

DETACHMENT OF ELECTRONS IN SLOW COLLISIONS BETWEEN NEGATIVE IONS AND ATOMS

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The nonstationary quantum problem in which the bound state of a quantum system at first approaches and then merges for a certain time with the continuous energy spectrum, due to the slow variation of an external parameter, is considered in the general form. Such conditions may occur during collisions between negative ions and atoms. The energy distribution of the emitted particles (electrons) is obtained and the probability that the system remain in the bound state (i.e., no electron is detached from the negative ion) is determined.

1. INTRODUCTION

THE nonstationary problem in which, under slow variation of the external parameters of the system, two discrete energy states turn out to be close to each other and a transition occurs between them, has been thoroughly analyzed in quantum mechanics. To calculate the probability of such a transition it is possible to employ in different cases the formulas of Landau and Zener^[1], Landau and Teller^[2], Zener and Rosen^[3], and others^[4].

None of these formulas are usable, however, if, following the change in the external parameters, the discrete energy level comes close to and then merges with the continuous spectrum of the energy operator. In this case we deal with interaction not between two but between an infinite number of energy levels, so that attempts to consider such a process in the two-level approximation and to use, for example, the Landau-Zener formula are incorrect. This can be seen at least from the fact that in this case there is no limitation preventing the merging of the levels, similar to the Neumann-Wigner theorem for discrete levels. A specific example of such a merging process are collisions of a negative ion A^- with an atom B , when for a distance R between nuclei, smaller than some critical value R_0 , the bound state of the negative molecular ion AB^- vanishes. This is known to take place, for example, in $H^- + H$ collisions, since there is no negative ion of the compound atom He^- .

Hirschfelder et al.^[5] calculated the term of the negative molecular ion H_2^- and found the value of R_0 for which the term H_2^- crosses the term H_2 ,

but the part of the curve given in that paper for H_2^- with $R < R_0$ is meaningless. It is obvious that in a variational calculation of such a term, as the number of varying parameters increases we should simply approach the boundary of the continuous spectrum, that is, to the term H_2 , and the deviation from this term determines only the error made in the choice of the function, and nothing else. There is no justification whatever for assuming that such a curve determines, for example, the position of some quasistationary state against the background of a continuous spectrum.

We note that merging is impossible when atoms or positive ions collide, since the removed electron is in this case in the effective Coulomb field and there is an infinite number of discrete states near the boundary of the continuous spectrum.

It is obvious that when the distance between the nuclei becomes smaller than R_0 , the bound state disappears and the wave function of the weakly bound electron is a wave packet made up of the states of the continuous spectrum, which gradually spreads out. After the collision, with further increase in the distance between the nuclei to $R > R_0$, the bound state is re-established, part of the spread-out wave packet is again "captured" in the produced effective potential well, and the remaining part goes to infinity.

The questions that must be answered by the theory are as follows.

1. What is the probability w that the electron will remain in the bound state, that is, the probability that no ionization will take place?

2. What is the energy spectrum $W(E)$ of the electrons produced during ionization?

2. FORMULATION OF THE PROBLEM AND THE TRANSFORMATION TO THE INTEGRAL EQUATION

In order to proceed to consider the formulated problem concerning the merging, we note that if the discrete state is far from the boundary of the continuous spectrum and the change in the external parameters is slow (low velocity of colliding particles), then the wave function of the system develops adiabatically in time, and no transitions take place. The adiabaticity condition is violated only when the discrete level and the continuous spectrum are quite close to each other. In this case the ionization potential of the system is very low, and the wave function of the weakly bound electron has dimensions that are appreciably larger than the region in which the effective potential differs noticeably from zero. The electron is principally in that part of space in which no forces act whatever, so that its wave function satisfies the Schrödinger condition for a free particle. The presence of an effective potential well of variable depth can be taken into account by specifying the logarithmic derivative of the wave function as a function of t on the boundary of the well or, neglecting the well dimensions, at the origin. (Questions connected with the correctness of the assumptions made here and below, that is, the limits of applicability of the theory, will be discussed in Sec. 5 below.)

We thus arrive at the following formulation of the problem. It is required to solve the Schrödinger equation for the free electron under the assumption that the wave function is spherically symmetrical, that is, the equation

$$\left(-\frac{1}{2} \frac{\partial^2}{\partial r^2} - i \frac{\partial}{\partial t}\right) \psi(r, t) = 0, \quad (1)$$

under the boundary condition

$$\left(\frac{1}{\psi} \frac{\partial \psi}{\partial r}\right)_{r=0} = f(t). \quad (2)$$

The function $f(t)$ is shown in Fig. 1.

When $t \rightarrow \pm\infty$, $f(t) \rightarrow -\alpha < 0$, and the effective Hamiltonian has one discrete eigenvalue $-\alpha^2/2$ and an eigenfunction $\exp(-\alpha r)$. When $f = 0$ merging with the continuous spectrum takes place,

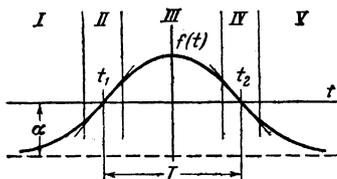


FIG. 1

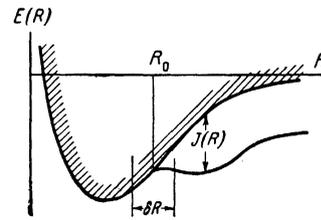


FIG. 2

and when $f \geq 0$, the bound state is missing, and at large f the eigenfunctions of the continuous spectrum are close to the diagonal functions of the free particle, that is, $\psi_k \approx \sin kr$.

It is seen from the foregoing that if we consider, as usual, the terms as functions of the internuclear distance, then the term corresponding to the bound state and the boundary of the continuous spectrum should come in contact at the point R_0 , as shown in Fig. 2; the crossing of the two curves at a certain nonzero angle is impossible. (We are referring all time to Σ -terms—states with identical symmetry.)

Another initial condition, supplementing (1) and (2), is the requirement that as $t \rightarrow -\infty$ the function $\psi(r, t)$ have the form

$$\psi(r, t) = V\sqrt{2\alpha} \exp\{-\alpha r + i\alpha^2 t/2\} \text{ as } t \rightarrow -\infty. \quad (3)$$

If we integrate (1) under conditions (2) and (3), then as $t \rightarrow \infty$ the function assumes the form

$$\psi(r, t) = cV\sqrt{2\alpha} \exp\{-\alpha r + i\alpha^2 t/2\} + R(r, t).$$

The square of the modulus of the coefficient c determines the probability w that the ionization will not occur and that the electron will remain in the bound state. The residual term $R(r, t)$ tends to zero for all r and represents a spreading wave packet. For large t this packet characterizes a free particle and consequently, expanding $R(r, t)$ in a Fourier integral, we obtain as $t \rightarrow \infty$ the momentum distribution of the emitted particles.

It is clear from qualitative considerations that the more abrupt the change in the function $f(t)$, the slower the decrease in the distribution of the emitted particles at large momenta and the faster the spreading of the wave packet. It is also clear that the principal role is played by the behavior of the function $f(t)$ near the merging points, that is, the derivatives $\dot{f}(t_1) = -\dot{f}(t_2)$, and in addition the time $T = t_2 - t_1$ during which there is no bound state. The value of $\dot{f}(t_1)$ can be expressed in terms of the velocity of the colliding particles (more accurately, in terms of $(dR/dt)_{R_0}$) and the derivative $(D^2 J/dR^2)_{R_0}$, where $J(R)$ is the electron binding energy in the system AB^- . These

quantities are thus characteristic parameters of the problem.

To continue the analysis, we change over from the semi-infinite interval $0 \leq r < \infty$ to the infinite interval $-\infty < x < \infty$, and replace r by x in (1). Then the boundary conditions (2) will be automatically satisfied if we introduce in (1) a potential $f(t)\delta(x)$ and consider a solution which is symmetrical in x . Indeed, integrating the equation

$$-\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} - i \frac{\partial \psi}{\partial t} = -f(t)\delta(x)\psi \quad (4)$$

with respect to x in a small interval from $-\epsilon$ to $+\epsilon$ and letting ϵ go to zero, we obtain precisely the condition (2).

We now write down a formal solution of (4), regarding the right half as an inhomogeneity and using the Green's function for the operator $-(1/2)\partial^2/\partial x^2 - i\partial/\partial t$ (which is similar to the Green's function for the heat-conduction equation):

$$G(x-x', t-t') = \frac{1}{\sqrt{2\pi(t-t')}} \exp\left[\frac{i(x-x')^2}{2(t-t')} + i\frac{\pi}{4}\right], \quad t' < t;$$

$$G = 0 \quad t' > t.$$

We obtain

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} e^{-3i\pi/4} \int_{-\infty}^t f(t') \psi(0, t') \times \exp\left[\frac{ix^2}{2(t-t')}\right] \frac{dt'}{\sqrt{t-t'}}. \quad (5)$$

Putting now $x = 0$, we arrive at an integral equation for the function $\Psi(t) = \psi(0, t)$

$$\Psi(t) = \frac{1}{\sqrt{2\pi}} e^{-3i\pi/4} \int_{-\infty}^t \frac{f(t') \Psi(t')}{\sqrt{t-t'}} dt'. \quad (6)$$

If we solve the resultant integral equation and obtain Ψ for specified $f(t)$ then, substituting the solution in the right half of (5), we obtain the unknown function $\psi(x, t)$.

However, Eq. (6) is apparently difficult to solve analytically if the function f has a form of any degree of complexity.

In the simplest case, when $f(t) = -\alpha = \text{const} < 0$, we have the obvious solution $\exp(i\alpha^2 t/2)$. It is easy to verify that in such a case this function actually satisfies the equation, and substitution in (5) gives the function (3).

If $f(t)$ depends linearly on t , then we can readily verify that the solution of (6) can be represented in the form of a contour integral. Yet, as can be seen for example from Fig. 1, it is precisely in the regions of interest to us—in the vicinities of the merging points—that the function $f(t)$

can be approximated in natural fashion by a linear function.

3. SOLUTION OF THE EQUATION WITH A LINEAR APPROXIMATION FOR $f(t)$

We put in (6)

$$f(t) = \beta t$$

and seek the solution of the equation in the form of a contour integral

$$\Psi(t) = \int_C e^{iut} Z(u) du. \quad (7)$$

The form of the integration contour will be made more precise subsequently. Substituting (7) in (6) and proceeding as usual, we obtain for Z the equation

$$Z(u) = -\frac{i\beta}{\sqrt{2u}} Z'(u).$$

Solving this equation we obtain an expression for Ψ :

$$\Psi = A \int_C \exp\left[\frac{i}{3\beta}(2u)^{3/2} + iut\right] du, \quad (8)$$

with both branches of the contour going to infinity in such a way that the exponential under the integral sign decreases.

Substituting the solution (8) in Eq. (5) for $\psi(x, t)$ and carrying out the necessary integrations, we obtain

$$\psi(x, t) = A \int_C \exp\left[\frac{i}{3\beta}(2u)^{3/2} + iut - \sqrt{2u}|x|\right] du.$$

In order to get rid of the ambiguity, we introduce the variable $v = \sqrt{2u}$. We then obtain an especially simple expression for ψ :

$$\psi = A \int_C \exp\left[\frac{i}{3\beta}v^3 - v|x| + i\frac{v^2}{2}t\right] v dv, \quad (9)$$

from which we see directly that ψ is a solution of the Schrödinger equation for the free particle. It is also easy to verify directly, by differentiating (9) with respect to the parameter and integrating by parts, that the boundary condition (2) is actually satisfied.

It remains only to determine the form of the contour C . To this end it is sufficient to consider the behavior of ψ at $x = 0$ and $t \rightarrow -\infty$. When $t \rightarrow -\infty$ we can write in the adiabatic approximation

$$\psi \sim \sqrt{-2\beta t} \exp[\beta t x + i\beta^2 t^3/6]. \quad (10)$$

At the same time the function ψ should de-

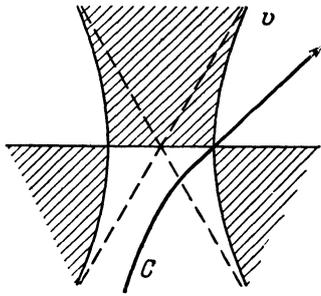


FIG. 3

crease for all x like some power of t as $t \rightarrow \infty$.

The saddle points for the integral (9) have at $x = 0$ values $v = 0$ and $v = -t$. The behavior of the modulus of the exponential under the integral sign is shown in Fig. 3, where the shaded regions are those in which the exponential is larger than unity in absolute value. The right and left saddle points correspond to values 0 and $-\beta t$, depending on which is larger. By calculating the integral by the saddle point method, we find that the integration contour has the form shown in Fig. 3. Then as $t \rightarrow -\infty$ the actual asymptotic form of Ψ is

$$\Psi(t) \sim \sqrt{2\beta t} \exp(i\beta^2 t^3/6), \quad (11)$$

and as $t \rightarrow \infty$ it decreases:

$$\Psi(t) \sim -i\sqrt{2/\beta^3 t^5}.$$

The constant A is equal to $(i\beta\pi)^{-1/2}$. Thus, the normalized wave function has a final form

$$\psi(x, t) = \sqrt{\frac{1}{i\beta\pi}} \int_C \exp\left(\frac{iv^3}{3\beta} + i\frac{v^2}{2}t - v|x|\right) v dv. \quad (12)$$

If $|x| \neq 0$, then the right-hand saddle point shifts downward and to the right, while the left-hand point shifts upward and to the left, and the asymptotic form of the function ψ is likewise easy to investigate. When $t \rightarrow -\infty$ we actually obtain the asymptotic form (10), but only for $x \ll \beta t^2$. For very large x , when $x \gg \beta t^2$, the function decreases more rapidly, in accordance with

$$\psi \sim \exp\left(-\frac{2}{3}\sqrt{-i\beta x^3}\right),$$

which is understandable, for at large values of x the changes that occur at the point $x = 0$ should not come into play instantaneously in noticeable fashion, so that the adiabatic approximation is not satisfied for very large x . When x is large the function "stores," as it were, the information on the value of $f(t)$ during the preceding instant of time, and accordingly decreases more rapidly.

An investigation of the asymptotic behavior for $x \neq 0$ and $t \rightarrow +\infty$ shows that the function actually decreases with increasing t for all values of x in

accordance with a power law. It is also easy to express the function $\psi(x, t)$ in terms of the Airy function (see, for example [6]). We obtain

$$\psi(x, t) = -2\beta^{-1/2} e^{i(\beta^2 t^3 + \pi)/12} \frac{\partial}{\partial x} \left\{ e^{\beta t x/2} \Phi \left[\beta^{1/2} e^{-i\pi/6} \left(x + \frac{i\beta t^2}{4} \right) \right] \right\}.$$

However, in the investigation it is more convenient to use expression (12) for ψ directly in the form of a contour integral.

4. MOMENTUM DISTRIBUTION OF THE EMITTED ELECTRONS AND IONIZATION PROBABILITY

We now consider the momentum representation for the function $\psi(x, t)$ as $t \rightarrow \infty$. To this end we multiply (12) by the normalized function of the free particle $\sqrt{2/\pi} \sin kx$ and integrate with respect to x from zero to infinity. We obtain

$$\varphi(k, t) = \sqrt{\frac{2}{i\beta\pi^2}} \int_0^\infty \sin kx dx \int_C \exp\left(\frac{iv^3}{3\beta} + i\frac{v^2}{2}t - vx\right) v dv.$$

If we now shift the contour C in such a way that it lies completely in the right half-plane, then we can interchange the order of integration and calculate the elementary internal integral with respect to x . We then get

$$\varphi(k, t) = \frac{1}{\pi} \sqrt{\frac{2}{i\beta}} \int_C \exp\left(\frac{iv^3}{3\beta} + i\frac{v^2}{2}t\right) \frac{kvdv}{v^2 + k^2}.$$

If we calculate this integral for $t \rightarrow \infty$ along the contour C which passes through the saddle point $v = 0$, then we obtain a function that tends to zero for all x . Such a contour, however, passes between the poles $v = \pm ik$, whereas the contour C' passes after deformation to the right of both poles. The difference between the two integrals, along the contour C' and the contour C , is the integral over the closed contour which encircles the pole $v = -ik$. Calculating the residue of the integrand, we obtain

$$\varphi(k, t) = \sqrt{\frac{2}{\beta}} k \exp\left\{-\frac{k^3}{3\beta} + \frac{ik^2}{2}t - i\frac{3\pi}{4}\right\} + \sqrt{\frac{2}{i\pi^2\beta}} \int_C \exp\left\{\frac{iv^3}{3\beta} + i\frac{v^2}{2}t\right\} \frac{kvdv}{v^2 + k^2}.$$

The part which is separated upon integration over the residue does not decrease as $t \rightarrow \infty$ and has the form of the momentum wave function of a free particle. It is this part which gives us the sought-for (and quite characteristic) momentum distribution of the electrons:

$$W(k) = \frac{2}{\beta} k^2 \exp\left\{-\frac{2k^3}{3\beta}\right\}. \quad (13)$$

The normalization condition

$$\int_0^{\infty} W(k) dk = 1$$

is automatically satisfied. The qualitative prediction made at the end of Sec. 2 is justified. Indeed, for large β , that is, when $f(t)$ varies very rapidly, the spectrum of the emitted electrons shifts towards the higher energies.

Using a linear approximation for $f(t)$, we obviously cannot calculate the ionization probability directly. Inasmuch as $f \rightarrow +\infty$ as $t \rightarrow +\infty$, the ionization occurs with unity probability, unlike in the real case. The momentum distribution obtained here for the emitted electrons is likewise meaningful only when the ionization probability for the real function is close to unity and the system is almost certain to disintegrate soon after reaching the critical distance R_0 , when the linear approximation is still valid.

In order to estimate nevertheless the probability that the electron will remain in a bound state after the collision, let us call attention to the fact that if we have two solutions, $\psi_1(r, t)$ and $\psi_2(r, t)$, which satisfy (1) and the boundary condition (2), but with different initial conditions, then the integral

$$\int_0^{\infty} \psi_2^* \psi_1 dr \quad (14)$$

does not depend on the time. We see therefore that the sought probability is the square of the modulus of such an integral, if we substitute for ψ_1 a function which satisfies condition (3) as $t \rightarrow -\infty$, and for ψ_2 a function which satisfies the same condition as $t \rightarrow +\infty$. The condition (3) denotes that the particle is completely in a bound state. Thus, the function ψ_1 corresponds to the usual formulation of the problem, and for ψ_2 we stipulate that the particle be in the bound state after the collision.

If we break up the entire interval $-\infty < t < +\infty$ of the variation of the function $f(t)$ into five parts (see Fig. 1), separating the vicinities of the merging points, then for the function ψ_1 we have in region I the usual adiabatic solution, which goes over smoothly in region II into the solution considered here, corresponding to the linear approximation f . In region III the latter goes over into the solution for the free particle. Analogously, for ψ_2 we can construct a solution which goes over in region V into the adiabatic solution, and in region III into the solution for the free particle, and calculate the integral (14) for some value of t in the region III. By the same token we eliminate the

need for continuing the function ψ_1 into the regions IV and V, and the function ψ_2 into the regions II and I.

It is more convenient to calculate this integral in the momentum representation. The function $\varphi_1(k, t)$ will be equal in region III to the first term of formula (26). For $\varphi_2(k, t)$ it is easy to obtain in the same manner, taking into account the shift of the time origin by an amount T , an expression

$$\varphi_2(k, t) = \sqrt{\frac{2}{\beta}} k \exp \left[-\frac{k^3}{3\beta} + \frac{ik^2}{2}(t - T) + i\frac{3\pi}{4} \right].$$

We get

$$w = \left| \int_0^{\infty} \varphi_2^* \varphi_1 dk \right|^2 = \left| \frac{2}{\beta} \int_0^{\infty} \exp \left(-\frac{2k^2}{3\beta} + \frac{ik^3}{2} T \right) k^2 dk \right|^2. \quad (15)$$

Calculating this integral for large T and neglecting the first term in the argument of the exponent, we obtain ultimately

$$w = 2\pi/\beta^2 T^3. \quad (16)$$

5. DISCUSSION OF THE REGION OF APPLICABILITY OF THE OBTAINED FORMULAS AND OF THE ASSUMPTIONS

We first define more precisely the meaning of the parameter β which enters into the final formulas. For $t < 0$ we have in the vicinity of the merging point

$$J = \frac{\beta^2 t^2}{2}, \quad \beta = \left(\frac{d^2 J}{dt^2} \right)_{t=0}^{1/2} = \left(\frac{d^2 J}{dR^2} \right)_{R_0}^{1/2} \left(\frac{dR}{dt} \right)_{R_0}.$$

Formulas (13) and (16) can then be rewritten, going over to arbitrary system of units and to an energy distribution, in the form

$$W(E) = \gamma E^{1/2} \exp \left(-\frac{2}{3} \gamma E^{3/2} \right), \quad \gamma = \frac{2^{3/2}}{h} \left(\frac{dR}{dt} \right)_0^{-1} \left(\frac{d^2 J}{dR^2} \right)_0^{-1/2}; \quad (17)$$

$$w = \frac{2\pi h}{T^3} \left(\frac{d^2 J}{dR^2} \right)_0^{-1} \left(\frac{dR}{dt} \right)_0^{-2}. \quad (18)$$

The quantity $(dR/dt)_0$ is the projection of the velocity of relative motion of the nuclei on the line joining them at the instant when the level merges with the continuous spectrum. For different estimates it can be replaced by V —the velocity of relative motion of the nuclei at infinity.

Let us proceed now to estimate the limits under which the theory is applicable. The distribution (13) has a maximum at $k = \beta^{1/3}$, which corresponds to an energy $\beta^{2/3}/2$. It is obvious that the particle can be regarded as free only in the case when the quantity $f^2/2$ with $f > 0$, analogous to the energy of the bound state with $f < 0$, is large compared with $\beta^{2/3}/2$, that is,

$$\frac{1}{2} f^2 = \frac{1}{2} \beta^2 t^2 \gg \frac{1}{2} \beta^{3/2}, \quad \beta^2 t^3 \gg 1.$$

The linear approximation for f should be valid up to the time \pm defined by this inequality. Replacing t by $\delta R/V$, where δR is the interval in the vicinity of R_0 in which the linear approximation is valid, we obtain

$$\left(\frac{d^2J}{dR^2}\right)_0 \frac{(\delta R)^3}{hV} \gg 1 \quad (19)$$

—a condition analogous to the known Massey adiabatic criterion.

From the very same condition it follows that in those cases when the theory is applicable the probability w is small, and also that the integral (15) can be calculated by the saddle point method. When $\beta^2 T^3 \sim 1$ the theory is not applicable, and the ionization probability can essentially differ from unity.

We note that the limitation indicated here on the side of large V is entirely connected with the approximate method used to solve the problem formulated in Sec. 2. If we solve this problem, for example, numerically, then for a real function $f(t)$ (and for a Volterra type equation such a solution can be obtained by the method of steps), we can get rid of this notation at the expense of complicating the calculations.

Another essential limitation is connected with the replacement of the system of the two atoms by a point singularity at the origin. It is obvious that such a replacement is valid only for the momenta corresponding to a wavelength which is appreciably larger than the dimensions of the system of the colliding atoms. We thus arrive at the inequality

$$\beta^{1/2} = m^{1/2} (hV)^{1/2} (d^2J/dR^2)_0^{1/2} \ll 2\pi h/R,$$

or

$$V^{1/2} \ll 2\pi R^{-1} m^{-1/2} h^{1/2} (d^2J/dR^2)_0^{-1/2}, \quad (20)$$

where R is the effective dimension of the system.

This limitation is quite important, because the velocity enters into the inequality in the $1/3$ power. Therefore even for very small velocities the left half can be only a few times smaller than the right half.

We can expect, however, that even under those conditions when this inequality is not satisfied, the energy distribution of the emitted electrons will be described by formula (17) at low energies, whereas the position of the maximum and the behavior at large energies will be determined by the finite dimensions of the colliding-particle system.

Finally, there exists an obvious limitation con-

nected with the classical analysis of the nuclei of the colliding atom and ion, on the side of low velocities. It is necessary here that the kinetic energy of relative motion of the atom be noticeably larger than the energy of detachment of the electron from the negative ion. Only in this case can we ascribe a definite relative velocity V to the nuclei in the vicinity of the merging point R_0 .

6. CONCLUSIONS

It is difficult to apply the results obtained directly to an interpretation of experimental data, since apparently there are no data on the energy of the electrons emitted in the reaction $A + B^- \rightarrow A + B + e$.

In addition, the quantity $(d^2J/dR^2)_0$, which enters into the theory, has not been calculated even for the simplest case $H^- + H$. Nonetheless, the theory considered makes it possible to make the following general predictions.

1. The average momentum of the emitted electrons decreases quite slowly (as $V^{1/3}$) with decreasing velocity of the colliding particles. This circumstance essentially limits the applicability of the theory, but the violation of the applicability conditions can lead (when $V \lesssim 1$) only to a shift of the average momentum to a value $k_0 = 2\pi h/R$, that is, at any rate it cannot lead to very small values of the momentum.

2. For small momenta k the distribution of the emitted electrons is isotropic and is proportional to the square of the momentum (energy). The momentum distribution for large k will follow formula (13) up to $k \sim k_0$.

3. The ionization probability is large if $\beta^2 R_0^3/V^3$ is much larger than unity. In this case the ionization cross section is determined by the geometrical cross section of the target of radius R_0 [7]. On the other hand, if this quantity is smaller than unity, then the theory is not applicable, the ionization probability for $R < R_0$ decreases, the adiabatic analysis is not suitable, and an essential contribution to the cross section is made also by the far collisions, in which the distance between the atoms does not decrease to R_0 .

4. The problem considered here, involving the "pushing out" of the bound state of a quantum system into the continuous spectrum by slow variation of the external parameter, is quite general in character. It is quite possible that the results obtained can be used in other cases.

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