INTERSECTION OF VACUUM POLE TRAJECTORIES

I. I. ROĬZEN and D. S. CHERNAVSKIĬ

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 5, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 46, 628-636 (February, 1964)

The possibility of intersection of several vacuum pole trajectories at the point $l = 1$ for $t = 0$ is discussed in connection with the problem of taking into account the contribution of many-particle states in the t-channel. The case of intersection of two trajectories is considered in great detail. It is demonstrated that this permits one to explain the experimental data on π^-p and pp scattering[1] and yields a definite relation between the total cross sections of the processes. A simple example is presented for which a nondecreasing asymptotic for the elastic scattering cross section can be obtained within the framework of the Regge method.

1. The recently obtained experimental data on pp and π^-p scattering[1] contradict the predictions made on the basis of the moving-pole method (MPM) [2,3]. This indicates that not all the assumptions used to obtain the aforementioned predictions are valid. It is obvious that at least some of them call for a review. The approach that seems most natural to us is to retain all the main premises of the MPM and forego the hitherto made assumption that there exists at $t = 0$ [1)] a single extreme-right vacuum pole in the l plane, determining the asymptotic value of the amplitude $A(s, t)$ as $s \to \infty$. In this connection a question worthy of investigation is that of the properties that will be possessed by the elastic scattering if it is assumed that the asymptotic amplitude $A(s, t)$ is determined in the region $|t/m_\pi^2| \ll 1$ not by one pole trajectory $l_0(t)$, but by several trajectories, $l_1(t)$ and $l_2(t)$ which converge to a single point $l_1(0) = l_2(0) = 1$ but which have at this point different slopes, i.e., $l_1'(0) \neq l_2'(0)$. Each pole makes a nonvanishing contribution to the total cross section. The main contribution to the elastic cross section at large s is made by the "slowest" pole. The possible existence of such intersecting trajectories is implied by many considerations which we shall discuss below.

Let us note the following important circumstance. If a pole of higher order is produced when the pole trajectories cross, then the cross sections of the processes will increase logarithmically with the energy. Indeed [2]

$$A^{\pm}(s, t) = \frac{1}{2}\sum_{l=0}^{\nu\leqslant b} (2l+1)(e^{i\pi l} \pm 1) f(l, t) P_l\left(1 + \frac{2s}{t - 4m^2}\right) + \frac{i}{4}\int_{b-i\infty}^{b+i\infty} dl \frac{(2l+1)}{\sin \pi l} (e^{i\pi l} \pm 1) f^{\pm}(l, t) \times P_l\left(1 + \frac{2s}{t-4m^2}\right) \quad (1)$$

and consequently

$$\sigma_{tot}(s) = \frac{16\pi}{s} \operatorname{Im} A(s, 0) = \frac{16\pi}{s} A_1(s, 0) \sim \ln s \quad (\text{for } s \to \infty). \quad (2)$$

For $n > 2$ such a dependence contradicts the Froissart theorem[4] [2)]. There is no contradiction for $n \leq 2$, but nonetheless the logarithmic increase in the cross section with the energy is not realistic. We shall not consider it. The cross sections can become constant when several trajectories cross only if the pole remains simple. We shall consider only this variant in what follows.

We shall attempt to ascertain whether a noncontradictory description of the experimental pp and π^-p scattering[1] is possible within the framework of this variant. We shall use the reaction-matrix (R-matrix) formalism[5,6]. The partial amplitudes of the different processes in the t channel are the elements of the T matrix, which is connected with the R matrix by the relation

$$T(l, t) = [1 - iR(l, t)\rho(t)]^{-1} R(l, t) = [i\rho(1 - iR\rho)]^{-1} + i\rho^{-1}, \quad (3)$$

[1)]We shall hence first use the terminology already established in the papers of Gribov and Pomeranchuk[2,3] without detailed explanations.

[2)]If we assume beforehand that the asymptotic expression has a Regge form, then $n \leq 1$.[7]

where

$$\rho_{ij} = [(t - 4m_i^2)/t]^{1/2}\,\delta_{ij} = \rho_i. \qquad (4)$$

The rank of T or R matrix is determined by the number of possible channels and is consequently equal to the number of branch points of the elements T_{ij} in the t plane[3]. The elements of the T and R matrices are real functions of their variables. In addition, as is well known[5], a unitary T matrix is equivalent to a Hermitian R matrix. The latter circumstance is compatible with the analytic properties of the T matrix in the t plane and denotes that there are no branch points in the right half plane of t for the R matrix elements.

We put

$$i\rho\,(1 - iR\rho) = C = (c_{ij}). \qquad (5)$$

Then

$$T_{ij} = C_{ji}/\det C + i/\rho_{ij} \qquad (6)$$

where C_{ji} is the cofactor of c_{ji}. The second term in the right half of (6) does not depend on l and will be therefore left out[4].

Let the element T_{11} and R_{11} correspond in the t-channel to the pion elastic scattering process, while the elements $T_{1j} = T_{j1}$ and $R_{1j} = R_{j1}$ to processes involving the transition of two pions into other binary states. We investigate the matrix $(\overline{T}_{ij}) = (\overline{C}_{ij}/\det C)$ obtained from (3) by putting $R_{1j} = 0\,(j \neq 1)$ and leaving the remaining elements of the R matrix the same as before. This matrix is of interest for the following reason. First, its elements (except for $\overline{T}_{11}$) have no branch point in the t-plane at $t = 4m_\pi^2$. Second, this matrix corresponds to a set of Feynman diagrams in which there are no two-pion states in the t-channel at all[5]. This matrix thus describes the contribution made to the amplitude by exchange of heavier particles only. The contribution of such processes to the amplitude will not vanish asymptotically, provided when $t = 0$ the elements $\overline{T}_{ij}$ will have a pole at the point $\overline{l}(0) = 1$, i.e., if for $t = 0$ and $l = 1$ we have $C_{11}(0, 1) = 0$. If at the same time a simple (not multiple) root exists for det C at the point $t = 0$ when $l = 1$, then the element $T_{11} = C_{11}/\det C$ has no pole at this point. This variant leads at best to an appreciable predominance of pions and is not realistic.

Thus, the elements of the T matrix (including the element T_{11}) and of the T matrix can have simultaneously poles at $t = 0$ and $l = 1$ only if the zero of det C is of higher multiplicity than the zero of C_{11}. The latter denotes that at the point $l = 1$ for $t = 0$ several pole trajectories of the matrix T intersect. This, in our opinion, can serve as a leading consideration for assuming that also in a real case allowance for the many-particle unitarity condition in the t-channel can lead to an analogous result if it is assumed that the contribution to the amplitude A (s, t) due only to these processes does not vanish asymptotically as $t \to 0$ and $s \to \infty$.

Let us consider the question of the relations that exist between the cross sections in our variant. From the condition of absence of a multiple pole and from formula (6) it follows that

$$\det C = C_0(l, t) \prod_{k=1}^{n} [l - l_k(t)], \qquad (7)$$

$$C_{ij}(l, t) = C_{0ij}(l, t) \prod_{k=1}^{n-l} [l - l_{kij}(t)]. \qquad (8)$$

The dependence on the crossing pole trajectories is written out here in explicit form. It must be noted that when $t = 0$ the number n of the crossing poles cannot be larger than the rank of the R-matrix (see the appendix). This indicates that the variant under consideration (with crossing of several poles) is possible only if account is taken (albeit indirectly) of the higher intermediate states.

Let us represent the elements T matrix in the form

$$T_{ij}(l, t) = \frac{C_{0ij} \prod_{k=1}^{n-1} [l - l_{kij}(t)]}{C_0 \prod_{k=1}^{n} [l - l_k(t)]} = \frac{1}{C_0} \sum_{k=1}^{n} \frac{C_{ij}^{(k)}}{l - l_k(t)}, \qquad (9)$$

$$C_{ij}^{(k)} = \frac{C_{0ij} \prod_{k'=1}^{n-1} [l - l_{k'ij}(t)]}{\prod_{k' \neq k}^{n} [l - l_{k'}(t)]}, \qquad (10)$$

so that

[3] Strictly speaking, the R-matrix formalism is applicable only to binary reactions. To take into account the many-particle intermediate states in the t-channel it is necessary to consider a matrix of infinite rank. This circumstance will apparently not influence the character of the results obtained in the present paper if the series which determine the determinant and the minors of the C-matrix (see below) converge and the number of crossing poles of the T matrix does not increase as the rank tends to infinity.

[4] This is always valid in a region close to the pole in l, if at the same time t does not tend to the threshold of one of the processes (where the corresponding $\rho_{i0} \to 0$). In the region $t \to 0$ which is of interest to us, all the terms $\rho_i^{-1} \to 0$).

[5] This can be readily verified by considering directly a two-channel process (i.e., a matrix of rank two), to which later-type Feinman diagrams correspond.

$$C_{0ij}(t=0,\ l=1)=\sum_{k=1}^{n} C_{ij}^{(k)}(t=0,\ l=1). \quad (11)$$

From the relation [8]

$$C_{ij}C_{i'j'}-C_{ij'}C_{i'j}=M_{iji'j'}\det C. \quad (12)$$

where $M_{iji'j'}$ is the minor obtained by crossing out the rows i and i′ and columns j and j′ from (9), it follows that

$$C_{ij}^{(k)}(l_k(t),t)\,C_{i'j'}^{(k)}(l_k(t),t) - C_{ij'}^{(k)}(l_k(t),t)\,C_{i'j}^{(k)}(l_k(t),t)=0. \quad (13)$$

Relation (13) is completely analogous to the well known relation between the residues [6], except that now it is no longer a relation between cross sections (see the appendix on this matter), since only the quantities C_{0ij} ($l = 1$, t = 0) are related directly with the cross sections.

We shall henceforth be interested in R-matrix elements that describe $\pi\pi$, πp and pp interactions, which we shall denote by $T_{\pi\pi}$, $T_{\pi p}$, and T_{pp} respectively. The associated cofactors of the C-matrix will be denoted by $C_{\pi\pi}$, $C_{\pi p}$, and C_{pp}. For these, relation (13) takes the form

$$C_{\pi\pi}^{(k)}C_{pp}^{(k)}-(C_{\pi p}^{(k)})^2=0. \quad (13')$$

If we denote the contribution to the total cross section of the process from the k-th pole by $\sigma^{(k)}$, then it follows from (13′) that

$$\sigma_{\pi\pi}^{(k)}\sigma_{pp}^{(k)}-(\sigma_{\pi p}^{(k)})^2=0. \quad (13'')$$

We now discuss the situation with pp and π^-p scattering. In [9], where the question of describing pp and π^-p interactions from a different point of view was discussed, it was already shown that two poles are sufficient for the purpose. They should "move" at t = 0 with essentially different "velocities," namely:

$$\alpha = l_2'(0)/l_1'(0) \lesssim 0.1 \div 0.15 \quad (14)$$

and play essentially different roles. In the pp interaction the contribution of the "fast" pole $l_1(t)$ should be predominate, while in π^-p interaction its contribution should be small (or even much smaller) than the contribution of the slower pole $l_2(t)$. This means that when $|t/m_\pi^2| \ll 1$ we have

$$\beta(t)=C_{\pi p}^{(1)}/C_{\pi p}^{(2)}<1, \quad (15)$$

$$\gamma(t)=C_{pp}^{(1)}/C_{pp}^{(2)}\gg 1. \quad (16)$$

From (13′), (15) and (16) it follows that when $|t/m_\pi^2| \ll 1$

$$C_{\pi\pi}^{(1)}/C_{\pi\pi}^{(2)}=\beta^2(t)/\gamma(t)<1/\gamma(t)\ll 1, \quad (17)$$

i.e., in $\pi\pi$-interaction the predominant role should be played by the slowly moving pole.

From (13″), (15), and (16) we can obtain a relation between the total cross sections. It takes the form

$$\sigma_{\pi\pi}\sigma_{pp}=\frac{(1+\gamma(0))(\gamma(0)+\beta^2(0))}{\gamma(0)(1+\beta(0))^2}\sigma_{\pi p}^2 \gtrsim \frac{\gamma}{4}\sigma_{\pi p}^2. \quad (18)$$

In a perfectly analogous manner we can consider both relations between different processes and the case n > 2. When n > 2 the picture does not change qualitatively if there are two distinctly separated groups of poles, some of which "move rapidly" at t = 0 and the others move "slowly."

It must be noted that, in our opinion, a somewhat strange situation occurs in the assumed variant if we consider the relation between the residues at $t \gtrsim 4m_\pi^2$. There both poles have an imaginary part, as can be readily verified on the basis of the unitarity condition

$$C_{\pi\pi}^{(1)}/C_{\pi\pi}^{(2)}\Big|_{t=4m_\pi^2}=\operatorname{Im} l_1/\operatorname{Im} l_2\Big|_{t=4m_\pi^2}. \quad (19)$$

On the other hand, on the basis of the dispersion relations for l_1 and l_2

$$l_{1,2}=1+\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\operatorname{Im} l_{1,2}(t')\,dt'}{t'(t'-t)}, \quad (20)$$

it is natural to expect $l_1'(0)/l_2'(0) \sim \operatorname{Im} l_1/\operatorname{Im} l_2|_{t\sim 4m_\pi^2}$ [6]. However, for $t = 4m_\pi^2$ we obtain in this case $C_{\pi\pi}^{(1)}/C_{\pi\pi}^{(2)} \sim \alpha \gg 1$, whereas according to [9] we have $C_{\pi\pi}^{(1)}/C_{\pi\pi}^{(2)}|_{t\to 0} \ll 1$. This is possible only if the dependence of the residues $C_{\pi\pi}^{(1,2)}$ on t in the interval $0 \le t < 4m_\pi^2$ is very sharply pronounced.

It must be emphasized, however, that these considerations are purely qualitative in character and do not touch upon the formal aspect of the proposed model.

2. The model considered above, together with its obvious generalization to the case of large (but finite) number of crossing poles, makes it possible to understand the different behavior of various elastic differential cross sections and the limited although arbitrarily large (depending on the number of channels) region of energy, and is characterized by the facts that: (a) sooner or later the important factor in elastic scattering becomes

[6]We note that this relation is valid only if $l_{1,2}$ are functions of class R (i.e., $\operatorname{Im} l_{1,2}/\operatorname{Im} t \geq 0$) and the main contribution to the integrals (20) is made by the region $t \sim 4m_\pi^2$. Neither is rigorously proved.

in all processes the "slow" pole l_{sl} and (b) the diffraction cone of all the elastic processes now is down in final analysis, albeit very slowly, and the cross sections for elastic interaction decrease logarithmically[7)].

$$\sigma_{el} \sim \sigma_0 / [Q + l'_M(0)\ln s/s_0].$$

We shall now discuss briefly a simple example which leads to qualitatively different results. We assume that the partial amplitude $f(l, t)$ in the d-channel has in the l-plane, in addition to the extreme right pole at the point $l = l_0(t)$ (the properties of which were investigated by Gribov and Pomeranchuk[2,3] one or several additional pairs of complex-conjugate poles[8)] with vacuum quantum numbers, the trajectories which pass when $t = 0$ through the point $l = 1$, i.e.,

$$f(l, t) = \frac{r_0(l, t)}{l - l_0(t)} + \frac{1}{2}\sum_{k=1}^{N}\left\{\frac{r_k(l, t)}{l - l_k(t)} + \frac{r_k^*(l^*, t^*)}{l - l_k^*(t^*)}\right\} + f_r(l, t), \quad (21)$$

where $f_r(l, t)$ is a function that is regular in l,

$$r_k(l, t) = r_k^*(l^*, t^*), \quad (22)$$

$$l_k(t) = 1 + i\bar{l}_k(t), \qquad l_k^*(t^*) = 1 - i\bar{l}_k^*(t^*), \quad (23)$$

with

$$\bar{l}_k(t) = \bar{l}_k^*(t^*), \qquad \bar{l}_k(0) = 0. \quad (24)$$

The amplitude (more accurately, its imaginary part) of elastic scattering the s-channel $A_1(s, t)$, corresponding to (21), is of the form

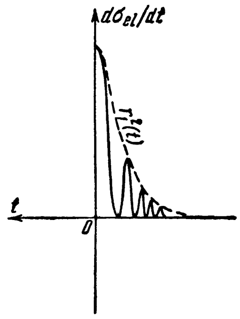
$$A_1(s, t) \sim r_0(l_0(t), t)s^{l_0(t)} + s\sum_{k=1}^{N} r_k(l_k(t), t)\cos[\operatorname{Im} l_k(t)\ln s]. \quad (25)$$

For simplicity we confine ourselves to the case $N = 1$ and one channel. Taking (22) and (23) into consideration, we get

$$\sigma_{tot}(s) = \frac{16\pi}{s}A_1(s, 0) \sim 16\pi\{r_0(1, 0) + r_1(1, 0)\}, \quad (26)$$

$$d\sigma_{el}(s, t) \geqslant \frac{1}{s^2}A_1^2(s, t) \sim \{r_0 s^{2(l_0(t)-1)} + 2r_0 r_1 s^{l_0(t)-1}\cos[\operatorname{Im} l_1(t)\ln s] + \frac{1}{2}r_1^2(t)[1 + \cos(2\operatorname{Im} l_1(t)\ln s)]\}\,d\Omega. \quad (27)$$

It follows from (27) that the differential (and consequently also the total) elastic cross section does not decrease as $s \to \infty$, but is likewise not purely diffractional. It has the form shown in the figure, with a diffraction envelope $r_1^2(t)$. We note that if there are more than two conjugate poles, then the sharp "oscillations" shown in the figure smooth out and approach the diffraction curve. In addition, even in the example analyzed, the experimental observation of these poles would be possible only if $\operatorname{Im} l_1(t)\ln s \sim 1$ at not too small a value of the ratio $r_1^2(t)/r_1^2(0)$.



We emphasize that the presence of a pole at the point $l = l_0(t)$ is necessary within the framework of the example considered in order to satisfy the Gribov-Pomeranchuk theorem[3] that $A_1(s, t)$ and all its derivatives with respect to t are positive in the interval $0 \leq t < 4m_\pi^2$. To satisfy this theorem it is sufficient to satisfy the relation

$$r_0(0)[l'_0(0)]^{2k} \geqslant |r_1(0)[l'_1(0)]^{2k}|, \quad k = 0, 1, \ldots \quad (28)$$

In conclusion it must be noted that in such a generalization of the moving-pole method there are already contained new arbitrary parameters. In this connection, the method ceases to be attractive and is little use for any predictions.

The situation simplifies if among the crossing poles there is one 'fast' pole and a group of slow ones. Then it is sensible to regard the contribution from the aggregate of the latter in a phenomenological manner, as was done in[9].

The authors are grateful to E. L. Feĭnberg for valuable advice and continuous interest in the work.

APPENDIX

We multiply the first column and the first row of the matrix $C = i\rho(1 - iR\rho)$ by ρ_1. We denote the resultant matrix by R. It is clear that

$$\det \overline{R} = \rho_1^2 \det C. \quad (A.1)$$

---

7)It must be emphasized, however, if we choose $\alpha \sim 0.1$, $l'_{s1}(0) \sim 0.1\ m_p^{-2}$, and $Q \sim 1.6\ m_p^{-2}$[10], the region of energies s in which this effect appears shifts quite far away: $s > s_0 \exp\{Q/l'_{s1}(0)\} \sim m_p^2 \times 10^6$.

8)These poles need no longer of necessity be real functions of t.

At the same time replacement of ρ_1 by $-\rho_1$ affects only the element R_{11}, so that[9)]

$$\det \bar{R}^{\mathrm{I}} - \det \bar{R}^{\mathrm{II}} = 2i\rho_1^3 C_{11}^{\mathrm{I}} \tag{A.2}$$

and consequently

$$\det C^{\mathrm{I}} - \det C^{\mathrm{II}} = 2i\rho_1 C_{11}^{\mathrm{I}}, \tag{A.3}$$

i.e.,

$$\det C^{\mathrm{II}} \sim -2i\rho_1 C_{11} \sim -2i\rho_1 C_{011}^{\mathrm{I}} (l-1)^{n-1} \tag{A.4}$$

as $l \to 1$.

Since $C_{1j}^{\mathrm{I}} = -C_{1j}^{\mathrm{II}}$, $j \neq 1$ [10)], we have according to (6), (8), and (A.4)

$$T_{i1}^{\mathrm{II}} = C_{01i}^{\mathrm{I}} / 2i\rho_1 C_{011}^{\mathrm{I}} \underset{\substack{l\to 1\\ t\to 0}}{\longrightarrow} 0, \tag{A.5}$$

i.e., these elements are regular in l at the point $t = 0$, $l = 1$.

We shall show that if conditions (7) and (8) are satisfied there is a pole in l at this point for at least one element $T_{i_0 j_0}^{\mathrm{II}}$ with $i_0, j_0 \neq 1$. Indeed, for these elements

$$C_{ij}^{\mathrm{I}} - C_{ij}^{\mathrm{II}} = 2i\rho_1 M_{11ij} (-1)^{i+j}. \tag{A.6}$$

We note that the aggregate of all the minors M_{11ij} is the aggregate of all the minors of the matrix whose determinant is C_{11}. The absence of a pole in the second sheet would denote that all

$$M_{11ij} \sim C_{11} \sim (l-1)^{n-1}, \tag{A.7}$$

which is impossible.

Theorem: If all the minors $M_{ij}(l)$ turn simultaneously into zeroes of multiplicity n_{ij}, then the determinant turns into a zero of multiplicity $n > \min n_{ij}$.

Proof: 1) Let at least some of the elements of a three-row matrix not vanish at this point. With them we have for this matrix

$$M_{ij} M_{i'j'} - M_{ij'} M_{i'j} = (-1)^{i+j+i'+j'} a_{pq} d_3, \tag{A.8}$$

where a_{pq} is the matrix element which does not vanish (it is obtained by crossing out lines i, i′ and columns j, j′, i.e., $p \neq i, i'$, $q \neq j, j'$), and d_3 is the determinant of the matrix. We see therefore that for a three-row matrix the theorem is correct.

We now consider a four-row matrix. For it the relation of type (1) takes the form

$$M_{ij} M_{i'j'} - M_{ij'} M_{i'j} = (-1)^{i+j+i'+j'} M_{iji'j'} d_4. \tag{A.9}$$

It is known from the foregoing that for a given M_{ij} there exists a minor $M_{iji'j'}$ which vanishes more slowly than M_{ij}. Therefore d_4 vanishes more rapidly than the M_{ij} which vanishes most slowly. It is easy to show by analogy that the statement of the theorem is valid for a matrix of rank n if it holds for a matrix of rank $n-1$. This proves the theorem.

2) If all elements of the matrix also vanish at this point, then the proof of the theorem is trivial.

3) From (A.9) it follows that if $d \sim (l-1)^n$ and all $M_{ij} \sim (l-1)^{n-1}$, then $M_{iji'j'} \gtrsim (l-1)^{n-2}$. Examining the following minors of type $M_{iji'j'i''j''}$ we verify analogously that they have zeros of order not smaller than $(l-1)^{n-3}$. Continuing this process further, we verify that the matrix elements vanish not more slowly than $(l-1)^{(n-n_0+1)}$, where n_0 is the rank of the matrix. But then $d < (l-1)^{(n-n_0+1)n_0}$ hence $n > (n-n_0+1)n_0$, i.e., $n \leq n_0$ if $n_0 > 1$.

The case $n_0 = 1$ calls for a special analysis. In this case R, T, and ρ are c-numbers. We expand $R(l, t)$ in powers of $x = l - 1$ at the point $x = 0$:

$$R(x, t) = R_0(t) + R_1(t)x + \ldots \tag{A.10}$$

To find near the vicinity of $t = 0$ n roots of the equation

$$1 - i\rho R = 0$$

crossing at $t = 0$ and vanishing at this point, it is sufficient to retain the $(n+1)$-st term in the expansion (A.10). The conditions for the vanishing of all the roots of the equation thus obtained are

$$\lim_{t\to 0} \frac{R_k}{R_n} = 0 \quad k < n, \qquad \lim_{t\to 0} \frac{1 - i\rho R_0}{i\rho R_n} = 0. \tag{A.11}$$

But this contradicts the requirement that the total cross section be constant, according to which

$$\lim_{t\to 0} \frac{R_{n-1}}{i\rho R_n} = \mathrm{const}.$$

Thus, the crossing of the finite number of poles at $n_0 = 1$ is impossible.

4) If we now express the corresponding amplitudes in terms of their values on the second sheet, namely

$$T_{1j_0}^{\mathrm{I}} = T_{1j_0}^{\mathrm{II}} / (1 - 2i\rho T_{11}^{\mathrm{II}}),$$

$$T_{i_0 j_0}^{\mathrm{I}} = T_{i_0 j_0}^{\mathrm{II}} + 2i\rho T_{i_0 1}^{\mathrm{II}} T_{1j_0}^{\mathrm{II}} / (1 - 2i\rho T_{11}^{\mathrm{II}}), \tag{A.12}$$

then, taking the proved theorem into consideration, we can readily verify that the known relation between the total cross sections[2,6] is satisfied only

[9)]The Roman numeral indicates the number of the sheet on the Reiman surface.

[10)]By virtue of the symmetry of the matrix $C = i\rho(1 - iR\rho)$ this pertains also to the transposed elements.

if the residue at the pole $T^{II}_{i_0 j_0}$ at $t = 0$, $l = 1$ vanishes.

[1] Foley, Lindenbaum, Love, Ozaki, Russel, and Yuan, Phys. Rev. Lett. **10**, (8), 376 (1963).

[2] V. N. Gribov, JETP **42**, 1260 (1962) and **41**, 1962 (1961), Soviet Phys. JETP **15**, 873 (1962) and **14**, 1395 (1962).

[3] V. N. Gribov and I. Ya. Pomeranchuk, JETP **42**, 1141 (1962), Soviet Phys. JETP **15**, 788 (1962).

[4] M. Froissart, Phys. Rev. **123**, 1053 (1961).

[5] R. Oehme, Zs. Physik **162**, 426 (1961).

[6] Gribov, Ioffe, Rudik, and Pomeranchuk, JETP **42**, 419 (1962), Soviet Phys. JETP **15**, 292 (1962).

[7] I. B. Khriplovich, Preprint, Siberian Division, Academy of Sciences, U.S.S.R.

[8] A. G. Kurosh, Kurs vyssheĭ algebry (Course of Higher Algebra), Fizmatgiz, 1962.

[9] E. L. Feinberg and D. S. Chernavskiĭ, JETP **45**, 1252 (1963), Soviet Phys. JETP **18**, 861 (1964).

[10] Diddens, Lillethun, Manning, Taylor, Walker, and Wetherell, Phys. Rev. Lett. **9**, 108, 111 (1962).

Translated by J. G. Adashko
89