THE $\pi^+ + p \to \pi + \pi + N$ REACTION NEAR THRESHOLD

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Reactions in which two π-mesons are produced as a result of π^+-meson-proton collision near the threshold are considered. The differential cross sections of these reactions are obtained with an accuracy in terms of the third order with respect to the relative moments of the particles produced.

GRIBOV et al[1-5] proposed a method of determining the scattering amplitudes of unstable particles at zero energy by analysis of reactions with production of three particles near threshold. It was shown that if the corresponding particles do not interact resonantly, the amplitude of such reactions can be represented in the form

$$f(k_{12}k_{13}k_{23}) = \sum_{n=0}^{\infty} f_n(k_{12}k_{13}k_{23}),$$

where k_{il} is the momentum of relative motion of the particles i and l, E is the total kinetic energy for the production of particles, and each function $f_n(k_{12}k_{13}k_{23})$ is a quantity on the order of $E^{n/2}$. An explicit form of the terms up to third order was obtained for neutral particles.

In the first of the cited papers[1], the technique developed for the neutral case is applied to the reactions

$$\pi^+ + p \to \pi^+ + \pi^+ + n, \tag{1}$$

$$\pi^+ + p \to \pi^+ + \pi^0 + p \tag{2}$$

and the amplitudes and the cross sections are obtained accurate to quadratic terms. In the present paper we present expressions for the cubic part of the differential cross section of these reactions.

Since the amplitudes of these reactions do not have a relative phase shift, no terms linear in k_{il} arise in the formula for the cross section (see [1]). It is therefore meaningful to take into account by way of the first correction the third-order terms, particularly since the consideration of the cubic terms does not involve the appearance of new indeterminate constants.

The cubic parts of the probabilities of these reactions can be represented in the following simple form:

$$dW_1^{(3)}/d\Gamma = \tfrac{4}{5}|F_{31}|^2\{\xi_1(k_{13}^3 + k_{23}^3) + \xi_2[k_{13}(k_{12}^2 + k_{23}^2) + k_{23}(k_{12}^2 + k_{13}^2)] + \xi_3[k_{13}(k_{12}^2 - k_{23}^2) + k_{23}(k_{12}^2 - k_{13}^2)]\}$$

for $\pi^+ + p \to \pi^+ + \pi^+ + n$ and

$$dW_2^{(3)}/d\Gamma = \tfrac{1}{10}|F_{31}|^2\{\eta_1 k_{13}^3 + \eta_2 k_{23}^3 + \eta_3 k_{12}(k_{13}^2 - k_{23}^2) + \eta_4 k_{13}(k_{12}^2 + k_{23}^2) + \eta_5 k_{13}(k_{12}^2 - k_{23}^2) + \eta_6 k_{23}(k_{12}^2 + k_{13}^2) + \eta_7 k_{23}(k_{12}^2 - k_{13}^2)\}$$

for $\pi^+ + p \to \pi^+ + \pi^0 + p$.

The quantities F_{31} and k_{il} have the same meaning as in [1].

The coefficients ξ and η expressed in unit Compton wave-length of the pion are

$$\xi_1 = 0.023\,(a_2 + 0.152),$$
$$\xi_2 = -0.018\,(a_2 + 0.333),$$
$$\xi_3 = -0.023\,(a_2 - 0.022);$$
$$\eta_1 = -0.058\,(a_2 - 0.012),$$
$$\eta_2 = -0.039\,(a_2 + 0.766),$$
$$\eta_3 = 0.192\,a_2\,(a_2 + 0.385),$$
$$\eta_4 = 0.006\,(a_2 - 0.200),$$
$$\eta_5 = -0.018\,(a_2 + 0.115),$$
$$\eta_6 = 0.034\,(a_2 + 0.402),$$
$$\eta_7 = -0.111\,(a_2 + 0.051),$$

where a_2—$\pi\pi$-scattering amplitude at zero energy in a state with isotopic spin T = 2; the experimental values used for the pion nucleon scattering lengths in isotopic states of $1/2$ and $3/2$ are:

$$b_{1/2} - b_{3/2} = 0.26, \qquad b_{1/2} + 2b_{3/2} = 0.$$

The expressions for $dW_1^{(3)}/d\Gamma$ and $dW_2^{(3)}/d\Gamma$ satisfy all the symmetry properties indicated in [2]. The total probability of reactions (1) and (2) is obtained accurate to cubic terms by summing the results of the present paper and of [1].

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