# THEORY OF THE INTERACTION BETWEEN CHARGED PARTICLES AND A NONEQUILIBRIUM PLASMA

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The interaction of a charged particle with a nonequilibrium plasma is investigated. It is shown that the particle energy losses due to the excitation of plasma oscillations can become anomalously large as the plasma approaches an unstable state.

It is well known that critical fluctuations [1,2] (see the review in [3]) i.e., anomalously large fluctuations, can arise in a nonequilibrium plasma that is almost unstable. These critical fluctuations lead to an anomalously large cross-section for the scattering of light in such a plasma [1] and to anomalously large coefficients for the scattering of longitudinal waves and conversion of such waves into transverse waves.[4]

In the present work we investigate the interaction between charged particles and a nonequilibrium plasma. It is shown that the particle energy losses due to the excitation of collective plasma oscillations can be anomalously large 1) if the plasma is almost unstable.

The probability for transition of a particle from a state characterized by momentum **p** to a state characterized by a momentum **p′** is related to the fluctuations in plasma charge density by the familiar expression

$$dw = \left(\frac{4\pi ez}{\hbar q^2}\right)^2 \langle\rho^2\rangle_{\mathbf{q}\omega} \frac{d\mathbf{p}'}{(2\pi\hbar)^3}, \qquad (1)$$

where $\langle \rho^2 \rangle$ is the correlation function for the charge density, $\hbar\mathbf{q} = \mathbf{p}' - \mathbf{p}$ and $\hbar\omega = (p'^2 - p^2)/2\mu$ are the changes in particle momentum and energy, ez is the particle charge and $\mu$ is the particle mass.

For a plasma consisting of hot electrons moving with respect to cold ions, the charge density correlation function in the frequency region $q(T_i/M)^{1/2} \ll \omega \ll q(T/m)^{1/2}$ is given by [1]

$$\langle\rho^2\rangle_{\mathbf{q}\omega} = \frac{1}{2}\left(\frac{Tq^2}{\Omega}\right)^2 \frac{(qs)^2}{m\,|\omega - \mathbf{qu}|}\,\delta(\omega^2 - q^2s^2), \qquad (2)$$

where T and $T_i$ are the electron and ion temperatures and m and M are the electron and ion masses, **u** is the mean directed velocity of the electrons, $\Omega = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency and $s = (T/M)^{1/2}$ is the sound speed (T is the sum of electron and ion temperatures).

Using Eqs. (1) and (2) we can estimate the energy lost by the particle per unit time in the excitation of sound waves:

$$dP = \left(\frac{ezT}{\Omega}\right)^2 \frac{(qs)^2}{m}\left\{\frac{\delta(\mathbf{qv} + qs + \hbar q^2/2\mu)}{qs + \mathbf{qu}} - \frac{\delta(\mathbf{qv} - qs + hq^2/2\mu)}{qs - \mathbf{qu}}\right\}\frac{d\mathbf{q}}{2\pi\hbar}, \qquad (3)$$

where **v** is the particle velocity. The first term in this expression describes the induced radiation and the second describes the absorption of waves by the particle. (We are interested in plasma states that are nearly unstable, in which case the number of sound waves is large; hence we can neglect the additional term in Eq. (3) that takes account of the spontaneous emission which is independent of the radiation.)

Carrying out the integration over angle in Eq. (3) we can find the intensity of the Cerenkov radiation per unit frequency interval $dP/d\omega$. This quantity is found to be anomalously large if $v\cos\theta_0 \approx u \approx s$ ($\theta_0$ is the angle between v and u); in this case

$$\frac{dP}{d\omega} = \left(\frac{ezT\omega}{\Omega s^2}\right)^2 \frac{\omega s}{m\mu\,(v\cos\theta_0 - u)^2}. \qquad (4)$$

Integrating Eq. (4) over the frequency we determine the particle energy loss per unit time due to the excitation of sound waves:

$$P = \left(\frac{ezT}{2\Omega\tilde{a}^2}\right)^2 \frac{s}{m\mu\,(v\cos\theta_0 - u)^2}, \qquad (5)$$

where $\tilde{a}$ is equal to several Debye radii. If $(v\cos\theta_0 - u)^2 < \frac{1}{4} uv(m/\mu)(a/\tilde{a})^4$ we have $P > P_0$ where $P_0 = (ez\Omega)^2/v$ is the magnitude of

1)The interaction of particles with a plasma in which an external source produces the nonequilibrium wave distribution has been investigated by Tsytovich.[5]

the particle energy loss due to binary collisions neglecting the Coulomb logarithm).

It is evident that $dP/d\omega$ and P can become infinite only within the framework of the linear theory that has been used to obtain Eq. (2) for the fluctuations in charge density. Actually, however, nonlinear effects must lead to saturation of the critical fluctuations; as a result the intensity of the Cerenkov radiation remains finite. Hence, Eqs. (4) and (5) [and also Eqs. (8) and (10)] do not apply at very small values of $\mathbf{v}\cdot\mathbf{u} - u^2$ in which case the critical fluctuations giving the particle energy loss become so large that their proper description requires a nonlinear theory. In the present work we limit ourselves to the linear theory and do not attempt to find any limitation on the growth of the critical fluctuations nor to determine their ultimate amplitudes.

We now consider the interaction of a particle with a plasma which is traversed by a compensated beam of charged particles. It is assumed that the beam velocity u is greater than the thermal velocity of the plasma electrons but that the beam temperature $T_1$ is high enough so that the damping of the plasma Langmuir oscillations is determined primarily by the interaction with the beam electrons. Under these conditions, the charge density correlation function in the high-frequency region ($\omega \gg q(T/m)^{1/2}$) is of the form [2]

$$\langle \rho^2 \rangle_{\mathbf{q}\omega} = \frac{1}{2} T_1 \frac{q^2\Omega^2}{|\omega - \mathbf{qu}|} \delta(\omega^2 - \Omega^2). \tag{6}$$

It is well known that the existence of a hot beam always leads to the growth of short wave Langmuir oscillations that satisfy $|\mathbf{q}\cdot\mathbf{u}| > \Omega$. If nonlinear effects are neglected in the interaction between fluctuations Eq. (6) can be used in the wave-vector region in which plasma oscillations are still nongrowing. The energy lost by the particle per unit time in the excitation of Langmuir oscillations can be found from Eqs. (1) and (6):

$$dP = \left(\frac{ez\Omega}{q}\right)^2 T_1 \left\{ \frac{\delta(\mathbf{qv} + \Omega + \hbar q^2/2\mu)}{\Omega + \mathbf{qu}} - \frac{\delta(\mathbf{qv} - \Omega + \hbar q^2/2\mu)}{\Omega - \mathbf{qu}} \right\} \frac{d\mathbf{q}}{2\pi\hbar}. \tag{7}$$

This formula takes proper account of the interaction of the particle with nongrowing oscillations i.e., modes for which $|\mathbf{q}\cdot\mathbf{u}| < \Omega$.

Carrying the integration over angle in Eq. (7) we find the intensity of the Cerenkov radiation per unit wave number interval q. In the simplest case, in which $\mathbf{u} \parallel \mathbf{v}$, we have

$$\frac{dP}{dq} = \frac{(ez)^2 T_1 qu}{\mu (v - u)^2}. \tag{8}$$

Integrating Eq. (8) over q, we find the particle energy lost in the excitation of Langmuir oscillations

$$P = \left(\frac{ez}{\tilde{a}}\right)^2 \frac{T_1 u}{2\mu(v-u)^2}. \tag{9}$$

If the particle velocity v and beam velocity u are approximately the same P is very large and can exceed the energy loss due to binary collisions as well as the energy loss due to excitation of Langmuir oscillations in the absence of the beam. This is the case when

$$(v-u)^2 < u^2 (mT_1/\mu T)(a/2\tilde{a})^2.$$

If the angle $\theta_0$ between **v** and **u** is different from zero and $q \approx \Omega/u$, then

$$\frac{dP}{dq} = \frac{(ez)^2 T_1 qu}{\mu (v\cos\theta_0 - u)^2}. \tag{10}$$

In contrast with Eqs. (5) and (9) the expression for the particle energy loss due to the excitation of Langmuir oscillations P at $\theta_0 \sim 1$ does not contain the resonance denominator $(\mathbf{u}\cdot\mathbf{v} - u^2)^2$. In this case the value of P is proportional to a large parameter that characterizes the cutoff of the amplitude of the critical fluctuations by nonlinear effects; thus, in this case P is large.

The difference in the structure of the expressions for the particle energy loss due to excitation of Langmuir oscillations when $\theta_0 \sim 1$ and when $\theta_0 \ll 1$ is related to the fact that the particle interacts strongly with modes for which $\mathbf{q}\cdot\mathbf{v} \approx \Omega$ whereas the amplitude of the oscillations is anomalously large for the condition $\mathbf{q}\cdot\mathbf{u} \approx \Omega$. Hence, it is only when the conditions $u \approx v$ and $\theta_0 \ll 1$ are satisfied that all modes with which the particle interacts will be anomalously large, regardless of their wave vectors.

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