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THEORY OF BREMSSTRAHLUNG IN THE PRESENCE OF A MEDIUM

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Scattering and bremsstrahlung of a fast charged particle in a medium are considered. The probability of scattering accompanied by a given radiation energy loss is calculated with double logarithmic accuracy.

SUMMATIONS of radiation corrections to the scattering processes of high energy electrons have been carried out in a number of papers.[1,2] It has been shown that the scattering cross section for processes accompanied by energy losses in the form of radiated photons which do not exceed a certain threshold value falls off in a characteristic manner as the electron energy increases. The derivation of this result is based on taking into account soft virtual and real photons. However, it is well known that in a medium the emission of such photons at high energies of the scattered particle is suppressed [3]. It is, therefore, of interest to determine to what extent the conclusion of references [1,2] is preserved in the presence of a medium. As we shall see, the effect remains, although it can be significantly weakened. In our calculations instead of summing diagrams we shall utilize a simpler method.

Since the basic effect results from the interaction of the particle with the field of the fast photons the motion of the particle is regarded as being given in the first approximation. Moreover, the interaction of soft photons with the medium may be described macroscopically by introducing the dielectric permittivity ϵ, which in future we shall treat as a real positive quantity that does not explicitly depend on the temperature. We expand the potential of the photon field $A_\mu(x)$ in terms of the creation and annihilation operators $c^+_{\mathbf{k},\mu}$, $c_{\mathbf{k},\mu}$. It can easily be shown that if one uses the gauge in which the Green's function of the electromagnetic field is diagonal one obtains

$$A_i(x) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}\varepsilon}} \times (c_{\mathbf{k},i}e^{i\mathbf{kx}-i\omega_{\mathbf{k}}t} + c^+_{\mathbf{k},i}e^{-i\mathbf{kx}+i\omega_{\mathbf{k}}t}),\quad i=1,2,3,$$

$$A_0(x) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}\varepsilon}} \times (c_{\mathbf{k},0}e^{i\mathbf{kx}-i\omega_{\mathbf{k}}t} + c^+_{\mathbf{k},0}e^{-i\mathbf{kx}+i\omega_{\mathbf{k}}t}). \qquad (1)$$

Here $c_{\mathbf{k},\mu}$ and $c^+_{\mathbf{k},\mu}$ obey the usual commutation relations for the electromagnetic field, while $\omega_{\mathbf{k}} = k/\sqrt{\epsilon}$.

Now utilizing the general formula relating the chronological and the normal products the expression for the scattering matrix

$$S = T\exp\left\{i\int d^4x\, j_\mu(x) A_\mu(x)\right\}$$

can be written in the form [4]

$$S = \exp\left\{-\tfrac{1}{2}\iint d^4x\, d^4y j_\mu(x) D^c_{\mu\nu}(x-y) j_\nu(y)\right\} \times N\exp\left\{i\int d^4x j_\mu(x) A_\mu(x)\right\} \qquad (2)$$

or in the momentum representation

$$S = \exp\left\{-\frac{1}{2(2\pi)^4}\int d^4k j^*_\mu(k) D^c_{\mu\nu}(k) j_\nu(k)\right\} \times N\exp\left\{\frac{i}{\sqrt{V}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}\varepsilon}}\left[j^*_i(\mathbf{k},\omega_{\mathbf{k}})c_{\mathbf{k},i} + j_i(\mathbf{k},\omega_{\mathbf{k}}) \times c^+_{\mathbf{k},i} - \frac{1}{\sqrt{\varepsilon}}(j^*_0(\mathbf{k},\omega_{\mathbf{k}})c_{\mathbf{k},0} + j_0(\mathbf{k},\omega_{\mathbf{k}})c^+_{\mathbf{k},0})\right]\right\}. \qquad (3)$$

Here $D^c_{\mu\nu}(x-y)$ is defined as the difference of the chronological and the normal products of the pair of operators $A_\mu(x)$ and $A_\nu(y)$. The normal product can be most conveniently chosen in such a way that its average over the ground state of the system is equal to zero. Then $D^c_{\mu\nu}(x-y)$ is the usual causal Green's function for soft photons in a medium at zero temperature [5]. In the momentum representation we have

$$D^c_{ik} = i\frac{\delta_{ik}}{\varepsilon k_0^2 - k^2 + i\delta},\quad i,k=1,2,3;$$

$$D^c_{00} = \frac{i}{\varepsilon}\frac{1}{\varepsilon k_0^2 - k^2 + i\delta},\quad D^c_{i0} = 0. \quad (4) \qquad (4)$$

Taking (4) into account the first exponential factor in (3) can finally be brought to the following form omitting an inessential numerical phase

$$\exp\left\{-\frac{1}{2V}\sum_{\mathbf{k}}\frac{1}{2\omega_{\mathbf{k}}\varepsilon}\left[j_i^*(\mathbf{k},\omega_{\mathbf{k}})\,j_i(\mathbf{k},\omega_{\mathbf{k}}) - \frac{1}{\varepsilon}j_0^*(\mathbf{k},\omega_{\mathbf{k}})\,j_0(\mathbf{k},\omega_{\mathbf{k}})\right]\right\}. \quad (5)$$

By utilizing the scattering matrix so obtained we can now calculate in the usual manner the probability of scattering of a charged particle accompanied by the emission of a definite number of photons. Since $c_{\mathbf{k},\mu}$ and $c^+_{\mathbf{k},\mu}$ commute when either $\mathbf{k}$ or μ are different, we can treat each momentum and polarization separately. After a fairly cumbersome, but in principle straightforward calculation we obtain that the probability of scattering without emission of photons of momentum $\mathbf{k}$ and polarization i averaged over the statistical Gibbs ensemble at a temperature T is equal to

$$\exp\left(-\rho\,\mathrm{cth}\frac{\omega_{\mathbf{k}}}{2T}\right) I_0\left(\rho/\mathrm{sh}\,\frac{\omega_{\mathbf{k}}}{2T}\right). \quad (6)^*$$

Here I_0 is the zero order Bessel function of imaginary argument

$$\rho = |j_i(\mathbf{k},\omega_{\mathbf{k}})|^2/2V\omega_{\mathbf{k}}\varepsilon.$$

In order to obtain the probability of the process of scattering in the course of which no photons from a given set of momenta and polarizations are emitted it is sufficient to multiply together expressions which relate to each momentum and polarization from this set separately.

We must now determine under what conditions it is justified to utilize expression (3) for the scattering matrix. In order to do this we note that it is a limiting case of the general functional solution [6] for the scattering of a particle with a sufficiently large momentum transfer if in the latter we neglect the polarization term and take into account only the contribution of the soft virtual and real photons whose momentum is considerably smaller than the momentum of the scattered particle. In this calculation the total contribution of virtual photons corresponds to the first exponential factor in (3), and the total contribution of real photons naturally corresponds to the second exponential factor. The polarization of the medium was taken into account above by the introduction of ϵ. As Ter-Mikaelyan [3,7] has shown such a description is effective for the process under consideration up to the highest frequencies if the energy of the scattered particle is sufficiently high. The polarization of the vacuum gives only singly logarithmic terms and is therefore not taken into account. As regards the softness of the photons, as will be seen from the final result the upper limit for the momentum of the soft photons in the doubly logarithmic approximation can be taken right up to the particle energy E. The contribution of virtual photons of momentum of the order of or greater than E does not contain double logarithms in the case under consideration and is also not taken into account in our approximation. Thus, in calculations with doubly logarithmic accuracy utilization of (3) turns out to be justified for the description of the whole scattering process.

In accordance with (6) we can now write for the probability of scattering with a sufficiently high momentum transfer in the course of which photons are radiated only up to the frequency ω_0, with $\omega_0 \gg T$,

$$w = \exp\left\{-\frac{1}{(2\pi)^3}\int_{\omega_0}^{E} k^2\,dk\int d\Omega\,\frac{1}{\sqrt{2\omega_{\mathbf{k}}\varepsilon}}\left[j_i^*(\mathbf{k},\omega_{\mathbf{k}})\,j_i(\mathbf{k},\omega_{\mathbf{k}}) - \frac{1}{\varepsilon}j_0^*(\mathbf{k},\omega_{\mathbf{k}})\,j_0(\mathbf{k},\omega_{\mathbf{k}})\right]\right\}. \quad (7)$$

The absence in this expression of a contribution from frequencies $\omega < \omega_0$ follows directly from the unitarity of the S-matrix, if we remember that the motion of the particle is regarded as being given.

To continue with the calculation we must now assume an expression for $\epsilon(\omega)$ and $j_\mu(\mathbf{k})$. We shall assume that $\omega_0 \gg \omega_{at}$, where ω_{at} is the characteristic frequency of the atomic electrons, and we shall take for $\epsilon(\omega)$ its limiting value $\epsilon(\omega) = 1 - 4\pi Ne^2/m\omega^2$. In finding the current $j_\mu(\mathbf{k})$ we shall take the deflection of the particle as a result of scattering to be instantaneous. Then the integral over the angles in (7) can be easily evaluated by using Feynman's parametrization and with doubly logarithmic accuracy we obtain for (7):

$$w = \exp\left\{\frac{e^2}{2\pi^2}\int_{\omega_0}^{E}\frac{d\omega}{\omega}\ln\left(\frac{m^2}{E^2}+\frac{4\pi Ne^2}{m\omega^2}\right)\right\}. \quad (8)$$

From the last expression it can be seen that the effect of the medium is significant for [7]

$$4\pi Ne^2/m\omega_0^2 \gg m^2/E^2. \quad (9)$$

When condition (9) is satisfied we obtain finally with doubly logarithmic accuracy:

$$w = \exp\left\{-\frac{e^2}{\pi^2}\ln\frac{E}{m}\ln\frac{E}{\omega_0}\right\}\exp\left\{\frac{e^2}{2\pi^2}\ln^2\frac{\sqrt{4\pi Ne}\,E}{m^{3/2}\,\omega_0}\right\}. \quad (10)$$

The first factor in (10) provides a cutoff of the cross section in the case of scattering in vacuo [1,2]. The second factor describes the reduction of this effect due to the influence of the medium. The first term in its expansion was obtained by the perturbation theory method in [7]. (The expansion of (10) contains in comparison with reference [7] an additional factor ½.) For reasonable values of

*cth = coth; sh = sinh.

the quantities appearing in (10) the polarization of the medium does not compensate for the falling off of the cross section with energy.

We also note the following circumstance. Depending on the conditions of the experiment it might turn out more convenient to deal not with the limiting frequency ω_0, but with the maximum allowable radiation energy loss ΔE. In this case one must simply replace ω_0 by ΔE in (10). This substitution in (10) introduces an error which lies outside the limit of accuracy of the doubly logarithmic approximation.

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