SOVIET PHYSICS JETP VOLUME 19, NUMBER 5 NOVEMBER, 1964

WEAK INTERACTIONS AT HIGH ENERGIES

M. P. REKALO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor December 18, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 46, 1891-1896 (May, 1964)

Processes of weak production of mesons in the interaction of high-energy neutrinos with nucleons are investigated on the basis of a hypothesis concerning the singularities of the partial amplitude as a function of the angular momentum. An expression for the differential cross section is obtained, and also a number of isotopic relations which depend on the isotopic spin of the leading pole.

1. INTRODUCTION

WEAK interactions involving hadrons [1] are modified to a considerable degree by strong interactions. For example, the processes of weak production of mesons in the reactions

$$\nu + N \rightarrow e^- (\mu^-) + N + \pi,$$
$$\nu + N \rightarrow e^- (\mu^-) + Y + K \qquad (1)$$

are essentially determined by the nature of the interaction of the mesons and baryons in the final state. Treatments of these processes with the strong interactions taken into account have been carried out in a number of papers [2-7] on the basis of dispersion relations, with the unavoidable approximations and restrictions which affect the applicability and validity of the dispersion approach.

In the present paper the processes (1) are investigated in the region of high energies of the incident neutrinos. The influence of the strong interactions is taken into account on the basis of a hypothesis about the singularities of the partial amplitude as a function of the angular momentum. We consider that asymptotic region of the processes (1) in which the mesons are produced at angles $\sim 180°$ in the center-of-mass system (c.m.s.) of the final baryon and meson. If we assume that the extreme right-hand singularity of the partial amplitude is a pole, then the asymptotic behavior of the amplitude in this region is determined by a pair of complex-conjugate poles of fermion nature. [8] This leads to an oscillatory behavior of the amplitude, but the oscillations are not manifested in the differential cross sections of such processes as $\pi + N \rightarrow \pi + N$, $\gamma + N \rightarrow N + \pi$, $\gamma + N \rightarrow \gamma + N$. [9-11] It is interesting to settle the question as to whether the oscillations will be manifested in the differential cross sections of the processes (1), in which, owing to the weak interaction, parity is not conserved.

2. THE STRUCTURE OF THE MATRIX ELEMENT

In the lowest order in the weak-interaction constant the matrix element for the processes (1) can be put in the following form:

$$M = \tfrac{1}{2} G \cdot 2^{-1/2} \bar{u}(p') \gamma_\mu (1 + \gamma_5) u(p) \bar{u}(p_2) J_\mu^{(+)} u(p_1), \qquad (2)$$

where p is the four-momentum of the neutrino and p′ is that of the lepton that is produced. The operator $J_\mu^{(+)}$ is the current operator of the strongly interacting particles and can be written as the sum of vector and axial vector currents:

$$J_\mu^{(+)} = J_\mu^{(V)} + J_\mu^{(A)}. \qquad (3)$$

In what follows we neglect the masses of the leptons; then in this approximation we get

$$\tfrac{1}{2} G \cdot 2^{-1/2} \bar{u}(p') \gamma_\mu (1 + \gamma_5) u(p) k_\mu \equiv \varepsilon_\mu k_\mu = 0, \qquad (4)$$

with $k = p - p'$. Therefore the matrix element (2) corresponds formally to the matrix element for the production of mesons in the interaction between nucleons and vector mesons whose polarization state is characterized by the vector ϵ_μ. We expand the quantities $\epsilon J^{(V)}$ and $\epsilon J^{(A)}$ in terms of the invariant combinations made up of ϵ_μ, the Dirac matrices, and the momenta [2]:

$$\varepsilon_\mu J_\mu^{(V)} = \sum_{i=1}^{6} A_i^{(V)} M_i^{(V)} (\varepsilon, k, p_1, p_2),$$
$$\varepsilon_\mu J_\mu^{(A)} = \sum_{i=1}^{6} A_i^{(A)} M_i^{(A)} (\varepsilon, k, p_1, p_2), \qquad (5)$$

where the $M_i^{(A,V)}$ are as follows:

$$M_1^{(V)} = i\gamma_5\hat{\varepsilon}\hat{k}, \quad M_2^{(V)} = i\gamma_5\varepsilon p_2, \quad M_3^{(V)} = i\gamma_5\varepsilon p_1,$$
$$M_4^{(V)} = \gamma_5\hat{\varepsilon}, \quad M_5^{(V)} = \gamma_5\hat{k}\varepsilon p_2, \quad M_6^{(V)} = \gamma_5\hat{k}\varepsilon p_1,$$
$$M_i^{(A)} = i\gamma_5 M_i^{(V)}. \qquad (6)$$

The amplitudes (5) and (6) describe the production of π and K mesons if the parity of the system KNY is -1. If the parity of this system is positive, the combinations $M_i^{(A)}$ and $M_i^{(V)}$ must be interchanged. The invariant amplitudes are determined by the strong interactions, and therefore we can assume that they satisfy the Mandelstam representation. This assumption is verified in perturbation theory.

3. THE ASYMPTOTIC BEHAVIOR OF THE AMPLITUDE AT HIGH ENERGIES

Let us consider the asymptotic behavior of the amplitudes so constructed for $s = -(k + p_1)^2 \to \infty$, $u = -(k - p_2)^2 = \text{const} < 0$, which corresponds to the production of mesons at angles $\sim 180°$ in the c.m.s. of the baryon and meson. According to the theory of complex angular momenta, this asymptotic behavior is determined by the extreme right-hand singularity of the partial amplitude in the crossed channel as a function of the angular momentum. We consider the simple possibility that the singularity on the right, which determines the asymptotic behavior, is a simple pole. Because of the fermion nature of the leading pole the asymptotic behavior of the amplitudes is determined by the contribution of two poles at complex-conjugate points. Keeping only the contribution from these main poles, we get

$$J_\mu^{(V)} = \gamma_\mu (a_1 + ia_2\hat{k})(i\hat{f} - \sqrt{u})\gamma_5\Delta_j + \gamma_\mu (a_1^* + ia_2^*\hat{k})(i\hat{f} + \sqrt{u})\gamma_5\Delta_{j^*},$$
$$J_\mu^{(A)} = \gamma_\mu (b_1 + ib_2\hat{k})(i\hat{f} - \sqrt{u})\Delta_j + \gamma_\mu (b_1^* + ib_2^*\hat{k})(i\hat{f} + \sqrt{u})\Delta_{j^*}, \qquad (7)$$

where

$$\Delta_j = \frac{s^{j-1/2} \pm (-s)^{j-1/2}}{\cos \pi j}, \qquad \Delta_{j^*} = \frac{s^{j^*-1/2} \pm (-s)^{j^*-1/2}}{\cos \pi j},$$
$$f = p_2 - k; \qquad (8)$$

the signs $\pm$ correspond to different signatures of the leading pole; j describes the trajectory of the pole; and $a_{1,2}$ and $b_{1,2}$ are real functions which depend on $u^{1/2}$ and determine the residues of the partial amplitudes.

We note that the asymptotic structure (7) corresponds to the sum of the contributions of two Feynman pole diagrams; $\gamma_\mu(a_1 + ia_2\hat{k})$ corresponds to the vertex where the vector particle is absorbed, $(i\hat{f} - u^{1/2})\Delta_j$ corresponds to the propagation function of the "reggeon," and γ_5 corresponds to the vertex where the pseudoscalar meson is emitted.

4. THE DIFFERENTIAL CROSS SECTION

By means of Eqs. (7) and (8) we calculate the differential cross section, averaged over the polarizations of the initial particles and summed over those of the final particles:

$$d\sigma = \eta |\overline{M}|^2 \delta^4 (k + p_1 - p_2 - q)(2\pi)^4 \prod_{i=1}^{3} \frac{d^3q_i}{2\varepsilon_i (2\pi)^3}, \qquad (9)$$

where $\eta = 1/wE$, E is the energy of the neutrino, w is the total energy of the neutrino and the nucleon in the c.m.s., and

$$|\overline{M}|^2 = \varepsilon_{\mu\nu} T_{\mu\nu}; \qquad (10)$$

$$\varepsilon_{\mu\nu} = -\frac{1}{4} G^2 [p_\mu p'_\nu + p_\nu p'_\mu - \delta_{\mu\nu} pp' + \varepsilon_{\mu\nu\rho\sigma} p_\rho p'_\sigma],$$
$$T_{\mu\nu} = \frac{1}{4} \mathrm{Sp}\, J_\mu (i\hat{p}_1 - M_1)\overline{J}_\nu (i\hat{p}_2 - M_2) = T_{\mu\nu}^{(VV)} + T_{\mu\nu}^{(AA)} + T_{\mu\nu}^{(AV)}. \qquad (11)$$

Here (VV) denotes the contribution of the vector part of the current, (AA) that of the axial-vector current, and (AV) that from the interference between the currents;

$$T_{\mu\nu}^{(VV)} = \tfrac{1}{2}\delta_{\mu\nu} us\{|\alpha_1^+|^2 + |\alpha_1^-|^2 + (k^2 - 2kp_2)(|\alpha_2^+|^2 + |\alpha_2^-|^2) + 2M_2 \,\mathrm{Re}\,(\alpha_1^+\alpha_2^{+*} + \alpha_1^-\alpha_2^{-*})\} - 2\sqrt{-u}$$
$$\times \mathrm{Im}\,(\alpha_1^-\alpha_2^{+*} + \alpha_1^+\alpha_2^{-*})[2kp_2 p_{1\nu} p_{2\mu} + s(p_{2\mu}p_{2\nu} + \tfrac{1}{2}u\delta_{\mu\nu})]$$
$$-4p_{1\mu}p_{2\nu}\sqrt{u}\,\mathrm{Re}[M_2(\alpha_1^-\alpha_1^{+*} + k^2\alpha_2^-\alpha_2^{+*}) - kp_2(\alpha_1^-\alpha_2^{+*} + \alpha_2^-\alpha_1^{+*})];$$
$$\alpha_1^\pm = a_1\Delta_j \pm a_1^*\Delta_{j^*}, \qquad \alpha_2^\pm = a_2\Delta_j \pm a_2^*\Delta_{j^*}. \qquad (12)$$

The last term in Eq. (12), proportional to $p_{1\mu}p_{2\nu}$, contracts only with the pseudotensor part of $\epsilon_{\mu\nu}$, and thus gives the part of the differential cross section that is responsible for the nonconservation of parity.

To get $T_{\mu\nu}^{(AA)}$ we have only to replace $\alpha_i^\pm$ by $\beta_i^\pm$ in $T^{(VV)}$ and change the sign of the interference terms of the type $\alpha_i^+\alpha_k^{-*}$. Finally,

$$T_{\mu\nu}^{(AV)} = 2\sqrt{-u}\,\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{Im}\,\{(k^2 p_{1\rho}p_{2\sigma} - M_2^2 k_\rho p_{1\sigma} + sk_\rho p_{2\sigma})$$
$$\times(\alpha_1^-\beta_2^{+*} + \alpha_1^+\beta_2^{-*} + \alpha_2^-\beta_1^{+*} + \alpha_2^+\beta_1^{-*}) + 2M_2(\tfrac{1}{2}sk_\rho p_{2\sigma}$$
$$- kfk_\rho p_{1\sigma})(\alpha_2^+\beta_2^{-*} + \alpha_2^-\beta_2^{+*}) + M_2 p_{2\rho}p_{1\sigma}(\beta_1^-\alpha_1^{+*} + \beta_1^+\alpha_1^{-*}$$
$$+ k^2\beta_2^+\alpha_2^{-*} + k^2\beta_2^-\alpha_2^{+*})\} + 2u\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{Re}\,\{k_\rho p_{1\sigma}M_2(\alpha_1^-\beta_2^{+*}$$
$$+ \alpha_1^+\beta_2^{-*} + \alpha_2^-\beta_1^{+*} + \alpha_2^+\beta_1^{-*}) + sk_\rho p_{2\sigma}(\beta_2^-\alpha_2^{-*} + \beta_2^+\alpha_2^{+*})$$

$$- p_{1\rho}p_{2\sigma}(\beta_1^+\alpha_1^{-*} + \beta_1^-\alpha_1^{-*} - k^2\beta_2^+\alpha_2^{+*} - k^2\beta_2^-\alpha_2^{-*})\}$$
$$+ 4\sqrt{-u}\,p_{2\mu}\,\varepsilon_{\nu\alpha\beta\gamma}k_\alpha p_{1\beta}p_{2\gamma}\,\mathrm{Im}\,(\alpha_1^+\beta_2^{-*} + \alpha_1^-\beta_2^{+*}$$
$$+ \alpha_2^+\beta_1^{-*} + \alpha_2^-\beta_1^{+*}). \quad (13)$$

On contraction with the symmetric part of $\epsilon_{\mu\nu}$ the last term in Eq. (13) gives a nonzero contribution, unlike the other terms in Eq. (13), which contract only with the antisymmetric part of $\epsilon_{\mu\nu}$. In the expressions (12) and (13) only the asymptotic main terms have been retained; also it is assumed that k^2 (the "mass" of the vecton) is a finite quantity, i.e., the lepton is produced in the direction of the momentum of the initial neutrino.

For the calculation of the quantities involved in Eqs. (12) and (13) we use the following relations:

$$\mathrm{Re}\,\alpha_{1,2}^+ = \delta_{(\pm)}\rho_{1,2}\cos(j''\zeta + \varphi_{1,2} \pm \omega)\,s^{j'-1/2},$$
$$\mathrm{Im}\,\alpha_{1,2}^+ = (\mp)\,\rho_{1,2}\cos(j''\zeta + \varphi_{1,2})\,s^{j'-1/2};$$
$$\mathrm{Re}\,\alpha_{1,2}^- = (\mp)\,\rho_{1,2}\sin(j''\zeta + \varphi_{1,2})\,s^{j'-1/2},$$
$$\mathrm{Im}\,\alpha_{1,2}^- = -\delta_{(\pm)}\rho_{1,2}\sin(j''\zeta + \varphi_{1,2} \pm \omega)\,s^{j'-1/2}, \quad (14)$$

where*

$$\rho_{1,2}e^{i\varphi_{1,2}} = a_{1,2}, \quad \mathrm{tg}\,\omega = \mathrm{sh}\,\pi j''/\cos\pi j',$$
$$\delta_{(\pm)} = (\mathrm{ch}\,\pi j'' \pm \sin\pi j')/(\mathrm{ch}\,\pi j'' \mp \sin\pi j')$$

and analogous relations also hold for $\beta_{1,2}^{\pm}$.

Then with the use of the relations (14) we get

$$|\alpha_{1,2}^+|^2 + |\alpha_{1,2}^-|^2 = (1 + \delta_{(\pm)}^2)\,\rho_{1,2}^2 s^{2j'-1},$$
$$\mathrm{Re}\,(\alpha_1^+\alpha_2^{+*} + \alpha_1^-\alpha_2^{-*}) = (1 + \delta_{(\pm)}^2)\,\rho_1\rho_2\cos(\varphi_1 - \varphi_2)\,s^{2j'-1},$$
$$\mathrm{Im}\,(\alpha_1^-\alpha_2^{+*} + \alpha_1^+\alpha_2^{-*}) = -(1 + \delta_{(\pm)}^2)\,\rho_1\rho_2\sin(\varphi_1 - \varphi_2)\,s^{2j'-1},$$
$$\mathrm{Re}\,\alpha_{1,2}^-\alpha_{1,2}^{+*} = \delta_{(\pm)}\rho_{1,2}^2\sin\omega\,s^{2j'-1},$$
$$\mathrm{Im}\,(\alpha_1^+\beta_2^{-*} + \alpha_1^-\beta_2^{+*} + \alpha_2^+\beta_1^{-*} + \alpha_2^-\beta_1^{+*})$$
$$= (1 + \delta_{(\pm)}^2)\,[\rho_1\rho_2'\sin(\varphi_2' - \varphi_1)$$
$$+ \rho_1'\rho_2\sin(\varphi_1' - \varphi_2)]\,s^{2j'-1},$$
$$\mathrm{Re}\,(\alpha_1^+\beta_2^{-*} + \alpha_1^-\beta_2^{+*} + \alpha_2^+\beta_1^{-*} + \alpha_2^-\beta_1^{+*})$$
$$= 2\delta_{(\pm)}\sin\omega\,[\rho_1\rho_2'\cos(\varphi_1 - \varphi_2')$$
$$+ \rho_1'\rho_2\cos(\varphi_1' - \varphi_2)]\,s^{2j'-1}. \quad (15)$$

Thus we can conclude that in spite of the oscillating character of the amplitude the differential cross section for weak meson production is a monotonic function of the energy and angle. This is also true to the same extent for the part of the cross section that conserves parity and for the part that does not.

*tg = tan, ch = cosh, sh = sinh.

5. ISOTOPIC RELATIONS

The production of π mesons in the interaction of high-energy neutrinos with nucleons can occur in the following reactions:

$$1)\ \nu + p \to l^- + p + \pi^+,$$
$$2)\ \nu + n \to l^- + p + \pi^0,$$
$$3)\ \nu + n \to l^- + n + \pi^+; \quad (16)$$

with the respective cross sections σ_1, σ_2, σ_3.

Also the following are the possible antineutrino reactions:

$$1)\ \bar\nu + n \to l^+ + n + \pi^-\ (\bar\sigma_1),$$
$$2)\ \bar\nu + p \to l^+ + n + \pi^0\ (\bar\sigma_2),$$
$$3)\ \bar\nu + p \to l^+ + p + \pi^-\ (\bar\sigma_3). \quad (16a)$$

The ratios of the cross sections of these processes depend on the isotopic spin of the leading pole. If the "reggeon" has isotopic spin ½, then

$$\sigma_1 : \sigma_2 : \sigma_3 = 2 : 1 : 0, \quad \bar\sigma_1 : \bar\sigma_2 : \bar\sigma_3 = 2 : 1 : 0, \quad (17)$$

and $\sigma_i \neq \bar\sigma_i$ [12]; if the isotopic spin of the "reggeon" is 3/2 we get

$$\sigma_1 : \sigma_2 : \sigma_3 = 1 : 2 : 9, \quad \bar\sigma_1 : \bar\sigma_2 : \bar\sigma_3 = 1 : 2 : 9. \quad (17a)$$

These relations are of a more concrete character than the isotopic relations discussed in a paper by Nguyen Van Hieu,[12] which gives some inequalities between the cross sections. As was to be expected, the relations (17) and (17a) do not contradict these inequalities, which follow from the general properties of the isotopic structure of the weak interactions. The more concrete character of Eqs. (17) and (17a) is due to the pole nature of the asymptotic behavior considered.

Weak production of strange particles occurs in the following reactions:

$$1)\ \nu + p \to l^- + \Sigma^+ + K^+\ (\sigma_4),$$
$$2)\ \nu + n \to l^- + \Sigma^+ + K^0\ (\sigma_5)$$
$$3)\ \nu + n \to l^- + \Sigma^0 + K^+\ (\sigma_6),$$
$$4)\ \bar\nu + n \to l^+ + \Sigma^- + K^0\ (\bar\sigma_4),$$
$$5)\ \bar\nu + p \to l^+ + \Sigma^- + K^+\ (\bar\sigma_5),$$
$$6)\ \bar\nu + p \to l^+ + \Sigma^0 + K^0\ (\bar\sigma_6). \quad (18)$$

The asymptotic main pole in these reactions has strangeness −1 and can have isotopic spin 0 or 1. For isotopic spin 0 we have

$$\sigma_4 : \sigma_5 : \sigma_6 = 1 : 1 : 0, \quad \bar\sigma_4 : \bar\sigma_5 : \bar\sigma_6 = 1 : 1 : 0; \quad (19)$$

for isotopic spin 1 we get

$$\sigma_4 : \sigma_5 : \sigma_6 = 1 : 1 : 2, \quad \bar\sigma_4 : \bar\sigma_5 : \bar\sigma_6 = 1 : 1 : 2. \quad (19a)$$

In the asymptotic region the production of Λ hyperons in the processes $\nu + n \rightarrow l^- + \Lambda + K^+$, $\bar{\nu} + p \rightarrow l^+ + \Lambda + K^0$ occurs only through a ''reggeon'' with isotopic spin 1.

If we admit the existence of neutral weak currents, then π mesons can also be produced in processes such as

$$\nu + p \rightarrow \nu + p + \pi^0 \quad (\sigma_1'),$$
$$\nu + p \rightarrow \nu + n + \pi^+ \quad (\sigma_2'),$$
$$\nu + n \rightarrow \nu + n + \pi^0 \quad (\sigma_3'),$$
$$\nu + n \rightarrow \nu + p + \pi^- \quad (\sigma_4').$$

If the asymptotic behavior is determined by a pole with isotopic spin $1/2$, then there is the following connection between the cross sections:

$$\sigma_1' : \sigma_2' : \sigma_3' : \sigma_4' = 1 : 2 : 1 : 2, \qquad \sigma_1'/\sigma_1 = g^2/2, \qquad (20)$$

where g is the ratio of the weak neutral constant to the ordinary weak interaction constant.[12] If the leading pole has isotopic spin $3/2$, then

$$\sigma_1' : \sigma_2' : \sigma_3' : \sigma_4' = 2 : 1 : 2 : 1, \quad \sigma_1'/\sigma_1 = 8g^2. \qquad (20a)$$

According to the neutral-current hypothesis strange particles are produced in the reactions

$$\nu + p \rightarrow \nu + \Sigma^0 + K^+ \quad (\sigma_5'),$$
$$\nu + p \rightarrow \nu + \Sigma^+ + K^0 \quad (\sigma_6'),$$
$$\nu + n \rightarrow \nu + \Sigma^- + K^+ \quad (\sigma_7'),$$
$$\nu + n \rightarrow \nu + \Sigma^0 + K^0 \quad (\sigma_8').$$

If the reactions go through a pole with isotopic spin 0, then

$$\sigma_5' : \sigma_6' : \sigma_7' : \sigma_8' = 1 : 0 : 0 : 1, \qquad \sigma_5'/\sigma_4 = 2g^2; \quad (21)$$

for a ''reggeon'' isotopic spin of 1 we have

$$\sigma_5' : \sigma_6' : \sigma_7' : \sigma_8' = 0 : 1 : 1 : 0, \qquad \sigma_6'/\sigma_4 = 4g^2. \quad (21a)$$

6. CONCLUSION

The differential cross section for production of mesons at large angles (u = const) in the interaction between nucleons and high-energy neutrinos which are converted into leptons emerging forward (k^2 = const) is a monotonic function of energy and angle, in spite of the oscillating behavior of the amplitude for the process. The oscillating character of the amplitude is a consequence of the fermion nature of the pole which determines the asymptotic behavior in question. The isotopic relations obtained above are consequences of the fact that the ''reggeon'' has a definite isotopic spin. These relations are of a more general character, not dependent on the applicability of the Regge-pole theory[13]; they are valid for any case in which there is exchange of a state with a definite isotopic spin.

[1] L. B. Okun', Proc. Intern. Conf. on High-energy Physics at CERN, 1962, page 845.

[2] N. Dombey, Phys. Rev. **127**, 653 (1962).

[3] P. Dennery, Phys. Rev. **127**, 664 (1962).

[4] Nguyen Van Hieu, JETP **43**, 1296 (1962), Soviet Phys. JETP **16**, 920 (1963).

[5] N. Cabibbo and G. Da Prato, Nuovo cimento **25**, 611 (1962).

[6] M. Fukawa, Progr. Theoret. Phys. **28**, 885 (1962).

[7] Y. Yamaguchi, Progr. Theoret. Phys. **23**, 1117 (1960).

[8] V. N. Gribov, JETP **43**, 1529 (1962), Soviet Phys. JETP **16**, 1080 (1963).

[9] Gorshkov, Rekalo, and Frolov, JETP **45**, 285 (1963), Soviet Phys. JETP **18**, 199 (1964).

[10] M. P. Rekalo, JETP **45**, 595 (1963), Soviet Phys. JETP **18**, 409 (1964).

[11] Rekalo, Gorshkov, and Frolov, JETP **45**, 672 (1963), Soviet Phys. JETP **18**, 462 (1964).

[12] Nguyen Van Hieu, JETP **45**, 202 (1963), Soviet Phys. JETP **18**, 143 (1964).

[13] Gribov, Okun', and Pomeranchuk, Preprint, ITÉF, No. 152, 1963.

Translated by W. H. Furry
272