

PION RESONANCES ACCOMPANYING STRANGE PARTICLE PRODUCTION IN

7.5-GeV π^- -p INTERACTIONS

V. A. BELYAKOV, V. I. VEKSLER, N. M. VIRYASOV, E. N. KLADNITSKAYA, G. I. KOPYLOV, A. MIHUL,¹⁾ V. N. PENEV, E. S. SOKOLOVA, and M. I. SOLOV'EV

Joint Institute for Nuclear Research

Submitted to JETP editor December 30, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 1967-1978 (June, 1964)

The simultaneous production of pion resonances and strange particles was investigated. The simultaneous production of ρ^0 mesons and Λ -K pairs was observed in events characterized by charged particle multiplicity $n_S = 4$ and having cross sections $\sigma_{\rho^0} = 20 \pm 8 \mu\text{b}$. Cross sections for the production of ω and η resonances are presented. The 1340-MeV peak in the distribution of four-pion effective masses is discussed.

INTRODUCTION

THE present work, performed with a 24-liter propane bubble chamber,^[1] has continued the study of strange-particle^[2-5] and pion production in a 7.5-GeV/c π^- beam. We studied the pion resonances produced in the interactions

$$\pi^- + p \rightarrow \begin{cases} \Lambda(\Sigma^0) + K^0 + m\pi \\ \Lambda(\Sigma^0) + K^+ + m\pi \\ K^0 + \tilde{K}^0 + p(n) + m\pi \\ K^0 + K^- + p(n) + m\pi \\ \tilde{K}^0 + K^+ + p(n) + m\pi \\ K^0 + \Sigma^+ + m\pi \\ K^0 + \Sigma^- + m\pi \end{cases},$$

where m is the number of pions. The following criteria governed the selection of reactions with protons:

- 1) No proton evaporation tracks are observed in a star including an Λ hyperon or K meson (Λ -K or K - \tilde{K} pair).
- 2) The combined charge of all secondary particles is zero.
- 3) No more than one baryon is identified in an event.
- 4) The target mass^[6] is less than or equal to the nucleonic mass.
- 5) The missing mass^[7] $\omega = (\Delta E^2 - |\Delta p|^2)^{1/2}$, where

$$\Delta E = E_0 - \sum_{i=1}^n E_i, \quad \Delta p = p_0 - \sum_{i=1}^n p_i$$

are the energy and momentum differences between the primary and secondary particles, must be real.

Because of the primary-particle momentum spread the criterion (5) applies only to events in which

$$\Delta E > 0,$$

$$|p_0| - \left| \sum_{i=1}^n p_i \right| > 0.$$

In cases where the given conditions were not fulfilled, the primary-pion momentum was varied within the permissible range in calculating the missing mass. Whenever ω^2 remained negative for the maximum possible variation of the primary-particle momentum the corresponding event was excluded.

The effective masses of different pion combinations were computed from

$$M_{\text{eff}}^2 = (E_1 + E_2 + \dots + E_i)^2 - (p_1 + p_2 + \dots + p_i)^2,$$

where E_i and p_i are the energies and momenta of the created pions. Errors in determining the effective mass depend on the accuracy with which coordinates, momenta, and angles were measured in our chamber.^[8] The ratio $\Delta M_{\text{eff}}/M_{\text{eff}}$, where ΔM_{eff} is the error in the effective mass, is almost independent of M_{eff} . For different pion systems the rms relative errors are as follows:

No. of pions:	2	3	4
$[(\Delta M_{\text{eff}}/M_{\text{eff}})^2]^{1/2}$, %:	3.5	6	7

The accompanying table gives data regarding the numbers of events used in plotting the effective mass spectra for different multiplicities n_S . All events were divided into two groups. The first

¹⁾Institute of Atomic Physics, Rumanian Academy of Sciences, Bucharest.

Particle	n_s	No. of events	No. of events excluded on the basis of the missing mass and target mass	No. of retained events
Λ	2	255	24	231
	4	116	28	88
K^0	2	327	15	312
	4	162	18	144

group (Λ) consists of interactions for which either both strange particles of a pair ($\Lambda, K^{0,+}$) were registered or only the Λ . The second group includes interactions where either both K mesons ($K-\bar{K}$ pairs) are visible in the chamber, or only one K. V^0 events that cannot be identified uniquely were considered to be Λ hyperons on the basis of the conclusions reached in [8].

1. THE ρ MESON

Much experimental and theoretical work has been done on the production and properties of ρ mesons (see the bibliography in [10]). The ρ quantum numbers (J, P, I, G) = (1, -1, 1, 1) can be regarded as firmly established. Some publications [11] have emphasized the predominant production of ρ in events with small four-momentum transfer, as well as the ρ production accompanied by $N_{3/2,3/2}$ isobar production. [12] There is still insufficient information available regarding ρ production in π^- -p interactions involving strange-particle production. Peyrou et al. [13] have studied $K^0-\bar{K}^0$ and Λ^0 -K pair production by 10-GeV/c π^- mesons in hydrogen without discovering ρ mesons.

The present work presents proof of ρ^0 production accompanying strange-particle production. We have established that ρ^0 is produced only in events with charged-particle multiplicity $n_s = 4$. The contribution of ρ^0 mesons to the effective mass distribution of $\pi^+\pi^-$ combinations for the group of events in which a Λ is produced amounts to $11 \pm 2\%$, which corresponds to a cross section of $20 \pm 8 \mu b$. Figure 1 shows a histogram and an ideogram of the effective mass distribution of $\pi^+\pi^-$ combinations for events with $n_s = 4$ and Λ production. As a background curve Fig. 1 shows the smooth distribution of masses $M_{\pi^+\pi^-}$ in Λ -producing events. This distribution agrees well with a curve calculated from statistical theory (see Sec. 3).

The $M_{\pi^+\pi^-}$ distribution in K^0 -producing events also exhibits an energy corresponding to a ρ^0 meson (Fig. 2), which here comprises at most 5% of all events. If it is considered that approximately $1/3$ of events with registered K^0 mesons are among

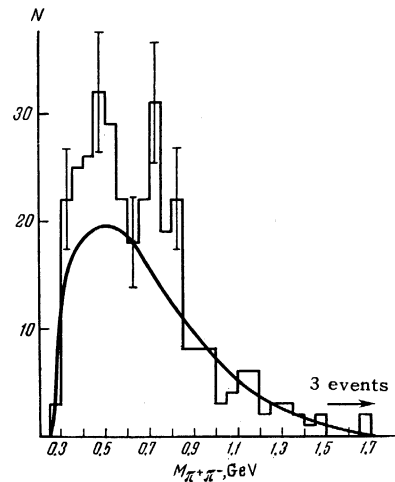


FIG. 1a. Effective mass spectrum of $\pi^+\pi^-$ combinations for $n_s = 4$ events with Λ hyperon. The curve was calculated from statistical theory, which furnishes a good description of the $M_{\pi^+\pi^-}$ spectrum.

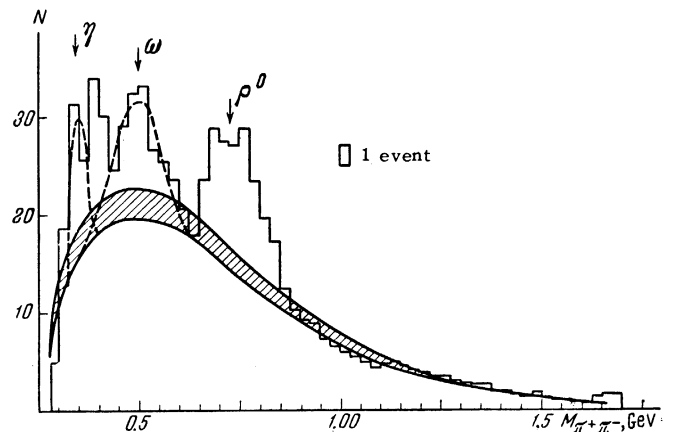


FIG. 1b. Ideogram of effective masses of $\pi^+\pi^-$ combinations. The dashed curves are η and ω resonance peaks. [12, 15] The background curve was normalized for events with $M > 0.8$ GeV. The shaded region represents the normalization error spread.

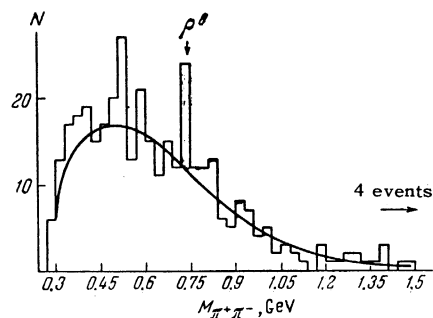


FIG. 2. Effective mass spectrum for 112 K^0 -producing events (367 combinations) with charged-particle multiplicity $n_s = 4$. The smooth curve is based on statistical theory and the $M_{\pi^+\pi^-}$ mass spectrum.

the Λ - K^0 pair-producing events,^[3] and we take into account the established percentage of ρ^0 productions accompanying Λ - K^{0+} pairs, it may be concluded that the ρ^0 peak of this distribution results from events in which Λ - K^0 pairs are produced. The position of the peak in the spectrum for stars with a Λ and the half-width calculated from the Breit-Wigner formula are $M_0 = 730$ MeV and $\Gamma'/2 = 80$ MeV. The natural width Γ was determined from $(\Gamma/2)^2 = (\Gamma'/2)^2 - \Delta M^2$, where Γ' is the experimental width and ΔM is the resolving power for the region of the peak at 26 MeV.

An analysis of events in the region of the ρ^0 peak yields the following results:

1. The production of ρ^0 mesons is entirely insensitive to the magnitude of transferred four-momentum $|\Delta|^2$. Figure 3 shows the $M_{\pi^+\pi^-}$ distributions for Λ -producing events ($n_s = 4$) involving different amounts of transferred momentum: a) $|\Delta| < 700$ MeV, b) $|\Delta| < 900$ MeV, and c) $|\Delta| < 1100$ MeV. If 650–850 MeV is taken to be the ρ^0 region, the contribution of this region is $31 \pm 9\%$ in case a, $32 \pm 6\%$ in case b, and $30 \pm 5\%$ in case c; these three values are thus seen to coincide. A similar result was obtained in an investigation of multiple pion production at 10 GeV/c.^[14] It has been noted in some publications that at low primary energies ρ^0 production occurs predominantly with low momentum transfer $|\Delta| < 400$ –500 MeV.

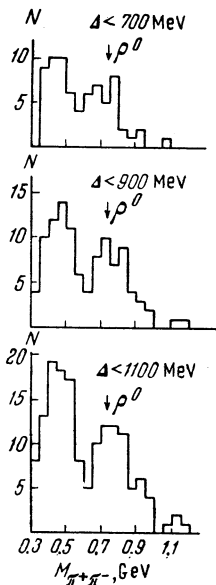


FIG. 3. $M_{\pi^+\pi^-}$ spectra for Λ -producing events ($n_s = 4$) with different amounts of momentum transfer between baryons.

2. The ρ^0 angular distribution in the pion-proton c.m.s. (Fig. 4) duplicates qualitatively the distribution of π^+ mesons accompanying Λ production,^[2]

but is peaked more strongly forward. The π^+ , π^- , and ρ^0 c.m.s. angular distributions are characterized by the following forward/backward ratios:

$$\begin{aligned} \vec{n}_{\pi^+}/\vec{n}_{\pi^-} &= 1.34 \pm 0.19, \\ \vec{n}_{\pi^-}/\vec{n}_{\pi^+} &= 1.43 \pm 0.17, \\ \vec{n}_{\rho^0}/\vec{n}_{\rho^0} &= 2.6 \pm 0.7. \end{aligned}$$

3. In the ρ^0 rest system the pions have an almost isotropic distribution (Fig. 5); according to the Kolmogorov-Smirnov test there is an 8% probability of coincidence with an isotropic distribution.

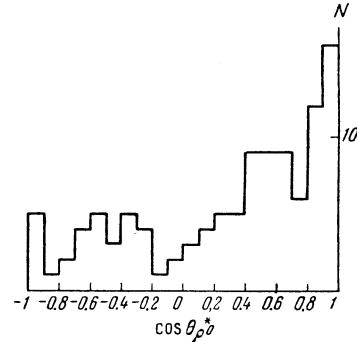


FIG. 4. ρ^0 angular distribution in π^- -p system ($n_s = 4$).

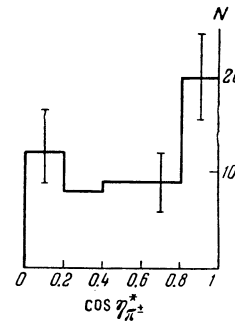


FIG. 5. Distribution of angles η^* for π mesons from ρ^0 decay in the ρ^0 rest system ($n_s = 4$).

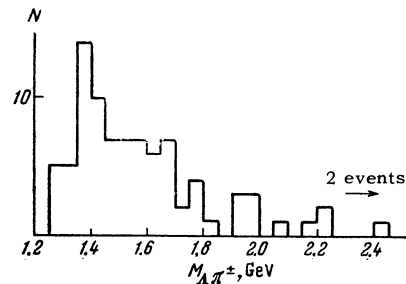


FIG. 6. $M_{\Lambda\pi^\pm}$ distribution for events where $650 < M_{\pi^+\pi^-} < 850$ MeV. ($M_{\pi^+\pi^-}$ for other pions; $n_s = 4$).

4. Our data do not conflict with the simultaneous production of ρ^0 and Y_1^* having a mass of 1380 MeV. Figure 6 shows the mass distribution of Λ - π^\pm combinations for all π^\mp not included in the region

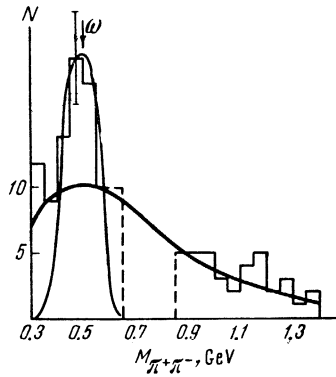


FIG. 7. $M_{\pi^+\pi^-}$ spectrum for ρ^0 -producing events ($650 < M_{\pi^+\pi^-} < 850$ MeV). Light smooth curve — $M_{\pi^+\pi^-}$ spectrum from ω meson events based on^[12]; thick curve — $M_{\pi^+\pi^-}$ distribution from Λ -producing events; $n_s = 4$.

650–850 MeV of the ρ^0 peak but produced along with π^\mp in this region. Investigations devoted to the search for ρ^0 show that in some events (as in.^[12]) where π^+ -p interactions occur a nucleonic isobar $N_{3/2,3/2}$ is produced along with ρ^0 .

5. The mean momenta of ρ^0 produced in our reactions were (in GeV/c units)

$$\bar{p}_\rho^* = 0.627, \quad \bar{p}_{\rho L}^* = 0.250, \quad \bar{p}_\rho = 2.67, \quad \bar{p}_{\rho\perp} = 0.423.$$

Here \bar{p}_ρ^* is the c.m.s. mean ρ^0 momentum, \bar{p}_ρ is the mean lab. system ρ^0 momentum, $\bar{p}_{\rho L}$ is the longitudinal c.m.s. momentum, and $\bar{p}_{\rho\perp}$ is the transverse momentum.

2. THE ω , η , AND δ MESONS

The low efficiency (10%) of γ registration in our chamber along with inexact knowledge of the primary-beam momentum prevented us from investigating directly the production of η and ω particles, for which we obtained only indirect in-

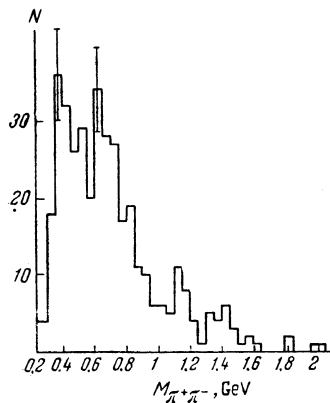


FIG. 8a. Mass spectrum of $\pi^+\pi^-$ combinations for $n_s = 2$, including events with a registered Λ hyperon and events with a K^0 meson.

formation. In the effective mass distribution of $\pi^+\pi^-$ combinations for $n_s = 4$ there is an excess of events above the phonon curves (Fig. 1) in regions corresponding to values of $M_{\pi^+\pi^-}$ associated with η and ω mesons. The values of $M_{\pi^+\pi^-}$ for π^\pm from η and ω decays lie in the range from $2m_\pi$ to $m_{\eta,\omega} - m_\pi$, i.e., from 0.28 to 0.64 GeV, for ω , and up to 0.31 GeV for η . Assuming that the given discrepancies actually are due to η and ω particles, we can estimate the contributions of these resonances. Using Dalitz plots from^[12,15], we obtained $M_{\pi^+\pi^-}$ distributions from η and ω decays, which were fitted to the experimental distribution along with the background curve and the Breit-Wigner curve for ρ^0 (Fig. 1b). The results show that a η meson decaying according to $\eta \rightarrow \pi^+\pi^-\pi^0$ is produced in $4 \pm 2\%$ of the events while $\omega \rightarrow \pi^+\pi^-\pi^0$ in $13 \pm 2\%$ of the events. The corresponding cross sections are $\sigma_\eta = 7 \pm 5 \mu\text{b}$ and $\sigma_\omega = 24 \pm 9 \mu\text{b}$.

If it is assumed that the excess of events in the $M_{\pi^+\pi^-}$ distribution around 500 MeV results from ω decay, our data reveal simultaneous ρ^0 and ω production. Indeed, this conclusion results from the $M_{\pi^+\pi^-}$ spectrum shown in Fig. 7 for events in which the effective mass of the two other pions is found in the ρ^0 region ($650 < M_{\pi^+\pi^-} < 850$ MeV). We estimated that the cross section for simultaneous ρ^0 and ω production is $\leq 10 \mu\text{b}$. Like ρ^0 production, ω production is insensitive to the amount of transferred momentum (Fig. 3).

The mass spectra of $\pi^+\pi^-$ combinations for charged-particle multiplicity $n_s = 2$ are identical for Λ - and K^0 -producing events, but differ entirely from the case of $n_s = 4$. Figure 8a shows the mass spectrum of $\pi^+\pi^-$ combinations for $n_s = 2$. Events are grouped around the masses ~ 400 and 600 – 700 MeV and the spectra are similar for stars with Λ or K^0 . The peak at 400 MeV can be accounted for by the presence of an excited hyperon Y^* in the $M_{\Lambda\pi^\pm}$ spectra, and also by an admixture of events with nonuniquely identified particles (i.e., when a K particle was taken to be a pion).

The maximum at $M_{\pi^+\pi^-} = 630$ MeV can be iden-

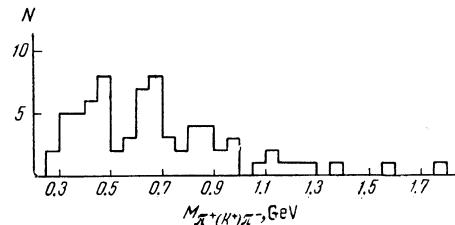


FIG. 8b. $M_{K^+\pi^-}$ spectrum calculated with K^+ assumed to be π^+ .

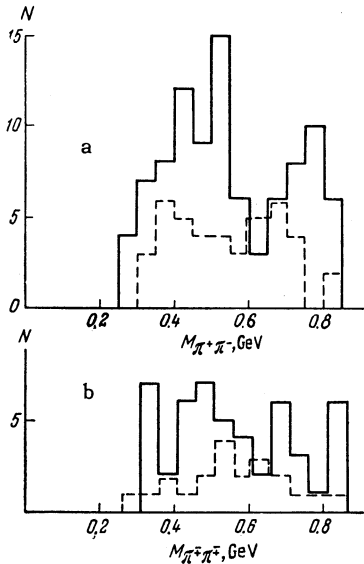


FIG. 9. $M_{\pi^+\pi^-}$ spectra of pions forming the 1340-MeV peak in the mass distribution of the 4π system. a - $1300 < M_{\pi^+\pi^-} < 1400$ MeV; b - $1400 < M_{\pi^+\pi^-} < 1600$ MeV.

tified with the previously observed ζ meson. Since the ζ mass varies in different investigations, Roos^[16] suggested that two different ζ mesons exist, with the masses $M_{\zeta^{0\pm}} = 565 \pm 10$ MeV and $M_{\delta^{0\pm}} = 625$ MeV. The proofs advanced for the existence of ζ^- and δ mesons are still insufficiently reliable; in^[11,12,17] these particles were not detected. Our results furnish no additional evidence for the existence of these mesons. Our data permit a different interpretation of the peak at $M_{\pi^+\pi^-} = 630$ MeV, —that the peak results from the influence of resonance in $K^+ - \pi^-$ or $\tilde{K}^0 - \pi^-$ systems at $M = 880$ MeV. The $M_{K^+\pi^-}$ distributions for $\Lambda - K^+$ pair-production events indicate a certain fraction of K^* resonances with the mass 880 MeV. Events producing this resonance lie in the region of the ~ 630 -MeV peak of the $M_{\pi^+\pi^-}$ distribution (Fig. 8b) if K^+ mesons are here replaced by π^+ mesons. An investigation of the $M_{K^0\pi^{\pm}}$ and $M_{K^{\pm}\pi^{\mp}}$ spectra showed that in $K - \tilde{K}$ pair-producing events with $n_S = 2$ there are no resonances in the $K - \pi$ system. Therefore the events forming the peak at ~ 630 MeV evidently belong to $\Lambda - K^{0+}$ pairs.²⁾

3. 4π SYSTEM

At the 1962 Geneva conference we reported^[9] the possibility of resonance in a four-pion system. The mass of this resonance was given as 1340 ± 70 MeV, and the proposed decay schemes were

²⁾Events with one registered K^0 can be divided as follows: $\frac{1}{3}$ — events with $\Lambda - K^0$ pairs, and $\frac{2}{3}$ — events with $K - \tilde{K}$ pairs.

$$(4\pi)^* \rightarrow \begin{cases} \rho^0 + 2\pi \\ \pi^+ + \pi^- + \pi^+ + \pi^- \end{cases}$$

Figure 9 shows the $M_{\pi^+\pi^-}$ and $M_{\pi^{\mp}\pi^{\mp}}$ distributions for π^{\pm} mesons forming a peak in the $M_{4\pi}$ distribution that is shown in Fig. 10 a,b ($1.3 < M_{4\pi} < 1.4$ GeV). Old statistical data were in an attempt used at a more critical verification of this peak and its properties.

A. It was necessary to demonstrate that the addition of background events does not result in a nonuniform distribution. Using ionization measurements, δ electrons, and an investigation of the missing mass, we were able to distinguish events which can be associated with $\Lambda - K^+$ pairs and with $K^0(\tilde{K}^0) - K^-(K^+)$ pairs. The mass distribution of $K^+\pi^-\pi^+\pi^-$ or $K^-\pi^+\pi^-\pi^+$ systems (where K^+ and K^- were regarded as pions) includes no accumulations in the region of the peak, which is also unaffected by $\pi^- - C$ interactions or interactions with quasi-free nucleons.

B. The background in the effective-mass distribution was evaluated by different methods. The statistical background for the effective-mass distribution of ν particles ($\nu = 1, 2, 3, \dots$) was represented by curves based on the expression^[20]

$$dW/dW_\nu = f(T) \left\{ \prod_i (2s+1)^{\sigma_i} / \prod_j N_j! \right\} \Omega_i^\nu dw(M_n M_\nu) / dM_\nu,$$

where $f(T)$, $\prod (2s+1)^{\sigma_i}$, and $\prod N_j!$ are coefficients taking into account isospin, spin, and the identity of particles. Phase-space curves of $dw(M_n M_\nu) / dM_\nu$ were calculated using the method proposed in^[20]. Here Ω_i is the Lorentz-contracted interaction volume, which for pions is given by

$$\Omega_{i\pi} = \frac{1}{(2\pi)^3} \frac{4\pi}{3} \left(\frac{\lambda}{m_\pi} \right)^3 \frac{1}{\gamma_c},$$

where m_π is the pion mass, γ_c is the Lorentz factor, and λ is a fitting parameter. Since we are here concerned with events in which strange particles are produced and are not making comparisons with other reactions, the choice of an interaction volume for strange particles does not play an essential role. The phase-space curves corresponding to $\Lambda - K$ and $K^0 - \tilde{K}^0$ pair-production channels were plotted on the basis of the experimental ratio between the numbers of $\Lambda - K^0$ and $K^0 - \tilde{K}^0$ pairs.³⁾

³⁾A detailed comparison between these results and calculations based on statistical theory, together with a discussion of the choice of the interaction volume for strange particles will be published later.

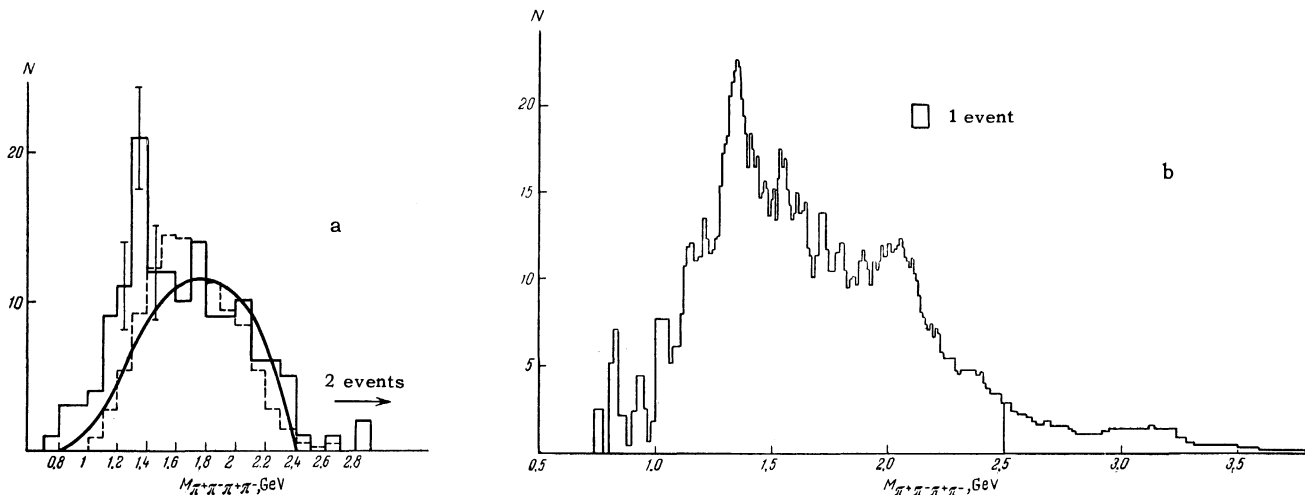


FIG. 10a. Effective-mass distribution of $\pi^+\pi^-\pi^+\pi^-$ system for 151 events. The smooth background curve is based on statistical theory; the dashed background histogram was calculated by the Monte Carlo method.

FIG. 10b. Ideogram of effective masses of $\pi^+\pi^-\pi^+\pi^-$ system.

Figure 10a shows the background distributions; the smooth curve was obtained from statistical theory and the dashed histogram was calculated by the Monte Carlo method. The difference between the two background curves can be attributed to the fact that the statistical calculation did not take into account the channels involving resonances,⁴⁾ which because of their large number make a considerable combined contribution. In the Monte Carlo calculation the effective masses were calculated using a random selection of momenta, energies, and angles from the experimental data. Resonance states could evidently influence this result. Figure 10a shows that the mass peak at 1340 MeV is outside the limits of 2–2.5 standard deviations.

C. The values of $M_{\pi^+\pi^-}$ for the events in the 1340-MeV peak region of the $M_{4\pi}$ distribution ($1.3 \text{ GeV} \leq M_{4\pi} \leq 1.4 \text{ GeV}$) are grouped about 550 MeV and the ρ^0 mass. The first of these two regions is the portion of the $M_{\pi^+\pi^-}$ spectrum which we are inclined to assign to ω production. It was considered how these two peaks can influence the $M_{4\pi}$ spectrum.

If two particles (2 and 3) out of three form a bound state, the properties of a Dalitz plot can be used to evaluate the influence of this bound state on the mass spectrum of particles 1 and 2. If the bound state of particles 2 and 3 with fixed mass

$m_{2,3}$ decays isotropically and the decay energy of these particles is small, the $m_{1,2}$ distribution can exhibit a narrow peak simulating a resonance of particles 2 and 3. Let us take a ρ^0 meson with $m_1 = 0.75 \text{ GeV}$ as the first particle and a 2π system with $m_2 = 0.55 \text{ GeV}$ as the second particle. The third particle will be a π^0 meson forming with the second particle a bound state, which is an ω meson having $m_{2,3} = 0.78 \text{ GeV}$; the minimum total energy of all three particles will then be $M = m_{2,3} + m_1 = 1.53 \text{ GeV}$. For this value of M the mass spectrum of particles 1 and 2 (in the $M_{4\pi}$ spectrum) can exhibit a peak at $m_{1,2} = 1.32 \text{ MeV}$ calculated from^[21]

$$m_{1,2}^2 = m_{2,3}m_1 + m_1^2 + m_2^2 + m_1 / m_{2,3}(m_2^2 - m_3^2).$$

If M is increased slightly to 1.6 GeV, the calculation gives the range $1.32 < m_{1,2} < 1.43 \text{ GeV}$.

Similarly, assuming that the mass spectrum of $\pi^\pm\pi^0$ combinations from ω decay is the same as the $M_{\pi^+\pi^-}$ spectrum and using the scheme

No. of particles	1	2	3
Type of particles	$\rho \rightarrow \pi^+\pi^-$	$\pi^+\pi^-$	$\pi^+\pi^0$
Mass, GeV	0.75	0.14	0.55
Bound state		$m_\omega = 0.78$	

our calculations show that in the $M_{\pi^+\pi^-\pi^+\pi^\pm}$ spectra we can expect a peak due to ρ^0 and ω mesons at mass $\sim 1000 \text{ MeV}$. The experimental spectrum (Fig. 11) does actually include a peak rising above the phase curve at $\sim 1000 \text{ MeV}$.

The foregoing argument furnishes additional support for the simultaneous production of ω and ρ^0 mesons in a single interaction. Moreover, the peak at $\sim 1340 \text{ MeV}$ can indicate a resonance state in a 5π system ($\pi^+\pi^-\pi^+\pi^-\pi^0$) corresponding to the

⁴⁾The statistical-theory calculations neglecting resonance channels do not agree with experiment^[4] with regard to the number of π^0 mesons. The result is changed by introducing channels with known resonances, and the mean number of π^0 mesons becomes close to the number obtained from γ rays.^[4]

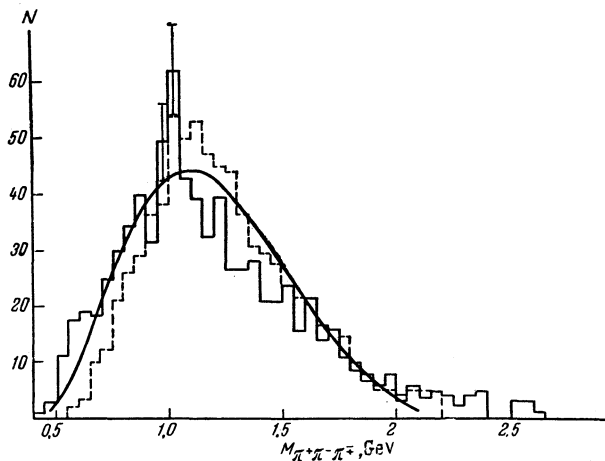
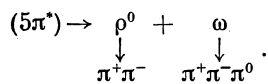


FIG. 11. Effective-mass distribution of 3π system (760 events). The curve was derived from statistical theory. The dashed histogram represents a Monte Carlo calculation.

resonance of ρ^0 and ω mesons. Assuming that this resonance exists, we transformed^[21] the $M_{4\pi}$ spectrum into a $M_{5\pi}$ spectrum and obtained the position of a possible $\rho^0\omega$ resonance. The maximum of this distribution is located in the 1600–1800-MeV region.

It can be asserted, in summation, that the 1340-MeV peak in the $M_{4\pi}$ distribution actually exists, with the following possible explanations:

- 1) Resonance occurs in a 4π system ($\pi^+\pi^-\pi^+\pi^-$) with isospin $T = 0$ and half-width $\Gamma/2 = 70$ MeV.
- 2) The peak results partially from simultaneous ρ^0 and ω production in four-prong stars with Λ -K pairs.
- 3) The maximum in the $M_{4\pi}$ distribution results from a narrow resonance with its mass in the 1600–1800-MeV range, with a decay cascade passing through ρ^0 and ω states:



We are greatly indebted to M. I. Podgoretskii and I. V. Chuvilo for assistance and valuable discussions, and also to Tu Yuan-Ts'ai, A. A. Kuznetsov, Kim Hi In, Nguyen Dinh Tu, and Wang Yung-Ch'ang for their participation in the first stage of this work. We are indebted to N. N. Govorun and N. F. Markova of the Computing Center of the Joint Institute for Nuclear Research, and to G. M. Korotkova, S. N. Komarova, and L. M. Zhukova for measurements and calculations.

¹ Wang, Solov'ev, and Shkobin, PTE No. 1, 41 (1959).

² Wang, Veksler, Tu, Kladnitskaya, Kuznetsov, Mihul, Nguyen, Penev, Sokolova, and Solov'ev, JETP 43, 815 (1962), Soviet Phys. JETP 16, 577 (1963).

³ Belyakov, Wang, Veksler, et al., JETP 44, 431 (1963), Soviet Phys. JETP 17, 294 (1963).

⁴ Belyakov, Wang, Viryasov, Tu, Kim, Kladnitskaya, Kuznetsov, Nguyen, Penev, Sokolova, and Solov'ev, JETP 44, 1474 (1963), Soviet Phys. JETP 17, 991 (1963).

⁵ M. I. Solov'ev, Proc. of the 1960 Annual International Conference on High Energy Physics at Rochester, Univ. of Rochester, 1961, p. 388.

⁶ N. G. Birger and Yu. A. Smorodin, JETP 37, 1355 (1959), Soviet Phys. JETP 10, 964 (1960).

⁷ Kopylova, Lyubimov, Podgoretskii, Rezaev, and Trka, JETP 44, 1481 (1963), Soviet Phys. JETP 17, 996 (1963).

⁸ Veksler, Viryasov, Vrana, Kim, Kladnitskaya, Kuznetsov, Nguyen, Solov'ev, Hofmokl, and Ch'eng, JETP 44, 84 (1963), Soviet Phys. JETP 17, 58 (1963).

⁹ V. A. Belyakov, Wang Yung-ch'ang et al., Proc. of the 1962 Annual International Conference on High-Energy Physics at Geneva, CERN, Geneva, Switzerland, 1962, p. 336.

¹⁰ M. Roos, Revs. Modern Phys. 35, 314 (1963).

¹¹ Anderson, Bang, Burke, Carmony, and Schmitz, Phys. Rev. Letters 6, 365 (1961), Revs. Modern Phys. 33, 431 (1961); Stonehill, Baltay, Courant, Fickinger, Fowler, Kraybill, Sandweiss, Sanford, and Taft, Phys. Rev. Letters 6, 624 (1961); Pickup, Robinson, and Salant, Phys. Rev. Letters 7, 192 (1961); Erwin, March, Walker, and West, Phys. Rev. Letters 6, 628 (1961).

¹² Alff, Colley, Gelfand, Nauenberg, Miller, Steinberger, Tan, Brugger, Kramer, and Plano, op. cit. ref.^[9], p. 50; Phys. Rev. Letters 9, 325 (1962).

¹³ Bigi, Brandt, Carrara, Cooper, de Marco, Macleod, Peyrou, Sosnovski, and Wroblewski, op. cit. ref.^[9], p. 597.

¹⁴ Fleury, Kayas, Muller, and Pelletier, op. cit. ref.^[9], p. 247.

¹⁵ Chretien, Bulos, Crouch et al., Phys. Rev. Letters 9, 127 (1962).

¹⁶ M. Roos, Phys. Rev. Letters 3, 242 (1963).

¹⁷ Selove, Hagopian, Brody, Baker, and Leboy, Phys. Rev. Letters 9, 272 (1962).

¹⁸ Peck, Jones, and Perl, Phys. Rev. 126, 1836 (1962).

¹⁹ R. Hagedorn, Nuovo cimento 15, 246 (1960).

²⁰ G. I. Kopylov, JETP 39, 1091 (1960), Soviet Phys. JETP 12, 761 (1961).

²¹ G. I. Kopylov, Preprint, Joint Inst. Nuclear Res. R-1368, 1963.