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A POSSIBLE METHOD OF DETERMINING THE MAGNETIC MOMENT OF THE Σ^+ HYPERON

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A method, in which no strong magnetic field need be used, is proposed for determining the magnetic moment of the Σ^+ hyperon. The method is based on the phenomenon of depolarization of positively charged particles in condensed media. It is shown that the Σ^+-hyperon magnetic moment can in principle be determined by analyzing the experimental data on asymmetry in the decay $\Sigma^+ \rightarrow p + \pi^0$ in flight and after stopping.

1. At the present time understandable interest attaches to measurements of the magnetic moment of the Σ^+ hyperon. However, the use of standard methods.for this purpose, in which spin precession in an external magnetic field is used, entails great difficulties, for the fields necessary to observe the precession have an intensity which lies at the limit of modern technical capabilities, owing to the small lifetime of the Σ^+ hyperon and the low value of the magnetic moment. In this connection we wish to call attention to another approach to this problem, which makes it possible to get along without an external magnetic field or to confine oneself to magnetic fields of the order of several thousand Gauss.

The main idea of this approach is to replace the external magnetic field by the internal atomic field. We are referring to the well known phenomenon of depolarization of positively charged particles in condensed media. In agreement with theory and experiment, the majority of slowed-down particles form hydrogen-like systems in the ground state. If we disregard many complicating factors for the time being, the depolarization is produced by the magnetic field of the electron which acts on the magnetic moment of the given particle. The degree of this action can be calculated theoretically.

2. The depolarization process was considered many times as applied to the muonium atom (see, for example [1-5]). So long as we have in mind polarization which is not averaged over the time, the entire reasoning can be repeated also for the Σ^+ hyperon (see also [5]). The transitions between the singlet and triplet states of the "sigmionium" atom, with zero spin projection on the initial direction of polarization, lead to time-dependent oscillations of the polarization in accordance with the law

$$\mathbf{P} = \mathbf{P}_0 \tfrac{1}{2} \ (1 + \cos \omega_0 t), \tag{1}$$

where ω_0 —magnitude of the hyperfine splitting, proportional to the magnetic moment of the Σ^+ hyperon ($\omega_0 = {}^{32}\!/_3\, \mu_\Sigma \mu_e / a_0^3$, where a_0 is the Bohr radius). In the case of the μ^+ meson, when the probability of decay per unit time is many times smaller than the frequency of the hyperfine splitting, we obtain after averaging over the time

$$\langle \mathbf{P} \rangle = \tfrac{1}{2} \langle \mathbf{P}_0 \rangle.$$

For the Σ^+ hyperon, the lifetime τ can turn out to be comparable with ω_0^{-1}, and the average polarization is described by the formula

$$\langle \mathbf{P} \rangle = \mathbf{P}_0 \frac{1}{2} \int (1 + \cos \omega_0 t) \frac{1}{\tau} e^{-t/\tau} dt$$
$$= \frac{1}{2} \mathbf{P}_0 \left(1 + \frac{1}{1 + \omega_0^2 \tau^2} \right), \tag{2}$$

The physical meaning of relation (2) is that half of the Σ^+ hyperons, which capture electrons with spin directions parallel to the direction of the initial polarization, retain all the polarization, while the other half of the Σ^+ hyperons, which capture electrons with opposite spin direction, do not have time to become completely depolarized, unlike the μ^+ mesons. Thus, $\langle \mathbf{P} \rangle > \mathbf{P}_0/2$.

It is of importance to us that relation (2) depends in explicit fashion on the hyperfine splitting of the "sigmionium," and consequently also on the magnetic moment of the Σ^+ hyperon. It is possible to measure in the experiment a quantity proportional to $|\langle \mathbf{P} \rangle|$, namely the asymmetry of proton emission in the $\Sigma^+ \rightarrow p + \pi^0$ decay, relative to the polarization direction. Knowing the ratio of the asymmetry coefficients in the decays after stopping and in flight, we can determine in principle the magnetic moment of the Σ^+ hyperon. In the approximation which we are using for the time being, this ratio takes the form

$$\beta = \frac{1}{2}\left(1 + \frac{1}{1+\omega_0^2\tau^2}\right). \quad (3)$$

It is clear from (3) that β does not depend on the absolute value of the initial polarization of the Σ^+ hyperon.

3. In the region $(\omega_0\tau)^2 \ll 1$ (small magnetic moments), formula (3) is insensitive to the value of $\omega_0\tau$. It is not excluded, however, that the magnetic moment of the Σ^+ hyperon is equal in order of magnitude to the magnetic moment of the proton or even exceeds it[1]. In this case $\omega_0\tau \sim 1$. In particular, if $\omega_0\tau = 1$ then $\beta = 3/4$, i.e., the deviation of β from unity and from $1/2$ becomes noticeable.

If $\omega_0\tau$ is large, the value of β becomes practically indistinguishable from $1/2$. In this case, to determine ω_0 it is advantageous to apply a weak longitudinal magnetic field. Then, β increases and its dependence on ω_0 is expressed by the formula

$$\beta' = \frac{1}{2(1+x^2)}\left(1 + 2x^2 + \frac{1}{1+(1+x^2)\,\omega_0^2\tau^2}\right), \quad (4)$$

$$x = eH/m_e c\omega_0 = \omega'/\omega_0, \qquad \omega' = eH/m_e c. \quad (4a)$$

When $\omega_0\tau \gg 1$ formula (4) goes over into the well known formula of Orear et al.[2], which is widely used in the analysis of μ^+-meson depolarization.

4. So far we have regarded the "sigmionium" atom as isolated, neglecting its interaction with the surrounding medium. Yet under real conditions this interaction may be appreciable, and formulas (3) and (4) become unsuitable for the analysis of the experimental results. Various complications that modify the depolarization scheme adhered to above, were investigated in detail for the case of muonium by many authors, particularly by Nosov and Yakovleva[3]. These complications include: the depolarization of the muonium electron by exchange with free electrons of the medium and with electrons of the atomic shells, interaction with the crystal fields, charge exchange, formation of a negative muonium ion with subsequent loss of an electron, etc. In addition, the depolarization of the μ^+ meson may be greatly influenced by the chemical interaction between the muonium and the atoms of the medium. The same factors are significant also in principle for Σ^+-hyperon depolarization. However, owing to the short Σ^+ lifetime, we can expect that the related depolarization does not have time to take place in many substances.

In order for the pure "sigmionium" depolarization mechanism to take place it is sufficient that the frequency ν of the electron spin flip, and the time τ_{chem} of the chemical relaxation, satisfy the inequalities

$$\nu \ll \omega_0, \qquad \nu \ll 1/\tau, \qquad \tau_{chem} \gg \tau \quad (5)$$

or[2] $\nu \ll 10^{10}$ sec^{-1} and $\tau_{chem} \gg 10^{-10}$ sec.

A detailed analysis[7] shows that inequalities (5) are apparently satisfied for emulsions, liquid argon, and krypton at "sigmionium" velocities on the order of 10^5—10^6 cm/sec. Judging from the aggregate of the data, they are satisfied also for many other dielectrics with diamagnetic properties[3], provided the latter either do not interact chemically with the hydrogen at all or interact not very actively. Estimates show that for "sigmionium" that slows down to velocities on the order of 10^6—10^5 cm/sec the value of ν does not exceed 10^6—10^8 sec^{-1}.

Yet the short lifetime of the Σ^+ hyperon can lead to complications of a different kind, which do not exist in the case of the μ^+ meson. The point is that at velocities on the order of 10^8 cm/sec the Σ^+ hyperon no longer forms a visible track, i.e., is treated as having stopped. At the same time, inequalities (5) can be satisfied only at velocities v_{Σ^+} for which the probability of ionization, exchange with excitation, and other inelastic processes that contribute to ν is sufficiently small ($v_{\Sigma^+} \sim 10^5$—10^6 cm/sec). As shown by simple es-

[1]The main contribution to the magnetic moment of the Σ^+ hyperon is apparently made by the virtual dissociation of the type $\Sigma^+ \to \Sigma^0 + \pi^\pm$. An analogous dissociation ($p \to n + \pi^+$) takes place also for the proton. There are therefore no grounds for assuming beforehand that the magnetic moment of the Σ^+ is appreciably smaller than that of the proton. In this connection it must be noted that dissociation with emission of a charged pion in a state with L = 1 is impossible in the case of the Λ^0 particle. This may be the reason for the low value obtained in recent experiments for the magnetic moment of the Λ^0[6].

[2]The limitations imposed by the inequalities (5) are more stringent for muonium: $\nu \ll 10^6$ sec^{-1} if $\tau_{chem} > \tau_{\mu+}$ or $\nu\tau_{chem} \ll 1$ if $\tau_{chem} < \tau_{\mu+}$. In the case of AgBr, $\nu\tau_{chem} \sim 10^2$[3], i.e., inequalities (5) are certainly not satisfied.

[3]In this case exchange of electrons with spin slip is practically impossible in elastic collision, since the concentration of the free electrons is negligible, and the spins of the atomic electrons are saturated. In metals and paramagnets, owing to the intensive exchange $\nu \gg 10^{10}$ sec^{-1} the bond between the electron and the Σ^+ hyperon is broken, and the Σ^+ hyperon is not depolarized at all. Therefore metallic and paramagnetic media are utterly unsuitable for our purpose.

timates, the time necessary for the Σ^+ hyperons to slow down in emulsions, liquid argon, or krypton from $v_{\Sigma^+} \sim 10^8$ cm/sec to $v_{\Sigma^+} \sim 10^6-10^5$ cm/sec amounts to approximately $10^{-11}-10^{-10}$ sec, i.e., it is comparable with τ. This causes a considerable fraction of the Σ^+ hyperons to be able to decay either in the free state, or under conditions in which the pure "sigmionium" mechanism of depolarization does not take place.

We note, however, that owing to the very abrupt (nearly exponential) decrease in the cross sections of the inelastic processes at velocities $v_{\Sigma^+} \ll 10^8$ cm/sec, the range of velocities where $\nu \sim \omega_0$ is rather narrow. The Σ^+ hyperon covers this velocity region within a time that is small compared with its lifetime (estimates yield t $\approx 10^{-12}$ sec^{-1}). At large velocities $\nu \gg 10^{10}$ sec^{-1}, and the Σ^+ hyperon decays practically unpolarized. At lower velocities, $\nu \ll 10^{10}$ sec^{-1}, the inequalities (5) are satisfied and since practically all the Σ^+ hyperons form in this case "sigmionium" atoms in the ground state [7], we can use formulas (3) and (4).

Starting from this, we can introduce an additional parameter f, which determines the fraction of the Σ^+ hyperons that have decayed prior to the start of the depolarization. Although the introduction of this parameter leads to a certain complication in the calculation formulas, its determination for its own sake is of physical interest. Taking the foregoing into account, we rewrite relation (3) in the form

$$\beta = f + \frac{1-f}{2}\left(1 + \frac{1}{1+\omega_0^2\tau^2}\right). \quad (6)$$

The two parameters f and $\omega_0\tau$ can in principle be determined by making use of the results of experiments with the magnetic field. In this connection, it is meaningful to consider the problem of the depolarization of Σ^+ in a "sigmionium" atom placed in a magnetic field of arbitrary direction.

5. The system of equations for the polarization parameters of the "sigmionium" takes the form (compare with [3])

$$\frac{dP_i^{(1)}}{dt} = -\frac{1}{2}\omega_0\varepsilon_{ist}T_{st}^{(1,2)} - \frac{e}{m_\Sigma c}\varepsilon_{ist}H_sP_t^{(1)}, \quad (7a)$$

$$\frac{dP_i^{(2)}}{dt} = \frac{1}{2}\omega_0\varepsilon_{ist}T_{st}^{(1,2)} + \frac{e}{m_e c}\varepsilon_{ist}H_sP_t^{(2)}, \quad (7b)$$

$$\frac{dT_{st}^{(1,2)}}{dt} = \frac{1}{2}\omega_0(\varepsilon_{stl}P_l^{(1)} - \varepsilon_{stl}P_l^{(2)}) - \frac{e}{m_\Sigma c}\varepsilon_{smn}H_mT_{nt}^{(1,2)} + \frac{e}{m_e c}\varepsilon_{tmn}H_mT_{sn}^{(1,2)} \quad (i, s, t, m, n = 1, 2, 3). \quad (7c)$$

Here $P^{(1)}$ —Σ^+-hyperon polarization vector, $P^{(2)}$ —electron polarization vector, T_{st} —electron and Σ^+-hyperon polarization correlation tensor, and ϵ_{ist} —absolutely antisymmetrical tensor.

We shall henceforth assume that the quantity x, introduced in (4), satisfies the inequality $xm_e/m_\Sigma \ll 1$ (the direct interaction between the Σ^+ hyperon and the external magnetic field is small compared with the interaction of the spins of the Σ^+ and electron). In this approximation, the terms $\epsilon_{ist}H_sP_t^{(1)}\times e/m_\Sigma c$ in (7a) and $\epsilon_{smn}H_mT_{nt}^{(1,2)}e/m_\Sigma c$ in (7c) can be discarded. We solve the approximate system of linear equations obtained in this fashion under the initial conditions

$$\mathbf{P}^{(1)}(0) = \mathbf{P}_0, \quad \mathbf{P}^{(2)}(0) = 0, \quad T_{st}^{(1,2)}(0) = 0. \quad (8)$$

As a result, we obtain the following expression for the time dependence of the vector of the Σ^+ polarization:

$$\begin{aligned}\mathbf{P}(t) &= \mathbf{n}(\mathbf{P}_0\mathbf{n})\frac{1}{2(1+x^2)}\{1 + 2x^2 + \cos\omega^{(1)}t\} \\ &+ \mathbf{m}(\mathbf{P}_0\mathbf{m})\left\{\frac{1}{4}\left(1 + \frac{x}{\sqrt{1+x^2}}\right)(\cos\omega^{(2)}t + \cos\omega^{(3)}t) + \frac{1}{4}\left(1 - \frac{x}{\sqrt{1+x^2}}\right)(\cos\omega^{(4)}t + \cos\omega^{(5)}t)\right\} \\ &+ \mathbf{l}(\mathbf{P}_0\mathbf{m})\left\{\frac{1}{4}\left(1 + \frac{x}{\sqrt{1+x^2}}\right)(\sin\omega^{(2)}t + \sin\omega^{(3)}t) + \frac{1}{4}\left(1 - \frac{x}{\sqrt{1+x^2}}\right)(\sin\omega^{(4)}t + \sin\omega^{(5)}t)\right\}. \quad (9)\end{aligned}$$

Here n —unit vector along the magnetic field, m —unit vector perpendicular to the direction of the magnetic field in the $(\mathbf{P}_0\mathbf{H})$ plane, $\mathbf{l} = \mathbf{m}\times\mathbf{n}$,

$$\omega^{(1)} = \omega_0\sqrt{1+x^2}, \quad \omega_{2,3} = \frac{\omega_0}{2}(x \pm 1 - \sqrt{1+x^2}),$$

$$\omega_{4,5} = \frac{\omega_0}{2}(x \pm 1 + \sqrt{1+x^2}).$$

After averaging over the time τ we obtain

$$\langle\mathbf{P}^{(1)}\mathbf{n}\rangle = (\mathbf{P}_0\mathbf{n})\frac{1}{2(1+x^2)}\left(1 + 2x^2 + \frac{1}{1+(1+x^2)\omega_0^2\tau^2}\right), \quad (10)$$

$$\langle\mathbf{P}^{(1)}\mathbf{m}\rangle = (\mathbf{P}_0\mathbf{m})\frac{1}{4}\left[\left(1 + \frac{x}{\sqrt{1+x^2}}\right)\times\left(\frac{1}{1+(\omega^{(2)}\tau)^2} + \frac{1}{1+(\omega^{(3)}\tau)^2}\right) + \left(1 - \frac{x}{\sqrt{1+x^2}}\right)\left(\frac{1}{1+(\omega^{(4)}\tau)^2} + \frac{1}{1+(\omega^{(5)}\tau)^2}\right)\right], \quad (11)$$

$$\langle\mathbf{P}^{(1)}\mathbf{l}\rangle = (\mathbf{P}_0\mathbf{m})\frac{1}{4}\left[\left(1 + \frac{x}{\sqrt{1+x^2}}\right)\times\left(\frac{\omega^{(2)}\tau}{1+(\omega^{(2)}\tau)^2} + \frac{\omega^{(3)}\tau}{1+(\omega^{(3)}\tau)^2}\right) + \left(1 - \frac{x}{\sqrt{1+x^2}}\right)\times\left(\frac{\omega^{(4)}\tau}{1+(\omega^{(4)}\tau)^2} + \frac{\omega^{(5)}\tau}{1+(\omega^{(5)}\tau)^2}\right)\right]. \quad (12)$$

We note that the coefficient of $\mathbf{P}_0 \cdot \mathbf{n}$ in (10) coincides with the right side of (4).

Formulas (11) and (12) are too cumbersome. However, they become much simpler if $x \gg 1$. Neglecting terms of order x^{-2}, but assuming as before that[4)] $x \ll m_\Sigma/m_e$, we obtain

$$\langle \mathbf{P}^{(1)}\mathbf{m}\rangle = 4\mathbf{P}_0\mathbf{m} / (4 + \omega_0^2\tau^2), \ \langle \mathbf{P}^{(1)}\mathbf{l}\rangle 0. \quad (13)$$

We see that (13) does not contain the magnetic field at all. This is connected with the fact that in the approximation under consideration the stationary states of the electrons are states with definite projection of the spin on the $\mathbf{H}$ direction; on the other hand, we neglect the direct action of the external magnetic field on the Σ^+-hyperon spins, assuming it to be small compared with the electron magnetic field. We note that inasmuch as half of the electrons have a spin projection $\hbar/2$ on the $\mathbf{H}$ direction, and half of them have a spin projection $-\hbar/2$ in the same direction, we have in this approximation symmetry with respect to the direction of the external magnetic field, and this leads to the equality $\mathbf{P}(t)\cdot\mathbf{m} = 0$[5)], which naturally remains valid also after averaging over the lifetime.

6. We now consider the possible experimental procedure for the determination of the magnetic moment of the Σ^+ hyperon. Assume that a beam of Σ^+ hyperons with a certain polarization $\mathbf{P}_0$ averaged over the emission angle[6)], is incident on the analyzer. The angular distribution of the decay protons is of the form

$$dW = (1 + \alpha\mathbf{D}_0\mathbf{k})\, d\Omega, \quad (14)$$

where $\mathbf{k}$—unit vector in the direction of proton emission.

We now choose the z axis along some direction $\mathbf{i}$ and integrate (14) over the azimuth angle. As a result we obtain

$$dW_\mathbf{i} = (1 + \alpha\mathbf{P}_0\mathbf{i}\cos\theta)\, d\cos\theta, \quad (15)$$

where $\cos\theta = \cos(\mathbf{k}, \mathbf{i})$. Thus, the asymmetry coefficient γ_i relative to some direction $\mathbf{i}$ is proportional to the projection of the polarization vector on this direction

$$\gamma_\mathbf{i} = \alpha\mathbf{P}_0\mathbf{i}. \quad (16)$$

We first consider the decays of the Σ^+ hyperons in the absence of an external magnetic field. In this case, regardless of the choice of $\mathbf{i}$, the ratio of the asymmetry coefficients in the decay after stopping and in flight is determined from (6). We then turn on the magnetic field $\mathbf{H}$ and determine the asymmetry coefficient of the decay in flight ($\gamma_\mathbf{H}^{(0)}$) and after stopping ($\gamma_\mathbf{H}^{(1)}$) relative to the $\mathbf{H}$ direction. It is easy to see that

$$\beta_\mathbf{H} = \frac{\gamma_\mathbf{H}^{(1)}}{\gamma_\mathbf{H}^{(0)}} = f + \frac{1-f}{2}\frac{1}{1+x^2} \times\left(1 + 2x^2 + \frac{1}{1 + (1+x^2)\,\omega_0^2\tau^2}\right). \quad (17)$$

Even this series of measurements is in principle sufficient to determine f and $\omega_0\tau$. The external magnetic field can in this case have an intensity on the order of several hundred Gauss, corresponding to $x \sim 1$.

We can proceed also in a different fashion, and measure the asymmetry coefficients $\gamma_\mathrm{t}^{\prime(1)}$ and $\gamma_\mathrm{t}^{\prime(0)}$ relative to a certain direction $\mathbf{t}$ perpendicular to $\mathbf{H}$, at an absolute value $H \sim 10^4$ G. Taking (13) into account, the ratio β_t will in this case have the form

$$\beta_\mathbf{t} = \frac{\gamma_\mathbf{t}^{\prime(1)}}{\gamma_\mathbf{t}^{\prime(0)}} = f + (1-f)\frac{4}{4 + (\omega_0\tau)^2}. \quad (18)$$

From (18) and (17) we can also determine f and $\omega_0\tau$. If it turns out that $\omega_0\tau \sim 2$–3, it is most advantageous to combine the measurements that lead to (18) and (17).

In conclusion we note that in principle it is possible to measure in analogous fashion also the magnetic moments of light hyperfragments.

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326

4)The intensity of the external magnetic field should in this case be of the order of 10^4 G.

5)For the projection on the vector $\mathbf{m}$ we will have

$$\mathbf{P}(t)\mathbf{m} = \mathbf{P}_0\mathbf{m}\cos(\omega_0 t/2).$$

6)Of course, the geometry of the experiment must be such as not to cause $\mathbf{P}_0$ to vanish.