THE ASYMPTOTIC MESON-NUCLEON SCATTERING AMPLITUDE

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In order to carry out the analytic continuation of the elastic scattering amplitude $F(k', p', k, p)$ of the reaction $p + k \rightarrow p' + k'$ (k, p, k', p', are the momenta of the meson and nucleon, with masses μ and M, respectively, before and after the scattering) one introduces the auxiliary retarded amplitude F^{ret}, which coincides with the physical amplitude in the physical region of the reaction:

$$F^{ret} = i\int d^4x \exp\left\{i\,\frac{k_1+k_2}{2}\,x\right\}\left\langle p'\left|\left[j\left(\frac{x}{2}\right)j\left(-\frac{x}{2}\right)\right]\right|p\right\rangle, \quad x \gtrsim 0. \tag{1}$$

Here $j(x)$ is the current of the meson field in the Heisenberg picture and the integration is in the upper half of the light cone. For the crossed reaction $p - k' \rightarrow p' - k$ the advanced amplitude F^{adv} plays a similar role. In the Breit coordinate system, which is defined by the condition $p' = -p'$, at fixed momentum transfer $t = (p' - p)^2 = -4p^2$:

$$F^{ret}(\omega; t) = t\int d^4x \exp\left[i\left(\omega x^0 - \mathbf{e}\mathbf{x}\sqrt{\omega^2 - \mu^2 + {}^1/_4\, t}\right)\right] \times \left\langle p'\left|\left[j\left(\frac{x}{2}\right)j\left(-\frac{x}{2}\right)\right]\right|p\right\rangle; \quad \omega = k^0, \tag{2}$$

where $\mathbf{e}$ is a unit vector perpendicular to the given vector $\mathbf{p}$. The one-particle matrix elements $\langle p' | j(x/2)j(-x/2) | p\rangle$ in the integrals (cf. [1,2]) are not ordinary functions, but so-called generalized functions or distributions. This is not just a mathematical rigorization, but represents the essence of the problem, for the main problem consists just in the determination of the form of these generalized functions. Bogolyubov, Medvedev, and Polivanov [1] (BMP), using a supplementary assumption about these generalized functions which was not connected with the physics, were the first to prove rigorously that $F^{ret}(\omega; t)$ is an analytic function of ω in the upper half-plane $\operatorname{Im}\omega > 0$ for $0 \geq t > -4\sigma\mu^2$ ($\sigma = M/(M + \mu)$). The BMP proof is very complicated, lengthy, and delicate. Several other authors [2], using the Dyson representation, have modified the BMP proof, but their proofs are still based on the same purely mathematical additional assumptions. The best value for $\sigma = {}^8/_3(M + \mu)/(M - \mu)$ has been obtained by Lehmann [2]. We introduce the asymptotic retarded amplitude 1)

$$F_{\infty}^{ret}(\omega; t) = i\int d^4x\, e^{i\omega(x^0 - \mathbf{e}\mathbf{x})}\left\langle p'\left|\left[j\left(\frac{x}{2}\right)j\left(-\frac{x}{2}\right)\right]\right|p\right\rangle, \quad x \gtrsim 0. \tag{3}$$

It can be seen directly that in the physical region of the reaction the amplitude $F_{\infty}^{ret}(\omega; t)$ asymptotically coincides with $F^{ret}(\omega; t)$ as $\omega \rightarrow \infty$, and hence also coincides with the physical amplitude of the reaction. In other words, for the analysis of the asymptotic behavior of the scattering amplitude in the physical region, one may replace the amplitude by $F_{\infty}^{ret}(\omega; t)$. We introduce the variable $\tau = (x^0 - \mathbf{e}\cdot\mathbf{x})(2)^{-1/2}$; then

$$F_{\infty}^{ret}(\omega; t) = i\int_0^{\infty} e^{i\omega\tau}\psi(\tau; t)\, d\tau, \tag{4}$$

where $\psi(\tau; t)$ is the result of integrating $\langle p' | j(x/2)j(-x/2) | p\rangle$ over the section $\tau = \text{const}$ of the light cone. We do not introduce any additional assumptions about the character of the generalized function $\psi(\tau; t)$. We only require that the causality principle be satisfied, and no action at a distance exist. It follows then that for any τ_1 and τ_2, $0 \leq \tau_1 < \tau_2 \leq +\infty$, the integral

$$\int_{\tau_1}^{\tau_2} e^{i\omega\tau}\psi(\tau; t)\, d\tau \tag{5}$$

will not transmit signals from points lying outside the interval $\tau_1 \leq \tau \leq \tau_2$. For ordinary functions this requirement is automatically fulfilled, but for generalized functions this constitutes a real restriction. One can prove that the most general class of generalized functions which satisfies this requirement is of the form:

$$\psi(\tau) = \sum_{\nu=0}^{\infty}\int_{-0,\,-1^{\nu}}^{\infty} \delta^{(\nu)}(\tau - \xi)\,\frac{d\sigma_{\nu}(\xi)}{\nu!}, \tag{6}$$

where

$$\sqrt[\nu]{s_{\nu}} \rightarrow 0 \quad \text{as} \quad \nu \rightarrow \infty, \qquad s_{\nu} = \int_0^{\infty} |d\sigma_{\nu}(\xi)|. \tag{7}$$

The space of test functions on which these generalized functions are defined as linear continuous functionals, consists of the functions $\varphi(\zeta)$, which are each analytic in some neighborhood of its own of the real positive semiaxis. The distance from the boundary C_{φ} of this neighborhood to the positive semiaxis is positive. At infinity the functions $\varphi(\zeta)$ are bounded. It follows from (6) and (7) that the result of the action of the generalized function $\psi(\tau)$ on the Cauchy kernel $(1/2\pi i)[1/(\zeta-\tau)]$ is a function which is analytic in ζ (off the positive semiaxis):

$$U_\psi(\zeta)=\sum_{\nu=0}^{\infty}\frac{1}{2\pi i}\int_0^\infty\frac{d\sigma_\nu(\tau)}{(\zeta-\tau)^{\nu+1}}. \qquad (8)$$

The Cauchy formula for the function $\varphi(\tau)$, in the test-function space, implies

$$(\psi,\varphi)=\int_{C_\varphi}U_\psi(\zeta)\,\varphi(\zeta)\,d\zeta. \qquad (9)$$

The retarded asymptotic amplitude $F_\infty^{\rm ret}(\omega;t)$ is, for any fixed $t\le 0$, just the Fourier transform of the generalized function $\psi(\tau;t')$, i.e., the value of this functional for the test function $i\exp(i\omega\tau)$, $\operatorname{Im}\omega>0$, belonging to the test function space. Equations (6)—(8) imply that this transform is an analytic function of ω in the upper half-plane $\operatorname{Im}\omega>0$ and increases slower than any exponential function $\exp(\epsilon|\omega|)$, $\epsilon>0$. According to [3] this fact implies that the amplitude $F_\infty^{\rm ret}(\omega;t)$ satisfies a dispersion relation with n subtractions if and only if the corresponding integral along the cut converges. The behavior of the function on the large circle need not be considered at all. The asymptotic advanced amplitude possesses similar properties. We note that just these properties are necessary in order to establish the asymptotic equalities between the cross sections for particles and antiparticles. Therefore all the results obtained in [4] and [5] acquire in terms of the asymptotic amplitudes a more direct character.

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1)For $x_0\to+\infty$, $\langle p'|j(x/2)\,j(-x/2)|p\rangle$ converges to $\langle p'|_{out}(x/2)\,j_{in}(-x/2)|p\rangle$. In Eq. (3) and following, $\langle p'|j(x/2)\,j(-x/2)|p\rangle$ will denote the difference between these two expressions. The Fourier transform of the pure asymptotic amplitude produces the pole terms in the amplitude.

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STUDY OF THE DECAY OF THE K_2^0 MESON INTO THREE NEUTRAL PIONS

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So far there are a few data on $K_2^0\to 3\pi^0$ decays [1,2]. We investigated these decays with the aid of a 570-liter freon bubble chamber [3], which was placed in a neutral-particle beam from the proton synchrotron of the Joint Institute of Nuclear Research. A description of the experiment and preliminary results were reported earlier [4].

Some 50,000 stereo photographs were taken during the irradiation. The possible cases of K^0-meson decay were classified as events with 3, 4, 5, or 6 electron-positron pairs directed approximately towards the same points, and also V-events. As a measure of the convergence of the quanta producing the pairs we chose the maximum distance h from the point of intersection of the trajectories of the two nearest γ quanta to the trajectories of the other γ quanta. The distribution of events with 6, 5, 4, and 3 electron-positron pairs relative to the convergence parameter h is shown in Figs. a, b, and c. In the construction of the histograms we used the largest of the values of h, measured in two projections. The distribution of events with three electron-positron pairs, obtained by the Monte Carlo method, is shown in Fig. d. A comparison of the histograms shown in the figure indicates that there exist definite physical causes that lead to the appearance of three and more electron-positron pairs whose vertices are directed approximately towards the same point.