# *ANGULAR DISTRIBUTION OF THE POLARIZATION OF ELASTICALLY SCATTERED FAST DEUTERONS*

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Elastic scattering of deuterons by complex nuclei is considered. Formulas for the vector and tensor polarizations are derived by employing the approximation of high energies and small angles, and by assuming that the spin-orbit part of the scattering phase shift is small. The results are compared with available experimental data, and the parameters characterizing the optical potential of the deuteron are determined.

LEVINTOV [1] proposed for the analysis of the polarization of high-energy nucleons a method based on the fact that the polarization is the ratio of quadratic functions of scattering-matrix parameters, so that to obtain the angular and energy dependence of the polarization it is not necessary to carry through the scattering-matrix parameter calculation to conclusion, it being sufficient to separate in them a common radial integral which cancels out in the final expression for the polarization. In the present paper we employ this method to calculate the polarization of deuterons.

If S = 1 the scattering matrix can be expanded in terms of the set of basis functions as follows [2]:

$$M = a(\theta) + b(\theta)N_iS_i + [c(\theta)(N_iN_j - {}^1\!/_3\delta_{ij}) + d(\theta)(D_iD_j - E_iE_j)]S_{ij},$$
$$S_{ij} = {}^1\!/_2(S_iS_j + S_jS_i - {}^2\!/_3\delta_{ij}), \tag{1}$$

where **N**, **D**, **E**—unit vectors in the directions of $\mathbf{k}_i \times \mathbf{k}_f$, $\mathbf{k}_f + \mathbf{k}_i$, and $\mathbf{k}_f - \mathbf{k}_i$, respectively. The scalar coefficients a, b, c, and d describe the scattering completely. In particular, all the average values of the irreducible tensor operators $T_{lM}$ characterizing the deuteron polarization [3] ($\langle iT_{11}\rangle$, $\langle T_{22}\rangle$, $\langle T_{20}\rangle$, $\langle T_{21}\rangle$), are expressed in terms of these parameters. With the aid of projection operators [4] we can express the parameters of the scattering matrix in terms of the scattering phase shifts $\Delta^{+,-,0}$ respectively for $j = l + 1$, $l - 1$, and $l$.

In the formulas obtained we replace summation over $l$ by integration with respect to the impact parameter $\rho$, and use for the Legendre polynomials Fock's asymptotic expression [5] ($0 < \theta < \pi/2$)

$$P_l(\cos\theta) \approx (\theta/\sin\theta)^{1/2}J_0(k\rho\theta),$$
$$P_l'(\cos\theta) \approx (\theta/\sin\theta)^{1/2}k\rho J_1(k\rho\theta), \tag{2}$$

where k—wave number and $J_0(k\rho\theta)$ and $J_1(k\rho\theta)$—Bessel functions of zero and first order. We relate the scattering phase shifts with the potential by the well known quasi-classical expression for high energies

$$\Delta^{+,-,0} = -\frac{k}{2E}\int_\rho^\infty \frac{U^{+,-,0}(r)}{(r^2-\rho^2)^{1/2}}\,r\,dr. \tag{3}$$

The optical potential is chosen in the form

$$U^{+,-,0}(r) = -\left[V_c e^{i\alpha}f(r) + \frac{1}{2}V_s\left(\frac{\hbar}{\mu c}\right)^2\frac{df(r)}{dr}(\mathbf{lS})\right];$$
$$\tan\alpha = \frac{\operatorname{Im}V(r)}{\operatorname{Re}V(r)},\quad \frac{\hbar}{\mu c} = 1.4\cdot10^{-13}\,\text{cm},$$
$$f(r) = \left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}. \tag{4}$$

The spin-orbit part of the potential is chosen pure real, and the tensor potentials are disregarded [6].

Using (3) and (4), we can separate the scattering phase shifts into central and spin-orbit parts ($\Delta_c$ and $\Delta_{lS}$). We assume that $\Delta_{lS} \ll 1$. We then obtain for the parameters a and b of the scattering matrix (in analogy with [1])

$$a(\theta) = iI, \tag{5}$$
$$b(\theta) = k^2A\theta e^{i\alpha}I, \tag{6}$$

where

$$I \equiv \left[k\left(\frac{\theta}{\sin\theta}\right)^{1/2}\int_0^\infty(1-e^{2i\Delta_c})\rho J_0(k\rho\theta)\,d\rho\right],$$
$$A \equiv \frac{V_s}{V_c}\left(\frac{\hbar}{\mu c}\right)^2.$$

The coefficients $c(\theta)$ and $d(\theta)$ depend on an integral of the type

$$\int_0^\infty F(\rho)\frac{d\Delta_c}{d\rho}\frac{1}{\rho}\,d\rho.$$

Recognizing that at high energies elastic scattering occurs essentially on the "edge" of the nucleus [7], i.e., the impact parameter varies within a narrow range $\rho \approx R$, $\Delta\rho/\rho \ll 1$ (the range of variation of $\rho$ is made narrower also because the only important impact parameters are those for which $df/dr \neq 0$), we can express this integral in terms of the integral that enters in the coefficient $b(\theta)$:

$$\int_0^\infty F(\rho)\frac{d\Delta_c}{d\rho}\frac{1}{\rho}\,d\rho \approx \frac{1}{R^2}\int_0^\infty F(\rho)\frac{d\Delta_c}{d\rho}\,\rho d\rho.$$

Now, using the recurrence relations for the Bessel functions, we obtain the following approximate expressions for $c(\theta)$ and $d(\theta)$:

$$c(\theta) = i(k\theta)^2 A e^{-i\alpha} I, \tag{7}$$

$$d(\theta) = iR^{-2} A e^{-i\alpha} I. \tag{8}$$

Substituting (5)—(8) in Stapp's formulas [2] for the polarization components, we obtain (using the usual approximations of high energies and small angles)

$$\langle iT_{11}\rangle = \frac{2}{\sqrt{3}}\frac{k^2 A\theta \sin\alpha}{[1 + {}^2/_3 (k^2A)^2\theta^2]}, \tag{9}$$

$$\langle T_{22}\rangle = -\frac{1}{2\sqrt{3}}\frac{k^2 A(\cos\alpha + k^2 A)\,\theta^2}{[1 + {}^2/_3 (k^2A)^2\theta^2]}, \tag{10}$$

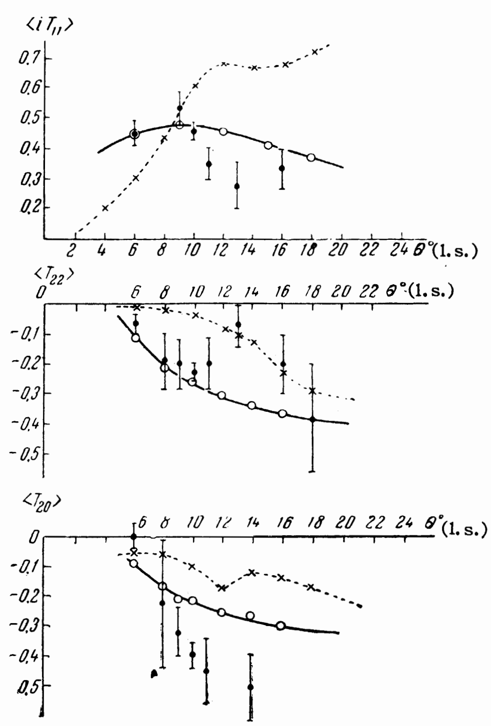


FIG. 1. Polarization of deuterons scattered by carbon, $E_d$ = MeV. Experimental data – [8], dashed curve – calculations of Zamick[10], solid line – calculation by formulas (9), (10), and (12).

$$\langle T_{21}\rangle = \frac{2}{\sqrt{3}}\frac{A}{R^2}\frac{[k^2 A + \cos\alpha]\,\theta}{[1 + {}^2/_3 (k^2A)^2\theta^2]}, \tag{11}$$

$$\langle T_{20}\rangle = \sqrt{{}^2/_3}\langle T_{22}\rangle. \tag{12}$$

Experimental data for $\langle iT_{11}\rangle$, $\langle T_{22}\rangle$, $\langle T_{20}\rangle$, $\langle T_{21}\rangle$ are available for $C^{12}$ at $E_d$ = 420 MeV and for $Be^9$ at $E_d$ = 410 MeV.[8] In addition, values of $\langle iT_{11}\rangle$ were measured at $E_d$ = 150, 120 and 90 MeV [9]. The comparison with experiment is made in the following manner: the parameters A and $\alpha$ are chosen to obtain the best agreement for $\langle iT_{11}\rangle$ (data of [8]). The result was (see Figs. 1 and 2)

$C^{12}$: $\langle iT_{11}\rangle_{max} = 0.48$, $\theta_{max} = 9°$, $\mathrm{Im}\,V(r) \approx \mathrm{Re}\,V(r)$, $V_s / V_c = 0.13$,

$Be^9$: $\langle iT_{11}\rangle_{max} = 0.50$, $\theta_{max} = 8°$, $\mathrm{Im}\,V(r) \approx \mathrm{Re}\,V(r)$, $V_s / V_c = 0.14$,

$(\langle iT_{11}\rangle_{max} = (\sin\alpha)/\sqrt{2}, \quad \theta_{max} = \sqrt{3/2}/k^2 A)$.

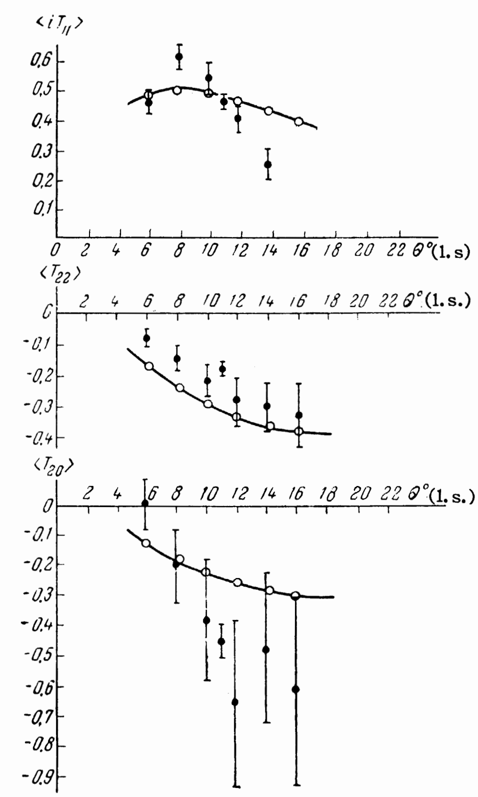


FIG. 2. Depolarization of deuterons scattered from beryllium, $E_d$ = 410 MeV. Experimental data – from[8], solid line – calculations by formulas (9), (10), and (12).

The parameters A and $\alpha$ were not further varied and the specified parameters were used to calculate $\langle T_{22}\rangle$, $\langle T_{20}\rangle$, $\langle T_{21}\rangle$ by formulas (10)—(12). The correct sign of the polarization was obtained in all cases. For $\langle T_{22}\rangle$ and $\langle T_{20}\rangle$ the agreement is satisfactory, $\langle T_{21}\rangle$ is found to be practically equal to zero, whereas in the experiment $\langle T_{21}\rangle \approx 10$ per cent.

The variation of $\theta_{max}$ with the deuteron energy was checked. A shift of the maximum of $\langle iT_{11} \rangle$ was actually observed as the energy was decreased from 400 to 90 MeV in accordance with the formula $\theta_{max} \sim 1/k^2A$, where $A \approx$ const.

We note that $\langle T_{21} \rangle \approx 0$ also in the impulse approximation [8] and in calculations with nucleon-nucleon amplitudes [10]; in addition, in the latter paper, as in our case, $\langle T_{22} \rangle = \sqrt{3/2} \langle T_{20} \rangle$.

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91