# PROPAGATION OF MAGNETOPLASMA WAVES IN A BISMUTH PLATE

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The propagation of waves in a plate situated in a strong magnetic field is considered. Expressions are given for the amplitudes of the waves reflected from and transmitted through the plate. The reflection coefficient at resonance, i.e., when a standing wave is established in the plate, differs significantly from unity.

It was shown by Khaĭkin, Édel'man, Mina, and the present author [1] that when a constant magnetic field H satisfying the conditions

$$\Omega \gg \omega, \qquad \Omega \gg k v_F, \tag{1}$$

is present, electromagnetic waves propagate in bismuth with velocity $\omega/k$ of order $c\Omega/\omega_n$; here $\Omega = eH/mc$ and $\omega_n$ are the cyclotron and plasma frequencies, respectively, and $v_F$ is the Fermi velocity. Because standing waves arise in the plate at definite values of H, the surface impedance depends on the field in an oscillatory fashion.

The calculations previously made [1] refer to an infinite space. Below we consider the field produced in a plate by a plane wave incident on one of its surfaces. The calculations are greatly simplified by the fact that when condition (1) is satisfied the relation between field and current is the same in the plate as in an infinite space. To verify this assertion it is sufficient to consider a semi-infinite space and verify that account of the boundary causes the appearance of terms of order $kv_F/\Omega \ll 1$.

Of most interest is the case when $\omega \gg k_z v_F$ (this condition is satisfied in strong fields $\Omega \gg \omega_n v_F/c$), because in the contrary case the waves decay. The decay only becomes small when H and k are disposed along symmetrical directions of the crystal. However, these special cases cannot be considered, since the waves with large k make a relatively small contribution to the impedance.

Thus, the problem reduces to solving Maxwell's equations*

$$c\,[\mathbf{k}\mathscr{E}] = \omega\mathscr{H}, \qquad c\,[\mathbf{k}\mathscr{H}] = -4\pi i\sigma\mathscr{E}$$

under the conditions that the tangential components of the field at the surfaces are continuous; $\mathscr{H}$ is the alternating magnetic field of the wave. The matrix $\sigma_{ik}$ $(i, k = x, y, z)$ was previously calculated [1] in a coordinate system with the z axis along H. Here, as in the work of Kaner and Skobov, [2] it is more convenient to use a system of coordinates in which the $\zeta$ axis is along the vector k and the constant field H lies in the $\eta\zeta$ plane. In this system $\mathscr{H}_\zeta = (\sigma\cdot\mathscr{E})_\zeta = 0$, and, having determined $\mathscr{E}_\zeta$ from the latter equality, we can consider only the field components perpendicular to the vector **k**. Then, the two-dimensional tensor $\bar{\sigma}_{\alpha\beta}$ $(\alpha, \beta = \xi, \eta)$ must be understood to mean the quantity $\sigma_{\alpha\beta} - \sigma_{\alpha\zeta}\sigma_{\zeta\beta}/\sigma_{\zeta\zeta}$. With an accuracy to terms like $\omega^2\Omega^{-2}\tan^2\theta$, the components of $\bar{\sigma}_{\alpha\beta}$ are related to the previously calculated components of $\sigma_{ik}$: [1]

$$\bar{\sigma}_{\xi\xi} = \frac{\sigma_{xx}\sigma_{zz} + \sigma_{xz}^2}{\sigma_{zz}},$$
$$\bar{\sigma}_{\xi\eta} = \bar{\sigma}_{\eta\xi} = \frac{\sigma_{zz}\sigma_{xy} - \sigma_{xz}\sigma_{zy}}{\sigma_{zz}\cos\theta},$$
$$\bar{\sigma}_{\eta\eta} = \frac{\sigma_{zz}\sigma_{yy} + \sigma_{yz}^2}{\sigma_{zz}\cos^2\theta}$$

where $\theta$ is the angle between **k** and **H**.

Since for given values of $\omega$ and H two waves with different wave vectors k and $\bar{k}$ exist in an infinite space, it follows that in a plate two pairs of waves arise traveling in opposite directions. If a field $\mathscr{E}^{(0)}$ is incident on a plate of thickness a normal to its surface, the amplitudes $\mathscr{E}^{(1)}$ and $\mathscr{E}^{(2)}$, respectively reflected from and transmitted through the plate, are given by the following expressions:

$$\mathscr{E}^{(1)} = \frac{\mathbf{A}(k, \bar{k})}{1 - ick\cdot 2\omega^{-1}\,\mathrm{tg}\,ka} + \begin{pmatrix} k \to \bar{k} \\ \bar{k} \to k \end{pmatrix} - \mathscr{E}^{(0)},$$

$$\mathscr{E}^{(2)} = \frac{\mathbf{A}(k, \bar{k})}{\cos ka - ick\cdot 2\omega^{-1}\sin ka} + \begin{pmatrix} k \to \bar{k} \\ \bar{k} \to k \end{pmatrix}, \tag{2}$$

*$[\mathbf{k}\mathscr{E}] = \mathbf{k} \times \mathscr{E}$.

where

$$A_\xi(k, \bar{k}) = \frac{(4\pi i\omega\bar{\sigma}_{\xi\xi}/c^2 - \bar{k}^2)\,\mathscr{E}_\xi^{(0)} + (4\pi i\omega\bar{\sigma}_{\xi\eta}/c^2)\,\mathscr{E}_\eta^{(0)}}{k^2 - \bar{k}^2},$$

$$A_\eta(k, \bar{k}) = \frac{k^2 - 4\pi i\omega\bar{\sigma}_{\xi\xi}/c^2}{4\pi i\omega\bar{\sigma}_{\xi\eta}/c^2} A_\xi(k, \bar{k})$$

are real for real values of $\mathscr{E}_\xi^{(0)}$ and $\mathscr{E}_\eta^{(0)}$, and do not depend on H and $\omega$. The expression for the amplitudes of the wave transmitted into the plate with, for example, wave vector k, differs from the first term in the formula for $\mathscr{E}^{(2)}$ by the factor $\frac{1}{2}e^{-ika}$.

Because $ck/\omega \sim \omega_n/\Omega \gg 1$ [the expansion in (2) is made in terms of this quantity], the variation of the reflected field with $\omega$ and H, which is determined by the factor $C = (1 - ick \cdot 2\omega^{-1}\tan ka)^{-1}$, is oscillatory. At exact resonance it is necessary to take the attenuation of the waves into account. One possible mechanism of attenuation has already been spoken of; however, in a strong field H the attenuation associated with scattering of the electrons is predominant. It can be taken account of by replacing $\omega$ by $\omega + i/\tau$ in the kinetic equation. In the spectrum of the wave a contribution $i/2\tau$ appears (for $\omega\tau \gg 1$), so that the imaginary part $k''$ is related to the real part $k'$ by the equation $k'' = k'/2\omega\tau$.

The resonances are defined by the condition $ak' = \pi n$ and are equidistant in the inverse field, since $k' \sim H^{-1}$. It is obvious from (2) that the reflection coefficient at resonance for large values of $\omega\tau$ differs significantly from unity. However, in experiment[1] this difference did not exceed 10%. Therefore, by neglecting unity in the expression for C, we obtain the following estimate for the reflection coefficient:*

*ctg = cot, sh = sinh.

$$1 - R \sim \frac{-4\omega}{ck'}\,\mathrm{Im}\,\mathrm{ctg}\,ka + (k \to \bar{k})$$

$$= \frac{(2\omega/ck')\,\mathrm{sh}\,(ak'/\omega\tau)}{\mathrm{sh}^2\,(ak'/2\omega\tau) + \sin^2 ak'} + (k \to \bar{k}).$$

For not too large numbers $\pi n/2\omega\tau \ll 1$, and we have at resonance $1 - R \sim a\omega^2\tau/cn^2$, which corresponds to the observed variation (see Fig. 2 of the paper by Khaĭkin et al.[3]) and gives $\omega\tau \approx 10$.

We draw attention to the polarizing property of the plate. Because when $\cos\theta \ll 1$ we have in one of the waves $k \sim (\cos\theta)^{-1}$, the transmitted wave is linearly polarized if the field H is parallel to the surface of the plate.

The experimentally investigated oscillations have a singularity not described by the formulae given here (the appearance of oscillations of the same frequency, but displaced with respect to the magnetic field by a constant phase; see the same Fig. 2 in [3]). It is possibly related to the geometry of the cavity used.

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1 Khaĭkin, Fal'kovskiĭ, Édel'man, and Mina, JETP **45**, 12 (1963), Soviet Phys. JETP **18**, 10 (1964).

2 É. A. Kaner and V. G. Skobov, JETP **45**, 610 (1963), Soviet Phys. JETP **18**, 419 (1964).

3 Khaĭkin, Édel'man, and Mina, JETP **44**, 2190 (1963), Soviet Phys. JETP **17**, 1470 (1963).

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264