angle for azimuthal angle $\varphi = 0$, for altitude 3860 m above sea level. Curves 1, 2, and 3 are for the respective distances 0, 100, and 400 m from the axis of the shower. The solid curves correspond to a primary proton with energy $4.5 \times 10^6$ BeV, and the dashed curves to one with energy $1.5 \times 10^3$ BeV.

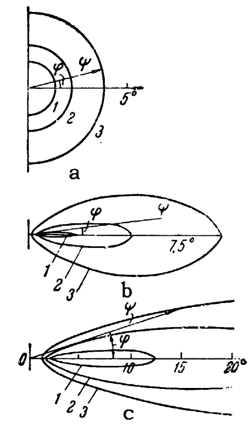


FIG. 8. Contours of equal intensity in the light flash at various distances from the axis of a shower arising from a primary proton with $E_{0p} = 4.5 \times 10^6$ BeV (3860 m above sea level). Curves 1, 2, and 3 correspond to the intensities $10^{-1}\, I_{max}(R)$, $10^{-2}\, I_{max}(R)$, and $10^{-3}\, I_{max}(R)$, and diagrams a, b, and c are for distances 0, 100, and 400 m from the axis of the shower.

with $E = 4.5 \times 10^{15}$ eV is considerably larger than that from such a shower at sea level. This difference is mainly due to the different distance of the registering device from the maximum of the shower. Thus the shape of the spot of light is sensitive to the height of the maximum of the shower, and at least in principle an analysis of the shape can be used also to determine the position of the maximum of a shower.

The present calculations have been made on the same assumptions as the calculations of the spatial distribution of the light made in [6], and therefore they can be checked directly by calculating the total luminous flux density

$$Q(R, E_0) = \int_0^{2\pi} \int_0^{10^\circ} I(E_0, R, \psi, \varphi) \sin\psi \, d\psi \, d\varphi \qquad (11)$$

at a given distance from the axis of the shower and comparing it with the results obtained in [6]. Calculations by Eq. (11) have been made for seal level for R = 100 m and R = 400 m. The results agreed with the results of [6] to an accuracy of several percent.

## CONCLUSION

The calculations that have been made enable us to draw the following conclusions:

1. Since the maximum intensity of the light from a shower does not coincide with the direction of arrival of the primary particle, in researches in which the determination of the angular coordinates of the primary particle is made by photographing the light flash from the shower one should seek improved accuracy in this determination by photo-

graphing the shower simultaneously from several positions.

2. If the distance from the axis of the shower to the detector is determined from independent data, then an analysis of the shape of the light flash from the shower and its total intensity gives information both about the initial energy of the primary particle and about the position in the atmosphere of the maximum of the shower, and can thus be used for the analysis of fluctuations in the development of showers in the atmosphere.

In conclusion I regard it as my pleasant duty to express my gratitude to A. E. Chudakov for suggesting this topic and for helpful discussions.

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