SOME FEATURES OF THE THERMOMAGNETIC EFFECTS IN FERROMAGNETIC METALS

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The normal thermoelectric power and the Nernst coefficient of ferromagnetic metals are considered for weak and strong magnetic fields when the Larmor notation frequency of the conduction electrons, ω, is respectively smaller and greater than the collision frequency $1/\tau$ of these electrons. It is shown that the special properties of the electron-magnon collision operator, referred to in earlier work,[1] lead to a violation of certain universal properties (which are characteristic of nonferromagnetic metals) of thermomagnetic coefficients.

1. INTRODUCTION

We showed earlier [1] that the electron-magnon collision operator of ferromagnetic metals has special symmetry properties considered as a function of the energy variables. Expressed in terms of the relaxation time these properties mean that this time is a rapidly varying function of $(\varepsilon - \zeta)$ (ε, and ζ are, respectively, the energy and chemical potential of electrons) in the r range $|\varepsilon - \zeta| \leq T$ (T is the temperature). Such properties of the collision operator lead to the possibility of the appearance of an electric current (a thermal current) in the presence of a temperature gradient (an electric field) in the zeroth-order approximation with respect to the degeneracy parameter T/ζ.

The present paper investigates the influence of these special features of the electron-spin wave collision operator on the thermomagnetic effects in weak ($\omega\tau \leq 1$) and strong ($\omega\tau \gg 1$) magnetic fields. However, it is assumed that $\mu H \ll T$ (μ is the effective magneton) and therefore the spin-wave spectrum is independent of the magnetic field. For this condition, the transport equation solution is of such a form that the quantity which plays the role of the relaxation time is also independent of the magnetic field.

It is known that in ferromagnets the galvanomagnetic and thermomagnetic coefficients consist of a normal component, due to the direct effect of the magnetic field on conduction electrons, and an anomalous component, due to the relativistic spin—orbital effect of the magnetization on these electrons. A theoretical comparison of the order of magnitude of these two components is very difficult and we can only say that, on cooling, the ratio of the anomalous to the normal component decreases.[2] On the other hand, there are no reliable experimental data on the thermomagnetic properties of ferromagnetic metals. In the present work, we restrict ourselves to the calculation of the normal part of the thermoelectric power and of the Nernst coefficient, which means that our results can be compared with experiment only under conditions such that the normal part can be

When $\omega\tau \ll 1$ (as in [1]), it is interesting to consider the case when the scattering of conduction electrons (which determines the electrical resistance) is mainly due to causes other than magnons. However, if only the non-magnon scattering is allowed for, we obtain the thermoelectric power $\alpha_{\parallel}$ and the Nernst coefficient $\alpha_{\perp}$ only in the first-order approximation with respect to the degeneracy parameter, although weak scattering on spin waves gives $\alpha_{\parallel}$ and $\alpha_{\perp}$ in the zeroth-order approximation with respect to the degeneracy parameter. It follows, therefore, that the spin-wave scattering may be important. If electrons are scattered mainly on defects the thermoelectric power [1] and the Nernst coefficient are inversely proportional to the first and second powers of defect concentration, respectively (instead of the zeroth and first powers in the case of non-magnon scattering).

In strong magnetic fields ($\omega\tau \gg 1$), the thermomagnetic coefficients $\alpha_{\parallel}$ and $\alpha_{\perp}$ of ferromagnetic metals exhibit the following features.

1) Although allowance for collisions in the

calculation of $\alpha_{\|}$ is important only in the second approximation with respect to the small parameter $(\omega\tau^{-1})$, the part of $\alpha_{\|}$ due to collisions appears in the zeroth-order approximation with respect to the degeneracy parameter and therefore it may be comparable to the part calculated without allowance for collisions in the first-order approximation with respect to the degeneracy parameter. The thermoelectric power will then depend strongly on the magnetic field intensity (without allowance for collisions with magnons, the value of $\alpha_{\|}$ is independent of the magnetic field).

2) For the usual types of scattering, $\alpha_{\perp}/\alpha_{\|} \approx (\omega\tau)^{-1} \ll 1$. When the collisions of electrons with magnons are allowed for, this relationship is not satisfied. Then the Nernst coefficient may be comparable with or greater than the thermoelectric power.

3) For the usual types of scattering, the product $(\alpha_{\perp}/\alpha_{\|})(\sigma_{\perp}/\sigma_{\|})$ in independent of temperature and magnetic field. This is not true if the scattering of electrons on magnons is allowed for.

2. GENERAL RELATIONSHIPS

In the isotropic case, in the presence of a magnetic field **H** along the z axis, a temperature gradient ∇T along the x axis and an electrical field **E** in the x-y plane, the components of the electrical current j and of the thermal current **Q** (when allowance is made for Onsager's relationships [3]) are

$$\begin{aligned} j_x &= \sigma_{\|}E_x + \sigma_{\perp}E_y - \beta_{\|}\,\partial T/\partial x, \\ j_y &= -\sigma_{\perp}E_x + \sigma_{\|}E_y - \beta_{\perp}\,\partial T/\partial x, \\ Q_x &= T\beta_{\|}E_x - T\beta_{\perp}E_y - \varepsilon_{\|}\,\partial T/\partial x, \\ Q_y &= T\beta_{\perp}E_x + T\beta_{\|}E_y - \varepsilon_{\perp}\,\partial T/\partial x, \\ j_z &= 0, \qquad Q_z = 0. \end{aligned} \tag{2.1}$$

If $j = 0$, then a thermoelectric field appears

$$E_x = \alpha_{\|}\,\partial T/\partial x, \qquad E_y = \alpha_{\perp}\,\partial T/\partial x; \tag{2.2}$$

$$\alpha_{\|} = \frac{\beta_{\|}\sigma_{\|} - \beta_{\perp}\sigma_{\perp}}{\sigma_{\|}^2 + \sigma_{\perp}^2}, \qquad \alpha_{\perp} = \frac{\beta_{\|}\sigma_{\perp} + \sigma_{\|}\beta_{\perp}}{\sigma_{\|}^2 + \sigma_{\perp}^2}. \tag{2.3}$$

The thermoelectric tensor α is, according to Eq. (2.3), related to the tensors σ and β, and to find the latter it is sufficient to calculate the electrical and thermal currents on the application of an electric field, for example, along the x-axis direction, and in the absence of a temperature gradient.

The electron distribution function in the presence of an electric field differs from the equilibrium Fermi function $n_0(\varepsilon)$, so that

$$\Delta n_{\pm} = n_{\pm} - n_0(\varepsilon) = -\,\mathbf{p}\mathbf{u}_{\pm}(\varepsilon)\,\partial n_0/\partial\varepsilon \tag{2.4}$$

($\pm$ refer to electrons with different spin directions; the notation is the same as in [1]). If we introduce as variables the energy (ε), the projection of the momentum along the magnetic field (p_z), and the azimuthal angle in the Larmor orbit (φ), then the transport equation for Δn has the form

$$-\frac{\partial n_0}{\partial\varepsilon}eE_x\frac{p_x}{m^*} - \omega\frac{\partial}{\partial\varphi}\Delta n + \left\{\frac{\partial\Delta n}{\partial t}\right\}_{\text{col}} = 0. \tag{2.5}$$

Here $v = \partial\varepsilon/\partial p$ is the electron velocity, $\omega = -eH/m^*c$ is the Larmor frequency, and $m^* = p/v$.

We shall consider separately the cases $\omega\tau \ll 1$ and $\omega\tau \gg 1$ (τ is the relaxation time of conduction electrons).

3. WEAK MAGNETIC FIELDS ($\omega\tau \ll 1$)

In this case $\sigma_{\perp}/\sigma_{\|} \approx \beta_{\perp}/\beta_{\|} \approx \omega\tau \ll 1$. Therefore, from Eq. (2.3), we have

$$\alpha_{\|} = \beta_{\|}/\sigma_{\|}, \qquad \alpha_{\perp} = \alpha_{\|}(\sigma_{\perp}/\sigma_{\|} + \beta_{\perp}/\beta_{\|}). \tag{3.1}$$

For the non-magnon scattering of electrons

$$\alpha_{\|} \approx \frac{1}{e}\frac{T}{\zeta}, \qquad \alpha_{\perp} \approx \frac{1}{e}\frac{T}{\zeta}\omega\tau \approx \frac{\sigma_{\perp}}{\sigma_{\|}}\alpha_{\|}. \tag{3.2}$$

We shall now consider the case when spin waves are among electron scatterers. Since $\omega\tau \ll 1$, the term with the magnetic field in the transport equation (2.5) is small and we shall allow for it by the iteration

$$\Delta n_{\pm} = \Delta n_{\pm 0} + \Delta n_{\pm 1} \tag{3.3}$$

($\mathbf{u}_{\pm} = \mathbf{u}_{\pm_0} + \mathbf{u}_{\pm_1}$). The quantities $\Delta n_{0,1}$ are defined by the equations

$$-\frac{\partial n_0}{\partial\varepsilon}eE_x\frac{p_x}{m^*} + \left\{\frac{\partial\Delta n_0}{\partial t}\right\}_{\text{col}} = 0, \tag{3.4}$$

$$-\omega\frac{\partial}{\partial\varphi}\Delta n_0 + \left\{\frac{\partial\Delta n_1}{\partial t}\right\}_{\text{col}} = 0. \tag{3.5}$$

Since Δn_0 is defined by Eq. (3.4) without bringing in the magnetic field, its solution is known: [1] u_0 is directed along the x axis so that Δn_0 determines $\beta_{\|}$, $\sigma_{\|}$, and also the thermoelectric power $\alpha_{\|}$, which remains the same as in the absence of a magnetic field.

The Nernst coefficient $\alpha_{\perp}$ is related to $\beta_{\perp}$ and $\sigma_{\perp}$, which are calculated with the help of Δn_1. As shown in [1], the symmetry of the electron-magnon collision operator is such that the tensor β may appear even in the zeroth-order approximation with respect to the degeneracy parameter (and not in the first order as in other types of scattering). Nevertheless, as shown in our earlier work, if electrons are scattered only on magnons, the thermoelectric power is found to be of the same order as $\alpha_{\parallel}$ in Eq. (3.2).

It is easily shown that in this case

$$\alpha_{\perp} \approx \omega\tau_s^p\alpha_{\parallel} \approx \sigma_{\perp}\alpha_{\parallel}\ \sigma_{\parallel}, \tag{3.6}$$

where τ_s^p is the momentum relaxation time for electrons scattered on magnons [cf. [1], Eq. (4.10)]. Thus the results, both for the thermoelectric power and for the Nernst coefficient, are the same as for other scatterers.

We shall consider in greater detail the more interesting case when electrons are scattered on spin waves less than on other scatterers. In this case (cf. [1], Sec. 5)

$$u_{\pm 0} = u^d_{\pm 0} + u'_{\pm 0}. \tag{3.7}$$

Here $u_0^d = eE\tau_d/m^*$ (τ_d is the relaxation time for the non-magnon scattering of electrons) is the drift velocity of electrons without allowance for collisions with magnons, and u_0' is the change of this velocity when such collisions are allowed for. Then, although $u_0' \ll u_0^d$, u_0' is not an even function of $x = (\varepsilon - \zeta)/T$ and therefore it produces a thermal current in the zeroth approximation with respect to the degeneracy parameter. This thermal current is due to the quasi-odd part of the drift velocity

$$w'_{-0}(x) = \frac{1}{2}[u'_{-0}(x) - u'_{+0}(-x)],$$

and

$$-\frac{\partial n_0}{\partial x} w'_{-0}(x) = \gamma\frac{\tau_d}{\tau_s}\frac{d}{dp}(pu_0^d)\int L_-(xx')\,dx' \approx \gamma\frac{\tau_d}{\tau_s}u_0^d. \tag{3.8}$$

Substituting Eq. (3.7) into Eq. (3.5), we obtain (u_1 is directed along the y axis):

$$-\frac{u_{\pm 1}}{\tau_d}\left(-\frac{\partial n_0}{\partial x}\right) + \left(\frac{\partial u_{\pm 1}}{\partial t}\right)_s = \omega\frac{\partial n_0}{\partial x}(u^d_{\pm 0} + u'_{\pm 0}). \tag{3.9}$$

Since the term $(\partial u/\partial t)_s$ is small, we can use it in the iteration so that, in analogy with Eq. (3.7), we have

$$u_{\pm 1} = u^d_{\pm 1} + u'_{\pm 1},$$

where

$$u^d_{\pm 1} = \omega\tau_d(u^d_{\pm 0} + u'_{\pm 0}), \tag{3.10}$$

$$-\frac{\partial n_0}{\partial x}u'_{\pm 1}(x) = \tau_d\left(\frac{\partial u^d_{\pm 1}}{\partial t}\right)_s. \tag{3.11}$$

It is easily seen that because $\tau_d/\tau_s \ll 1$, it is sufficient to substitute $u^d_{\pm 1} = (\omega\tau_d)u^d_{\pm 0}$ into Eq. (3.11). Then it is convenient to write down first the result for the quasi-odd part

$$-\frac{\partial n_0}{\partial x}w'_{-1}(x) = \gamma\frac{\tau_d}{\tau_s}\frac{d}{dp}(\omega\tau_d p u_0^d)\int L_-(xx')\,dx'. \tag{3.11'}$$

Since u_0^d is independent of the energy in the zeroth approximation with respect to the degeneracy parameter, then the first term in Eq. (3.10) gives a contribution to $\beta_{\perp}$ only in the first approximation of this parameter (this term is responsible for $\sigma_{\perp} = \omega\tau_d\sigma_{\parallel}$). Thus that part of w^s_{-1} of the drift velocity, which is due to the scattering on spin waves and gives a contribution to $\beta_{\perp}$ in the zeroth approximation with respect to the degeneracy parameter, consists of Eq. (3.11′) and the second term of Eq. (3.10):

$$-\frac{\partial n_0}{\partial x}w^s_{-1}(x) = \left\{\omega\tau_d w'_{-0}(x)\left(-\frac{\partial n_0}{\partial x}\right) + \gamma\frac{\tau_d}{\tau_s}\frac{d}{dp}(\omega\tau_d p u_0^d)\int L_-(xx')\,dx'\right\} \tag{3.12}$$

Comparing Eqs. (3.12) and (3.8), we find

$$\beta_{\perp}/\beta_{\parallel} = 3\omega\tau_d\left(1 - \frac{1}{3}\frac{d(\ln p)}{d(\ln v\tau_d)}\right). \tag{3.13}$$

Since $\sigma_{\perp}/\sigma_{\parallel} = \omega\tau_d$, we find from Eq. (3.1) that

$$\alpha_{\perp} = 4\omega\tau_d\alpha_{\parallel}\left(1 - \frac{1}{4}\frac{d(\ln p)}{d(\ln v\tau_d)}\right), \tag{3.14}$$

where $\alpha_{\parallel}$ is given by Eq. (5.9) in [1]. From Eq. (3.14), we conclude that the usual relationship of Eq. (3.2) is retained between $\alpha_{\perp}$ and $\alpha_{\parallel}$, but the quantities $\alpha_{\perp}$ and $\alpha_{\parallel}$ themselves are different from the corresponding quantities for the "normal" types of scattering. In particular, in the case of scattering on defects, $\alpha_{\parallel}$ is as pointed out in [1]—inversely proportional to the defect concentration; $\alpha_{\perp}$, as indicated by Eq. (3.14), is then inversely proportional to the square of the defect concentration.

4. STRONG MAGNETIC FIELDS ($\omega\tau \gg 1$)

In this case

$$\sigma_{\perp}/\sigma_{\parallel} \approx \omega\tau \gg 1. \tag{4.1}$$

It will be shown that a relationship of the type of Eq. (4.1) is not satisfied by the tensor β if the scattering of electrons by spin waves is important. However, before dealing with this case, it is useful to consider the results for the "normal" types of scattering. In the latter case $\beta_{\perp}/\beta_{\parallel} \approx \omega\tau \gg 1$, so that Eq. (2.3) assumes the form

$$\alpha_{\parallel} = -\beta_{\perp}/\sigma_{\perp},$$
$$\alpha_{\perp} = -(\beta_{\parallel}/\beta_{\perp} + \sigma_{\parallel}/\sigma_{\perp})\alpha_{\parallel} \approx (\omega\tau)^{-1}\alpha_{\parallel} \ll \alpha_{\parallel}. \tag{4.2}$$

The following conclusions can be drawn from the foregoing formulas:

1. The temperature dependence and order of magnitude of the thermoelectric power remain

the same as in the absence of a magnetic field: $\alpha_{\parallel}$ is independent of the magnetic field because

$$\sigma_{\perp}(\beta_{\perp}) \approx (\omega\tau)^{-1}\sigma^{0}(\beta^{0}_{\parallel}),$$

where $\sigma^0(\beta^0_{\parallel})$ is the electrical conductivity (the coefficient $\beta_{\parallel}$) in the absence of a magnetic field.

2. The Nernst coefficient $\alpha_{\perp}$ is inversely proportional to the magnetic field and $\omega\tau$ is inversely proportional to the magnetic field and $\omega\tau$ orders of magnitude smaller than the thermoelectric power.

3. The product of these two ratios $(\alpha_{\parallel}/\alpha_{\perp})(\sigma_{\parallel}/\sigma_{\perp})$ is independent of temperature and magnetic field.

We shall now return to the case of electron scattering on magnons. Its characteristic feature is this: in the absence of other types of scattering (as well as in the absence of any scattering), the thermal current (the tensor β) may appear not in the first but in the zeroth—order approximation with respect to the degeneracy parameter. We shall represent the tensor β in the form

$$\beta_{\parallel} = \beta^{d}_{\parallel} + \beta^{s}_{\parallel}; \qquad \beta_{\perp} = \beta^{0}_{\perp} + \beta^{s}_{\perp}. \tag{4.3}$$

Here, $\beta^{d}_{\parallel}$ is due to all types of electron scattering, except scattering by spin waves; $\beta^{0}_{\perp}$ is the value of $\beta_{\perp}$ calculated without allowance for any collisions; and $\beta^{S}_{\parallel}$ and $\beta^{S}_{\perp}$ are the parts of $\beta_{\parallel}$ and $\beta_{\perp}$ which are related to the scattering by spin waves.

Since β^{S} appears in the zeroth approximation, and β^0, β^{d} in the first approximation with respect to the degeneracy parameter, the former should be allowed for specially in Eq. (2.3). Therefore, we have

$$\alpha_{\parallel} = \alpha^{1}_{\parallel} + \alpha^{s}_{\parallel}, \quad \alpha^{1}_{\parallel} = -\frac{\beta^{0}_{\perp}}{\sigma_{\perp}}, \quad \alpha^{s}_{\parallel} = \frac{\sigma_{\parallel}\beta^{s}_{\parallel} - \beta^{s}_{\perp}\sigma_{\perp}}{\sigma^{2}_{\perp}}; \tag{4.4}$$

$$\alpha_{\perp} = \alpha^{1}_{\perp} + \alpha^{s}_{\perp}, \quad \alpha^{1}_{\perp} = \frac{\beta^{d}_{\parallel}\sigma_{\perp} + \beta^{0}_{\perp}\sigma_{\parallel}}{\sigma^{2}_{\perp}}, \qquad \alpha^{s}_{\perp} = \frac{\beta^{s}_{\parallel}}{\sigma_{\perp}}. \tag{4.5}$$

When $\omega\tau \gg 1$, the collision term in the transport equation (2.5) is small and therefore we shall use it in the iteration. Then

$$\Delta n_{\pm} = \Delta n_{\pm 0} + \Delta n_{\pm 1} + \Delta n_{\pm 2} + \ldots$$

and correspondingly

$$u_{\pm} = u_{\pm 0} + u_{\pm 1} + u_{\pm 2} + \ldots,$$

where

$$-\frac{\partial n_0}{\partial \varepsilon} eE_x \frac{p_x}{m^*} - \omega \frac{\partial}{\partial \varphi}\Delta n_0 = 0, \tag{4.6}$$

so that $u_0 = -cE/H$,

$$-\omega\frac{\partial}{\partial\varphi}\Delta n_i + \left\{\frac{\partial}{\partial t}\Delta n_{i-1}\right\}_{st} = 0 \qquad (i = 1, 2, \ldots). \tag{4.7}$$

All the even approximations to the drift velocity, u_{2n}, are directed along the y axis and determine $\beta_{\perp}$ and $\sigma_{\perp}$; all the odd approximations, u_{2n+1}, are directed along the x axis and determine $\beta_{\parallel}$ and $\sigma_{\parallel}$. From now on, we shall omit the index of the vector component of u_k for the sake of brevity.

As is evident from Eqs. (4.4) and (4.5), $\alpha^{S}_{\perp}$ is determined by a correction to the first approximation with respect to collisions ($\beta^{S}_{\parallel}$), and $\alpha^{S}_{\parallel}$ by the correction to the second approximation ($\beta^{S}_{\perp}$). Moreover, according to Eqs. (4.4) and (4.5)

$$\alpha^{s}_{\parallel} = \alpha^{s}_{\perp}\sigma_{\parallel}/\sigma_{\perp} - \beta^{s}_{\perp}/\sigma_{\perp}, \tag{4.4'}$$

so that it is more convenient to calculate $\alpha^{S}_{\perp}$ first.

5. NERNST COEFFICIENT $\alpha^{S}_{\perp}$, DUE TO THE SCATTERING OF ELECTRONS ON MAGNONS ($\omega\tau \gg 1$)

We should first determine Δn_1 from the equation

$$-\omega\frac{\partial\Delta n_1}{\partial\varphi} + \left\{\frac{\partial\Delta n_0}{\partial t}\right\}_{col} = 0 \tag{5.1}$$

with Δn_0 taken from Eq. (4.6). The collision integral is the sum of the integrals representing $\{\partial\Delta n/\partial t\}_s$ and other scatterers $\{\partial\Delta n/\partial t\}_d$, which in this section will be allowed for by introducing a relaxation time τ_d, so that

$$\left\{\frac{\partial\Delta n}{\partial t}\right\}_{col} = -\frac{\Delta n}{\tau_d} + \left\{\frac{\partial\Delta n}{\partial t}\right\}_s. \tag{5.2}$$

Therefore

$$\Delta n_1 = \Delta n^{d}_{1} + \Delta n^{s}_{1}, \qquad u_1 = u^{d}_{1} + u^{s}_{1},$$

where

$$\Delta n^{d}_{1} = \frac{1}{\omega\tau_d}\frac{\partial n_0}{\partial\varepsilon}p_x c\frac{E_x}{H}, \qquad \text{i.e.,} \quad u^{d}_{1} = -\frac{1}{\omega\tau_d}c\frac{E_x}{H},$$

and Δn^{S}_{1} is given by the equation

$$\frac{\partial}{\partial\varphi}\Delta n^{s}_{1} = \frac{1}{\omega}\left\{\frac{\partial\Delta n_0}{\partial t}\right\}_s, \qquad \left\{\frac{\partial\Delta n}{\partial t}\right\}_s = \frac{\mathbf{p}}{T}\left\{\frac{\partial\mathbf{u}}{\partial t}\right\}_s; \tag{5.3}$$

the form of $\{\partial u/\partial t\}_S$ is given by Eq. (2.9) in [1].

Substituting Eq. (4.6) into Eq. (5.3), we obtain

$$\Delta n^{s}_{\pm 1} = c\frac{E_x}{H}\frac{p_x}{T}(\omega\tau_s)^{-1}\Big\{\mp\gamma\int L_{\pm}(xx')\,dx' \mp \gamma'\int L_{\pm}(xx')(x'-x)\,dx'\Big\}. \tag{5.4}$$

Since the contributions to the electrical and thermal currents are governed, respectively, by the quasi-even and quasi-odd parts of the drift velocity,[1] it is convenient to write down the expressions for them:

$$-\frac{\partial n_0}{\partial x}w^{s}_{+1} = -c\frac{E_x}{H}(\omega\tau_s)^{-1}\gamma'\int L_{-}(xx')(x-x')\,dx', \tag{5.5}$$

$$-\frac{\partial n_0}{\partial x} w^s_{-1} = -c\frac{E_x}{H}(\omega\tau_s)^{-1}\gamma\int L_-(xx')\,dx'. \quad (5.6)$$

From Eqs. (5.5) and (5.6) it is evident that the momentum relaxation time of electrons scattered on magnons is of the order of $\tau_S(\gamma')^{-1}$ for the quasi-even part of the drift velocity, and of order $\sim\tau_S\gamma^{-1}$ for the quasi-odd part (we should point out that $\gamma \approx T_C/\zeta$, $\gamma' \approx T/T_C$). According to [1], the contribution of the quasi-odd part of Eq. (5.6) to the thermal current is $(\gamma')^{-1}$ times greater than the contribution of the quasi-even part of Eq. (5.5), so that the latter can be neglected in the calculation of the thermal current.

Using Eq. (3.1) from [1], and substituting Eq. (5.6), we find the thermal current Q^S associated with the scattering on magnons:

$$Q^s_x = -\frac{2}{3}Tp^3(2\pi^2\hbar^3)^{-1}\frac{|m^*|}{m^*}\int_0^\infty dx\frac{\partial n_0}{\partial x}x$$

$$[w_-(x) - w_-(-x)] = -\frac{\pi^2}{6}c\frac{E_x}{H}T\frac{|m^*|}{m^*}\frac{\gamma}{\omega\tau_s}N \quad (5.7)$$

(N is the conduction electron density). Since $\sigma_\perp = -ecH^{-1}N$, then

$$\alpha^s_\perp = \frac{\pi^2}{6}\frac{1}{e}\frac{|m^*|}{m^*}\frac{\gamma}{\omega\tau_s}. \quad (5.8)$$

Because $\tau_S^{-1} \sim T$, therefore $\alpha^S_\perp \sim T$.

As is easily proved by means of Eq. (4.4), the order-of-magnitude value is

$$\alpha^1_\perp \approx \frac{1}{e}\frac{T}{\zeta}(\omega\tau)^{-1}, \quad (5.9)$$

where $\tau^{-1} \approx (\tau_d^{-1} + \gamma'\tau_S^{-1})$ is the total relaxation frequency of electrons. Comparison of Eqs. (5.8) and (5.9) allows us to conclude that $\alpha^S_\perp$ may be considerably greater than $\alpha^1_\perp$ and that the temperature dependence of $\alpha^S_\perp$ given by Eq. (5.8) is the same as for the "normal" case $\alpha^1_\perp$ given by Eq. (5.9) when the latter quantity is due to the scattering on lattice defects and is characterized by a relaxation time which is independent of temperature.

6. THERMOELECTRIC POWER $\alpha^S_\parallel$ DUE TO THE SCATTERING OF ELECTRONS ON SPIN WAVES ($\omega\tau \gg 1$)

The thermoelectric power $\alpha^S_\parallel$ can be represented in the form [cf. (4.4) and (4.5)]

$$\alpha^s_\parallel = -\beta^s_\perp/\sigma_\perp + \alpha^s_\perp\sigma_\parallel/\sigma_\perp. \quad (6.1)$$

The second term in Eq. (6.1) is calculated by means of a correction to the distribution function in the first-order approximation with respect to collisions, while the calculation of the first term requires iteration of the transport equation (4.7) to the second order, so that we have

$$-\frac{\partial n_0}{\partial x}u_2 = \frac{1}{\omega}\left[\left(\frac{\partial u_1}{\partial t}\right)_d + \left(\frac{\partial u_1}{\partial t}\right)_s\right]. \quad (6.2)$$

Since in the zeroth-order approximation with respect to the degeneracy parameter, the thermal current appears only due to the odd (as a function of x) part of the drift velocity, we shall be interested only in that part of u_2. The quantity u^d is an even function of x, and the operator $(\partial/\partial t)_d$ is even (in the sense of Eq. (3.9) in [1]), so that $(\partial u^d/\partial t)_d$ is also an even function of x. Therefore, it is sufficient to use that part of u_2 which is contained in the expression

$$-\frac{\partial n_0}{\partial x}u_2 = \frac{1}{\omega}\left[\left(\frac{\partial u^s_1}{\partial t}\right)_d + \left(\frac{\partial u^s_1}{\partial t}\right)_s + \left(\frac{\partial u^d_1}{\partial t}\right)_s\right]. \quad (6.3)$$

In subsequent calculations, it is essential to divide $(\partial/\partial t)_d$ into terms representing collisions with phonons $(\partial/\partial t)_f$ and with defects, the latter being represented by a relaxation time τ_D. Similarly,

$$u^d_1 = u^f_1 + u^D_1.$$

Neglecting the small terms of the order of γ or γ' compared with the other terms, and introducing the notation τ^ε_f, τ^p_f for the energy and momentum "relaxation times" of electrons scattered on phonons[1)], the quasi-odd part of the drift velocity ω_{-2} is given by:

$$-\frac{\partial n_0}{\partial x}w_{-2}(x) = \frac{1}{\omega\tau^\varepsilon_f}\int L_f(xx')[w^s_{-1}(x') - w^s_{-1}(x)]\,dx'$$

$$-\frac{1}{\omega\tau_D}w^s_{-1}(x)\left(-\frac{\partial n_0}{\partial x}\right) + \left(\frac{\gamma}{\omega\tau_s}u^D_1 - \frac{2}{\omega\tau_s}w^D_{-1}\right)$$

$$\times\int L_-(xx')\,dx' - \frac{1}{\omega\tau_s}\int L_-(xx')[w^s_{-1}(-x')$$

$$+ w^s_{-1}(x)]\,dx'\ \frac{1}{\omega\tau_s}\int L_-(xx')\left\{\frac{1}{2}\gamma\frac{d(\ln\omega\tau_s)}{d\ln p}[u^f_1(x')\right.$$

$$\left.- u^f_1(x)] - [w^f_{-1}(x') + w^f_{-1}(x)] + \frac{\gamma}{\omega\tau_s}u^f_1(x')\right\}dx'. \quad (6.4)$$

Since $w^{f,D}_{-1} \approx \gamma u^{f,D}_1$, the terms in braces in Eq. (6.4) (i.e., $\omega^{-1}(\partial u^f_1/\partial t)_S$) are of the order of

$$\frac{\gamma}{\omega\tau_s}u^f_1 \approx \frac{\gamma}{\omega\tau_s}\frac{1}{\omega\tau^p_f}u_0.$$

The first term in the right-hand part of Eq. (6.4), which originates from $\omega^{-1}(\partial u^S_1/\partial t)_f$, is of the order of

$$\frac{1}{\omega\tau^\varepsilon_f}w^s_{-1} \approx \frac{\gamma}{\omega\tau_s}\frac{u_0}{\omega\tau^\varepsilon_f}$$

[cf. (5.6)]. Since $\tau^p_f \gg \tau^\varepsilon_f$, the terms in braces

1)The form of L_f[4] is irrelevant in our case; all that is important is that $L_f(x, x')$ are simply numbers which are independent of temperature.

may be neglected as being small corrections of the order of $\tau_f^{\varepsilon}/\tau_f^{p}$. It can be shown that the second and third terms in the right-hand part of Eq. (6.4) are of the order of

$$\frac{1}{\omega\tau_D}\frac{\gamma}{\omega\tau_s}u_0 \approx \frac{1}{\omega\tau_D}w_{-1}^s,$$

and the fourth term is of the order of $w_{-1}^s/\omega\tau_s$. Thus

$$\frac{w_{-2}}{w_{-1}^s} \approx \left(\frac{1}{\omega\tau_D} + \frac{1}{\omega\tau_s} + \frac{1}{\omega\tau_f^{\varepsilon}}\right). \qquad (6.5)$$

The relationship (6.5) means that if electrons are scattered on phonons (magnons), then the ratio of the components of the tensor β in the second approximation to its components in the first approximation contains not $(\omega\tau_f^p)^{-1}$ $((\omega\tau_s^p)^{-1})$, but a much larger parameter $(\omega\tau_f^{\varepsilon})^{-1}$ $((\omega\tau_s)^{-1})$. Since

$$\beta_{\perp}^s/\beta_{\parallel}^s \approx w_{-2}/w_{-1}^s,$$

then, comparing Eqs. (6.1) and (4.5), we find that the first term in Eq. (6.1) is of the order of $\alpha_{\perp}^s w_{-2}/w_{-1}^s$.

Furthermore, since

$$\frac{\sigma_{\parallel}}{\sigma_{\perp}} \approx \left(\frac{1}{\omega\tau_D} + \frac{\gamma'}{\omega\tau_s} + \frac{1}{\omega\tau_f^p}\right),$$

it is easily seen that the second term in Eq. (6.1) is either equal in its order of magnitude to the first term (if the relaxation time for electron scattering on defects is less than the energy relaxation time for electron scattering both on phonons and magnons) or smaller than the first term (if the relaxation time for electron scattering on defects is longer than any one of these times). Thus for the thermoelectric power $\alpha_{\parallel}^s$, we have:

$$\alpha_{\parallel}^s = \left(\frac{J_1}{\omega\tau_D} + \frac{J_2}{\omega\tau_s} + \frac{J_3}{\omega\tau_f^{\varepsilon}}\right)\alpha_{\perp}^s, \qquad (6.6)$$

where $J_{1,2,3}$ are numerical parameters of the order of unity.[2)] The expression (6.6) contains all the information on the dependence of $\alpha_{\parallel}^s$ on temperature and magnetic field and on its order of magnitude (together with Eq. (5.8) of $\alpha_{\parallel}^s$). Since

$$\alpha_{\parallel}^1 \approx T/e\zeta, \qquad (6.7)$$

then in moderately strong magnetic fields $\alpha_{\parallel}^s$ may be comparable with $\alpha_{\parallel}^1$ and then the thermoelectric power $\alpha_{\parallel}$ will depend strongly on the magnetic field (in accordance with the law $\alpha_{\parallel} = A + B/H^2$).

Furthermore, a comparison of Eq. (5.8) with Eqs. (6.6) and (6.7) allows us to conclude that $\alpha_{\perp}$ may be comparable with or even greater than $\alpha_{\parallel}$ (the latter will occur if, in accordance with the above discussion, $\alpha_{\parallel}$ depends strongly on H).

Finally, it is easily seen that the ratios $\sigma_{\parallel}/\sigma_{\perp}$ and $\alpha_{\perp}/\alpha_{\parallel}$ are now essentially different and, therefore, $(\sigma_{\parallel}/\sigma_{\perp})(\alpha_{\parallel}/\alpha_{\perp})$ may depend both on temperature and on the magnetic field. Thus the properties of the thermomagnetic coefficents $\alpha_{\parallel}$ and $\alpha_{\perp}$ listed at the beginning of Sec. 4 do not apply to ferromagnetic metals in strong magnetic fields.

2)They are determined by integral of certain combinations of L_f and L_-, and can be evaluated. However, in view of the limitations of the isotropic energy spectrum model, we present no "exact" numerical formulas.

[1] L. É. Gurevich and G. M. Nedlin, JETP **45**, 576 (1963), Soviet Phys. JETP **18**, 396 (1964).

[2] L. É. Gurevich and I. N. Yassievich, FTT **4**, 2854 (1962), Soviet Phys. Solid State **4**, 2091 (1963).

[3] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Gostekhizdat, 1957.

[4] L. É. Gurevich and G. M. Nedlin, JETP **37**, 765 (1959), Soviet Phys. JETP **10**, 546 (1960).

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146