HYPERNUCLEI WITH TWO Λ PARTICLES AND THEIR DECAY

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The existence of bound states of two Λ particles with nucleons is discussed, and their binding energies are estimated on the basis of global symmetry. It is shown that a correlation should exist between the emission angles of the pions produced in the decay of such systems.

1. Although up to now the existence of hypernuclei containing two Λ particles (produced, for example, upon absorption of Ξ particles by nuclei[1]) has not been proved conclusively by experiment, a few theoretical considerations can be advanced in favor of the realization of bound systems of this type.

We eliminate from consideration the deformation of the core of the hypernucleus by Λ particles, which can only increase the binding energy of the system. Then, introducing the coordinates r_i ($i = 1, 2$) of the Λ particles relative to the center of the core, and excluding the motion of the center of inertia of the hypernucleus, we can write for the Hamiltonian of the system $\Lambda_1 + \Lambda_2$ + core

$$\hat{H} = \hat{H}_0 + V_{\Lambda\Lambda} + \hat{\mathbf{p}}_1\hat{\mathbf{p}}_2/M_0, \qquad (1)$$

where

$$\hat{H}_0 = \sum_{i=1}^{2} (\hat{\mathbf{p}}_i^2/2\mu + V_i(r_i)),$$

$\hat{p}_i$ and V_i are respectively the momentum and potential-energy operators of the i-th Λ particle in the field of the core; M_0 and μ are the mass of the core and the reduced mass of the Λ particles; $V_{\Lambda\Lambda}$—potential of ΛΛ interaction [1)].

For an arbitrary function $\Phi(r_i)$ we have

$$\int \Phi^* (\hat{H} + B_{2\Lambda})\, \Phi d\tau \geqslant 0$$

(where $B_{2\Lambda}$—binding energy of the ground state of the hypernucleus). From this, choosing for $\Phi(r_i)$ that eigenfunction of the Hamiltonian $\hat{H}_0$, which corresponds to the binding energy $2B_\Lambda$, we obtain

$$-\int \psi^2(r_1)\, \psi^2(r_2)\, V_{\Lambda\Lambda}(r_{12})\, d^3r_1\, d^3r_2 < \Delta B, \qquad (2)$$

since

$$\langle \hat{\mathbf{p}}_1\hat{\mathbf{p}}_2 \rangle = 0 \qquad (\Delta B = B_{2\Lambda} - 2B_\Lambda).$$

It can be shown by using the inequality resulting from (2), which is valid when $V_{\Lambda\Lambda}(r) \geq 0$, namely

$$-\Omega\psi^2_{max} < \Delta B \qquad (3)$$

[where $\Omega = \int V_{\Lambda\Lambda}(r)\, d^3r$ and ψ^2_{max}—maximum value of $\psi^2(r)$], that hypernuclei with two Λ particles can exist even in the case of a sufficiently strong repulsion between them. In fact, the condition for the energetic impossibility of the decay of the hypernucleus $X^A_{2\Lambda}$ with emission of a Λ particle is $\Delta B < B_\Lambda$ (B_Λ—binding energy of the hypernucleus X^{A-1}_Λ). An estimate for $He^6_{2\Lambda}$ leads to $\Omega < 15$ MeV-F^3. Account is taken here of the fact that $B_\Lambda = 3.1$ MeV[2] and that the potential in which the Λ particle moves is [3]

$$V_1(r) = \Omega^{(1)}_{\Lambda N} \left\{1 + \frac{R_1^2}{6}\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)\right\} \rho(r), \qquad (4)$$

where $\Omega^{(1)}_{\Lambda N}$ and R_1 are respectively the volume integral of the central spin-independent part of the ΛN interaction and its quadratic radius, $\rho(r) = 4a^{-3}\pi^{-3/2}\exp(-r^2/a^2)$ [4], and the parameter a (with allowance for the correction for the proper dimensions of the proton) is 1.16 F [1,5]. The value of ψ^2_{max} is bounded from above by $\psi^2(0) = \pi/2r_0^3$ for a particle moving in a rectangular well with infinitely high walls of depth $U_0 = (V_1(r))_{max}$. Its radius r_0 was determined from the condition of equality of the energy of the

1)The discussion that follows is based on the assumption that there is no repulsion center in the ΛΛ interactions. However, in the case when the radius of the center r_c is small compared with the dimension R_0 of the Λ-particle orbit, it can be shown that the existence of the center leads to the appearance of additional terms of order $(r_c/R_0)^3$ in the binding energy.

ground state to B_Λ and was found to be 2.1 F.

To estimate the order of magnitude of the energy ΔB we use global symmetry. Since the function of the two Λ particles in the doublet representation[6] is written in the form

$$\Lambda_1\Lambda_2 = \frac{1}{2} Y_1Y_2 + \frac{1}{2} Z_1Z_2 + \frac{1}{2}\Big\{2^{-1/2}\Big[2^{-1/2}\big(Y_1Z_2 + \Sigma_1^+\Sigma_2^-\big) + 2^{-1/2}\big(Y_2Z_1 + \Sigma_2^+\Sigma_1^-\big)\Big] + 2^{-1/2}\Big[2^{-1/2}\big(Y_1Z_2 - \Sigma_1^+\Sigma_2^-\big) + 2^{-1/2}\big(Y_2Z_1 - \Sigma_2^+\Sigma_1^-\big)\Big]\Big\}, \quad (5)$$

we can show that the potential of $\Lambda\Lambda$ interactions, neglecting the Σ and Ξ channels, is equal to the potential of the ΛN interaction in the singlet state and consequently should be short-range (one-pion exchange is forbidden in both cases). For numerical estimates the volume integral of the singlet ΛN interaction was taken to be $\Omega = -510$ MeV-F^3[5]. Taking into account the fact that for a δ-type potential we have in the framework of perturbation theory

$$\Delta B \approx -\Omega \int \psi^4(r)\, d^3r,$$

we obtained $\Delta B \approx 9$ MeV for the case of $He^6_{2\Lambda}$. $\psi(r)$ was approximated here by the function $\exp(-r^2/l^2)$, where l was taken from the calculations for the ordinary hypernuclei.

2. Let us consider the successive meson decay of a light hypernucleus with two Λ particles. In the case when the first to take place is three-particle decay, a correlation can exist between the two decays for nonzero spin of the intermediate hypernucleus, while in the case of two-particle decay it can exist when the spins of the initial and intermediate nuclei differ from zero. Among the nuclei with $Z < 3$, the latter is satisfied only for $H^5_{2\Lambda}$, which decays in accordance with the scheme

$$H^5_{2\Lambda} \to He^5_\Lambda + \pi^-, \quad He^5_\Lambda \to He^4 + p + \pi^-.$$

The hypernucleus decay was calculated in the usual fashion[7,8] on the basis of the impulse approximation, with allowance for the interaction in the final state and under the assumption that each of the decays occurs independently. Taking for the amplitude of the decay of the free Λ particle the expression $s(1 + \beta\sigma\cdot k/k_0)$[8] (where the experimentally-determined parameter[2)] β^2 is equal to 0.13[9] at $k_0 = 101$ MeV/c, k—pion momentum, and s—a constant of no importance for what follows), we obtain for the decay matrix element

$$M = s \int \psi_2^{-*}\chi_2\left(1 + \beta\frac{\sigma k}{k_0}\right)\psi_1\chi_1 d\tau, \quad (6)$$

where ψ_1 and ψ_2 are the spatial and χ_1 and χ_2 the spin parts of the wave functions of the initial and final states (ψ_2^- contains at infinity a plane and a converging wave).

We choose a reference system in which the intermediate nucleus He^5_Λ is at rest. After multiplying the matrix elements of the first and second decays and summing over the intermediate states in the resultant formula for the cross section, summation was carried out over the final spins and averaging over the initial spins. The result was the formula

$$\sigma \sim \sigma_0(k, \varphi)\left[1 - \frac{4\beta^2 pk/k_0^2}{(1 + \beta^2p^2/k_0^2)(1 + \beta^2k^2/k_0^2)}\cos\theta\right], \quad (7)$$

where $\sigma_0(k, \varphi)$—cross section of the second decay[8], p and k—momenta of the mesons of the first and second decays. From the energy balance of the first decay, neglecting the interaction of the Λ particles with each other, we have p = 134 MeV/c, θ—angle between p and k, φ—angle between the proton and the meson of the second decay.

To obtain the correlation between the first and second mesons, we integrate (7) with respect to φ and k. The energy distribution obtained after integrating with respect to φ can, in accordance with [8], have a sharp maximum near the upper threshold. In view of this, the integration in (7) can be reduced to a replacement of k by k_{max} = 95 MeV/c[8], corresponding to the maximum of the energy spectrum. Hence

$$\sigma \sim 1 - 0.47\cos\theta \qquad \beta^2 = 0.13,$$
$$\sigma \sim 1 - \cos\theta \qquad \beta^2 = 0.83.$$

The calculation of the decays

$$H^4_{2\Lambda} \to He^4_\Lambda + \pi^-, \qquad He^4_\Lambda \to He^3 + p + \pi^-$$

under the assumption that the spin of He^4_Λ is equal to unity[3)], leads in analogy with the case of $H^5_{2\Lambda}$ to the expression

$$\sigma \sim \sigma_0(k, \varphi)\left[1 - \frac{8\beta^2 pk/k_0^2}{(3 + 2\beta^2p^2/k_0^2)(1 + \beta^2k^2/k_0^2)}\cos\theta\right]. \quad (8)$$

(An important role in the derivation of (8) is played by the weak dependence of the phase shift of the scattering of the proton by He^3 on the total spin of the system in the low energy region[10]). Recognizing that p = 115 MeV/c and k_{max} = 90 MeV/c[11], we get

[2)]β is assumed real.

[3)]It is usually assumed that the spin of He^4_Λ is zero. In this case, as indicated above, there would be no correlation.

$$\sigma \sim 1 - 0.29 \cos\theta \quad \text{for } \beta^2 = 0.13,$$

$$\sigma \sim 1 - \cos\theta \quad \text{for } \beta^2 = 1.28.$$

We now consider the case when the first decay is a three-particle decay. In the presence of a core with zero spin in the sufficiently heavy hypernucleus, we can apparently neglect the presence of the Λ hyperon in the analysis of the interaction of the hypernucleus with the nucleon produced during the decay. An approximation of this type was used to calculate the decay

$$He^6_{2\Lambda} \to He^5_\Lambda + p + \pi^-, \qquad He^5_\Lambda \to He^4 + p + \pi^-.$$

Choosing a reference frame in which He^5_Λ is at rest, we obtain after summing and averaging over the spins

$$\sigma \sim \sigma_1\sigma_2\left[1 - \frac{4\beta^2 pk/k_0^2}{(1+\beta^2p^2/k_0^2)(1+\beta^2k^2/k_0^2)}\cos\theta\right], \qquad (9)$$

where σ_1 and σ_2 are the cross sections of the first and second decays in the chosen reference frame.

Following integration over the angles of emission of the first and second protons, σ_1 and σ_2 in (9) yield the energy distribution for the first and second mesons. The energy distribution for the second decay is known[8]. The distribution of the mesons of the first decay will be analogous, since the main part of the energy is carried away by the proton and the pion, so that the energy spectrum of the latter depends little on which of the particles the reference frame is fixed in—the He^5_Λ or $He^6_{2\Lambda}$. Putting $p_{max} = k_{max} = 95$ MeV/c, we get from (9)

$$\sigma \sim 1 - 0.35 \cos\theta \quad \text{for } \beta^2 = 0.13,$$

$$\sigma \sim 1 - \cos\theta \quad \text{for } \beta^2 = 1.$$

In conclusion we note that in spite of the small value of β^2, the correlation between the mesons in all three cases is appreciable and should be observed experimentally.

Note added in proof (30 March 1964). After this article went to press we learned of an experimental discovery of hypernuclei containing two Λ particles[12]. An estimate of $\Omega_{\Lambda\Lambda}$ on the basis of the experimental values of B_Λ leads to values[13] that are close to $\Omega_{\Lambda N}$ for the singlet state.

[1] R. H. Dalitz and B. H. Downs, Phys. Rev. **111**, 967 (1958); R. H. Dalitz, A series of lectures delivered at the summer school in theoretical physics in Bangalor, India, 1961.

[2] W. G. G. James, Nuovo cimento Suppl. **23**, 285 (1962).

[3] A. M. Kol'chuzhkin and N. N. Kolesnikov, JETP **38**, 996 (1960), Soviet Phys. JETP **11**, 716 (1960).

[4] Mayer-Berkhout, Ford, and Green, Ann. of Phys. **8**, 119 (1959).

[5] A. M. Kol'chuzhkin and N. N. Kolesnikov, Izv. Vuzov, Fizika No. **4**, 19 (1963), Paper at Fourth All-union Conference on Elementary Particle Theory, Uzhgorod, 1962.

[6] E. Wigner, Proc. Nat. Acad. Sci. USA **38**, 449 (1952). M. Gell-Mann, Phys. Rev. **106**, 1296 (1957). J. Schwinger, Ann. of Phys. **2**, 407 (1957). D. B. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957).

[7] R. D. Hill, Nuovo cimento **8**, 459 (1958).

[8] J. C. Tang, Nuovo cimento **10**, 780 (1958). J. Szymański, Nuovo cimento **10**, 834 (1958).

[9] J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963).

[10] B. H. Brandsen and N. N. Robertson, Phys. Rev. Soc. **72**, 770 (1958).

[11] V. A. Lyul'ka, JETP **39**, 471 (1960), Soviet Phys. JETP **12**, 331 (1960).

[12] M. Danysz et al., Phys. Rev. Lett. **11**, 29 (1963).

[13] R. H. Dalitz, Phys. Lett. **5**, 53 (1963).

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