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VERIFICATION OF THE PROPERTIES OF CERENKOV RADIATION IN UNIAXIAL CALCITE CRYSTALS

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Some experimental results of a study of a number of properties of the Cerenkov radiation produced in a uniaxial crystal of calcite by a 663-MeV proton beam ($\beta = 0.81$) are presented. The radiation properties were investigated in the following two particular cases of particle motion, (a) along and (b) perpendicular to the optical axis of the crystal. The predictions of the theory that in case (a) a single cone of extraordinary waves is excited and in case (b) two radiation cones are excited (one ordinary and one extraordinary) are confirmed experimentally. The conclusions of the theory regarding the directional properties (emission angles) of the radiation, the polarization and threshold properties, and the azimuth distribution of the radiation intensity are also in complete agreement with the theory.

INTRODUCTION

DESPITE the fact that the theory of the Cerenkov radiation in uniaxial crystals was first developed by Ginzburg[1] as early as in 1940, that is, soon after the development by Tamm and Frank[2] of the theory of Cerenkov radiation in isotropic media, and despite the subsequently published papers by Kolomenskiĭ[3], Kaganov and Sitenko[4,5], Pafomov[6], Tanaka[7], Bolotovskiĭ[8], Frank[9], Muzicar[10] and others, not one deduction of this theory has yet been subjected to an experimental verification. This situation is explained to some extent by the fact that the theory of the Cerenkov radiation is based on equations of both classical and quantum electrodynamics, and therefore there is in general never any doubt about the conclusions of the theory. Nevertheless, it must be recognized that these reasons are not sufficient to justify the present situation, particularly if we consider that there are many more possibilities for setting up suitable experiments at the present time than were available to Cerenkov, who investigated exhaustively the properties of radiation in an isotropic region approximately 30 years ago.

The present paper is devoted to a check on several properties of the Cerenkov radiation in a uniaxial calcite crystal which has highly anisotropic refractive indices for the ordinary (o) and extraordinary (e) waves: $n_o = 1.6584$, $n_e = 1.4864$ (for $\lambda = 5893\,\text{Å}$).

1. EXPERIMENTAL CONDITIONS

The experiments were made with the extracted proton beam from the synchrocyclotron of the nuclear problems laboratory of the Joint Institute for Nuclear Research. The average proton energy in the beam was 663 ± 3 MeV.

The high intensity of the proton beam and its good collimation (beam diameter 5 mm and collimator length 600 mm) made it possible to use the exceedingly simple scheme of Fig. 1 for all the experiments.

The proton beam passed through both the calcite slab and a planoconvex parabolic lens placed in optical contact with the crystal. The lens served not only to focus the radiation excited in the crystal, but also to extract the radiation in the case when total internal reflection prevented it from passing through the crystal-air boundary. In ad-

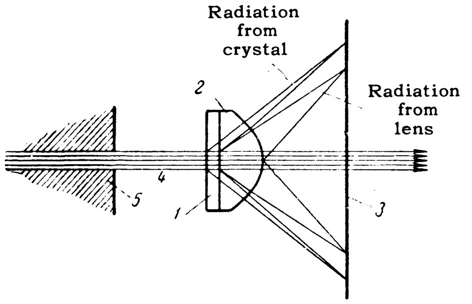

FIG. 1. Experimental setup: 1 – crystal, 2 – lens, 3 – photographic plate, 4 – proton beam, 5 – collimator.

dition, the Cerenkov radiation excited by the protons in the lens itself could serve as a good reference in the calculation of the angles of the radiation emitted in the crystal.

The calcite slab used in the experiments was cut parallel to the optical axis, 4.50 mm thick (1.22 g/cm^2). A 3.62 mm (10 g/cm^2) slab cut perpendicular to the optical axis was also used. The proton energy loss in such relatively thin slabs was 2.2 MeV-g^{-1}cm^2, so that the slowing down effects and also multiple scattering played no appreciable role in the determination of the radiation angles.

The Cerenkov radiation was registered with the aid of a special cassette either on a flat 18×25 cm negative "DS-2" color film (sensitivity 45 GOST units) or on "Agfacolor Negativ–Planfilm Ultra T" film (17° DIN) or else on black and white iso-orthochromatic plates with sensitivity 65 GOST units. Photography on color film made it possible to reduce the error in the determination of the radiation angles.

At an average proton beam intensity of 10^9 sec^{-1}, with a 5 mm collimator diameter, and a distance 17 meters from the exit window of the accelerator chamber, the exposure was 2—3 minutes, corresponding to a total of approximately 10^{11} protons.

2. PREDICTIONS OF THE THEORY AND EXPERIMENTAL RESULTS

For greater convenience and clarity we present below the theoretical predictions in parallel with the experimental results.

A check on the properties of the Cerenkov radiation was made in two particular cases: (a) particle moves along the optical axis of the calcite crystal (in this case, a plane-parallel slab cut perpendicular to the optical axis) and (b) motion of the particle perpendicular to the optical axis (calcite slab cut parallel to the optical axis).

a) Charged particle motion along the optical axis of a uniaxial crystal (Fig. 2). As predicted by the theory, in this case the ordinary waves cannot be emitted even if the particle velocity exceeds the threshold value. This is connected with the fact that the electric vector of ordinary waves in a uniaxial crystal should be perpendicular to the principal optical plane, a condition satisfied only when the ordinary waves are radiated in the direction of motion of the particle, but under these conditions the intensity of the radiation vanishes.

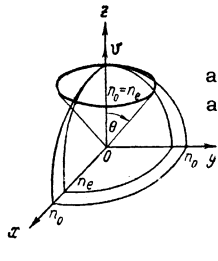

FIG. 2. Relative positions of the optical axis of the crystal z, the particle velocity, and the axes x and y.

It is clear from symmetry considerations that the cone of the extraordinary waves should be circular with uniform intensity distribution, and the polarization properties of the radiation should coincide with those for the isotropic case.

The correct formula for the extraordinary-wave radiation intensity when a particle moves along the optical axis was obtained in several papers[3-6] and is of the form[1)]

$$\frac{dW_e^{\parallel}}{dz} = \frac{e^2}{c^2}\int\limits_{n_0\beta>1}\left(1-\frac{1}{n_0^2\beta^2}\right)\omega\,d\omega, \tag{1}$$

where n_0 is the refractive index of the ordinary waves.

The radiation angle θ_e of the extraordinary waves when the particle moves along the optical axis of the uniaxial crystal is determined by the formula[1]

$$\cos^2\theta_e = \left[1+\frac{n_e^2}{n_0^2}\left(n_0^2\beta^2-1\right)\right]^{-1}, \tag{2}$$

where $\beta = v/c$ —particle velocity, and n_0, n_e — refractive indices of the ordinary and extraordinary waves.

At a proton velocity $\beta = 0.810$ ($E_p = 661$ MeV), the radiation angle, in accordance with formula (2), is $\theta_e = 38°46'$ for $\lambda = 5893$ Å. We note that in the comparison of the experimentally measured radiation angle θ_e with the calculated value it is necessary to take into account the fact that formula (2) determines the angle between the particle velocity and the normal **N** to the wave front, which does not coincide with the direction of energy propagation in an anisotropic medium **S**.

The angle θ_e' between the particle velocity **v** and the beam velocity **S** is determined in the following manner:

$$\mathrm{tg}\,\theta_e' = (n_0^2/n_e^2)\,\mathrm{tg}\,\theta_e, \tag{3}^*$$

where θ_e' and θ_e are measured from the crystal axis. In our case $\theta_e' = 44°27'$.

Inasmuch as the radiation angle $\theta_e = 38°46'$ is close to the angle of total internal reflection ($\theta_e^* = 42°17'$), the radiation leaves the plane-

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[1)] The corresponding formula in Ginzburg's paper[1] contains an error, as pointed out by Kolomenskii[3] and Kaganov[4]. Tanaka's formulas[7] are likewise incorrect[3].

*tg = tan.

FIG. 3. Cerenkov radiation photograph obtained with the setup of Fig. 1, for a proton beam passing through a calcite slab cut perpendicular to the optical axis (outer ring—extraordinary waves, inner ring—waves from the lens).

parallel slab at a glancing angle, and is difficult to register. Therefore the radiation was extracted by means of a plane-parabolic lens of focal distance $f = 22.7 \pm 0.1$ mm in the manner shown in Fig. 1. The average proton energy in the lens was (658.2 ± 3) MeV, corresponding to a proton velocity $\beta = 0.8091 \pm 0.0008$. At such a velocity, the Cerenkov radiation angle in the lens, for $\lambda = 5893$ Å ($n_D = 1.512 \pm 0.001$), was $\theta = 35°10' \pm 5'$, and the Cerenkov radiation angle for the extraordinary waves in the lens, with allowance for refraction on the crystal-lens boundary, was 40°59′.

Figure 3 shows a photograph of the Cerenkov radiation, obtained with the experimental setup shown in Fig. 1, the proton beam passing through a calcite slab cut perpendicular to the optical axis.[2] The outer ring is the ring of extraordinary waves emitted by the particle in the crystal, while the inner ring is the radiation produced in the lens. The distribution of the radiation energy in both rings with respect to φ is uniform, as should be the case.

The ratio of the ring diameters in the focal plane of the lens is determined by the ratio of the tangents of the radiation angles from the crystal and the lens in the lens itself, and its theoretical value is $k_{th} = 1.233$. The experimentally obtained ratio was $k_{exp} = 1.24 \pm 0.01$, corresponding to an extraordinary-wave radiation angle $\theta_e = 38°56' \pm 14'$. This agrees well with the theoretically expected value of θ_e (38°46′).

The threshold values of the Cerenkov radiation are determined in the case when the particle moves along the axis of the uniaxial crystal, as can be readily seen from (2), by the condition

$$(n_e^2/n_o^2)\,(n_o^2\beta^2 - 1) \geqslant 0. \qquad (4)$$

For the region of normal dispersion ($n_o, n_e > 0$), the inequality (3) assumes the same form as the threshold condition in the isotropic medium, $n_o\beta \geq 1$. Thus, for $n_o = 1.683$ ($\lambda = 4000$ Å) we have $\beta_n = 0.5942$.

The threshold radiation properties were verified for a particle moving along the calcite axis with a deuteron beam extracted from the synchrocyclotron of the Nuclear Problems Laboratory. The measured energy of the deuterons leaving the accelerator chamber [11] was (405 ± 2) MeV. At the entrance to the crystal, the deuteron velocity was $\beta_d = 0.5652$, that is, only $\Delta\beta = 0.029$ lower than the threshold value. An experiment performed with the setup of Fig. 1 but without a lens showed that no radiation was excited in the crystal.

The polarization properties of the radiation from a particle moving along the crystal axis, as noted above, should coincide with those which we get in the isotropic case.

The polarization properties were checked experimentally with the setup shown in Fig. 1 but with addition of a polarization filter between the

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[2] It is interesting to note that a strong long-duration fluorescence of yellow-orange color is visually observed at the point where the proton beam passes through the calcite crystal. It can be clearly seen even days later when the crystal is heated to 100°C.

lens and the photographic film. The photograph obtained under these conditions is shown in Fig. 4, from which it is seen that the radiation in an isotropic medium and radiation from a particle moving along the optical axis of a uniaxial crystal have identical polarization properties, that is, the electric vector of the radiation lies in the plane passing through the direction of emission of the radiation and the direction of the particle velocity.

In addition, an experiment performed with particles moving along the optical axis illustrated clearly the influence of the type of boundary through which the radiation leaves the crystal.

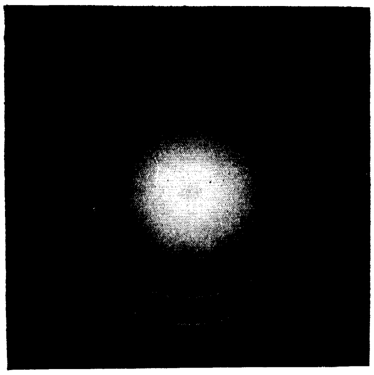

FIG. 4. Photograph of Cerenkov radiation excited by a particle passing along the optical axis of a calcite crystal and transmitted through a polaroid (outer arcs – extraordinary waves, inner arcs – from lens).

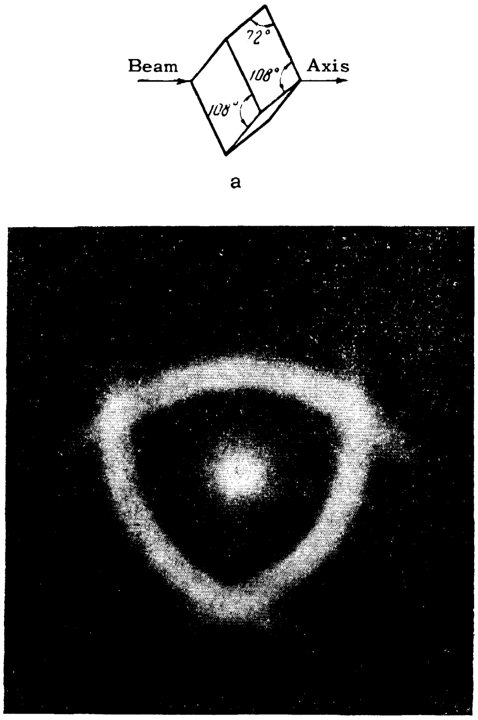

FIG. 5. a–Experimental setup for the study of the influence of the boundaries of a calcite crystal with natural outline on the form of the radiation emerging from the crystal: b–photograph of the Cerenkov radiation obtained by passage of a proton beam (β = 0.81) along the optical axis of the calcite crystal.

We are accustomed to see the Cerenkov radiation cone and intersect the plane perpendicular to the direction of motion of the particle in a regular ring, but the extent to which the radiation becomes unrecognizable if it is excited by particles in a calcite crystal having a natural outline (rhombohedral form) can be judged from the photograph of Fig. 5b, obtained with the experimental setup illustrated in Fig. 5a.

b) Charged particle motion perpendicular to the optical axis of a uniaxial crystal. In this case both extraordinary and ordinary waves can be excited. For ordinary waves the directivity properties are determined by the relations

$$\cos\theta_o = 1/n_o\beta. \tag{5}$$

The cone of the ordinary waves is circular, but with uneven distribution of the intensity with respect to the angle φ; this distribution is determined by the formula[1]:

$$\frac{dW_o^{\perp}}{dy} = \frac{e^2}{2\pi c^2}\int\limits_{n_o\beta>1}\left(1-\frac{1}{n_o^2\beta^2}\right)\frac{\cos^2\varphi\, d\varphi\omega d\omega}{\cos^2\varphi+(n_o\beta)^{-2}\sin^2\varphi}, \tag{6}$$

where the angle φ is measured from a direction perpendicular to the optical axis.

The cone of the extraordinary waves turns out to be not circular and to have an uneven intensity distribution relative to φ. The directivity of the Cerenkov radiation for the extraordinary waves is determined by the equation[1]

$$\cos^2\theta_e^{\perp} = \left[1+\frac{n_o^2(n_e^2\beta^2-1)}{n_o^2\cos^2\varphi+n_e^2\sin^2\varphi}\right]^{-1}, \tag{7}$$

from which we see that $\theta_e^{\perp}$ depends on φ.

Figure 6a shows the Huygens constructions (for β = 0.81) drawn in the two planes xy (φ = 0) and zy ($\varphi = \pi/2$).

From (7) we get the threshold condition for the excitation of the extraordinary waves:

$$\beta \geqslant n_e^{-1}. \tag{8}$$

The dependence of the radiation intensity of the extraordinary waves on the angle φ was first derived in correct form by Pafomov[5] [3]:

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[3] In Ginzburg's paper[1] the analogous formula is likewise incorrect (see, for example,[6,8]).

Table I

$\lambda_1 = 4000$ Å				$\lambda_2 = 5893$ Å			
φ	θ_e	θ_e'	θ_e''	φ	θ_e	θ_e'	θ_e''
0°	34°30′	34°30′	58°03′	0°	33°51′	33°51′	55°33′
90°	37°41′	31°27′	72°23′	90°	36°48′	30°31′	67°27′

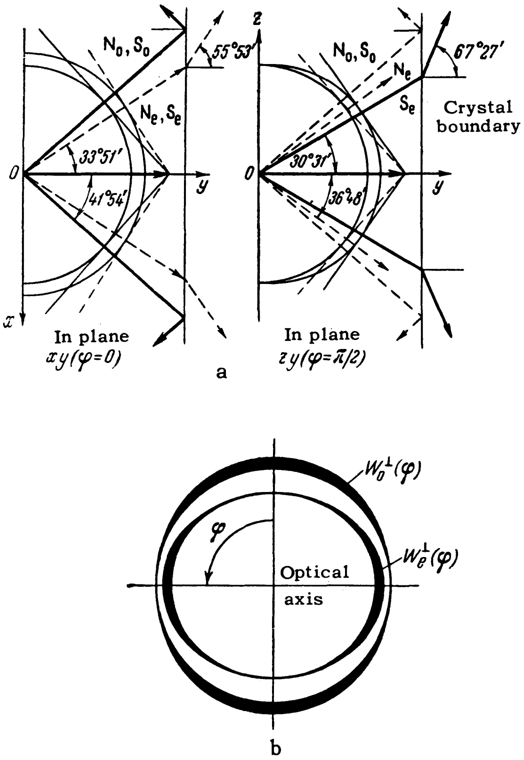

FIG. 6. a – Huygens constructions for the case of a particle with $\beta = 0.81$ moving perpendicular to the optical axis of a calcite crystal; construction in the planes xy ($\varphi = 0$) and yz ($\varphi = \pi/2$); b – theoretical distribution of the intensity of radiation as a function of the angle φ, obtained when a particle with $\beta = 0.81$ moves perpendicular to the optical axis of the calcite crystal.

$$\frac{dW_e^\perp}{dy} = \frac{e^2}{2\pi c^2} \int\limits_{n_e\beta>1} \left(1 - \frac{1}{n_e^2\beta^2}\right) \times \frac{\sin^2\varphi \, d\varphi n_e^2 \omega \, d\omega}{(n_e^2 \sin^2\varphi + n_o^2\cos^2\varphi)(n_o^2\beta^2\cos^2\varphi + \sin^2\varphi)}. \tag{9}$$

The approximate distribution of the Cerenkov radiation intensity with respect to the angle φ, calculated from (6) and (9) for a particle moving with $\beta = 0.81$ perpendicular to the optical axis of calcite, is shown in Fig. 6b.

The extraordinary-wave radiation angles (for $\varphi = 0°$ and $\varphi = \pi/2$) at $\lambda_1 = 5.893$ Å, $\lambda_2 = 4000$ Å, and $\beta = 0.810$ are listed in Table I.

The angles θ_e, θ_e', and θ_e'' listed in Table I denote respectively the angles between the particle velocity and the wave normal, the ray velocity, and the direction of the radiation in the air on leaving the crystal.

The directivity properties of the Cerenkov radiation for a particle moving perpendicular to the optical axis of the calcite crystal were verified in two experiments.

In the first experiment the radiation was registered on a flat color film without using a focusing lens. In this case only the extraordinary waves left the crystal, the ordinary ones remaining in the crystal because the radiation angle for the ordinary waves was $\theta_0 = 41°54'$ at $\lambda = 5893$ Å and was larger than the angle of total internal reflection.

Figure 7 shows a photograph of the radiation registered in this case on photographic film located 17.7 mm away from the surface of the crystal (facing the film). The experimentally determined angles θ_e (that is, the angles between the velocity and the wave normal) for $\lambda \approx 6000$ Å are:

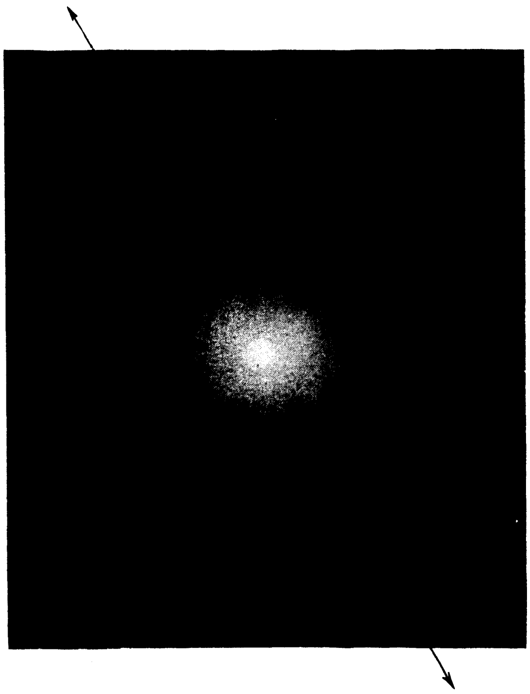

FIG. 7. Photograph of extraordinary Cerenkov radiation waves obtained with the setup of Fig. 1 (without a focusing lens).

$$\theta_e(\varphi = 0) = 33°14' \pm 1°, \qquad \theta_e(\varphi = \pi/2) = 35°44' \pm 50'.$$

These value are in satisfactory agreement with the calculated ones. The values of the angles for $\lambda = 4000\,\text{Å}$ were not calculated, since the width of the blue-violet part of the spectrum on the photograph turned out to be too large, owing to dispersion and owing to divergence in air.

The second experiment was carried out with the lens in the setup of Fig. 1, so that these angles could be determined more accurately. A photograph obtained in this case is shown in Fig. 8. The photograph shows, in addition to the "ellipse" of the extraordinary waves, also a circular (external) cone of ordinary waves with uneven distribution of the intensity. The radiation intensity distribution for the ordinary and extraordinary waves relative to the angle φ is in agreement with the predictions of the theory. Thus, for $\varphi = 0$ the intensity of radiation of the ordinary waves is maximal, while that of the extraordinary ones is equal to zero. Conversely, for $\varphi = \pi/2$ the intensity of the extraordinary is maximal, and that of the ordinary ones is equal to zero.

FIG. 8. Cerenkov radiation photograph obtained for a proton beam moving perpendicular to the optical axis of a calcite crystal (outer arcs – ordinary waves, "ellipse" – extraordinary waves solid ring – radiation from lens, the arrow indicates the direction of the optical axis of the crystal).

The radiation angle of the ordinary waves was determined also from the ratio of the diameters of the radiation rings from the lens and from the crystal. The experimental ratio of the diameters of the rings was found to be $k_e = 1.59 \pm 0.03$. The greater part of the error in this ratio is due principally to the difficulty of determining the position of the portion of the spectrum located near the line $\lambda = 5893\,\text{Å}$. The theoretical value of this ratio is $k_{th} = 1.527$. The experimental radiation angle of the ordinary waves found from these values is $\theta_o = 42°52' \pm 20'$ and exceeds the calculated value $\theta_o = 41°54'$ by approximately 1°. Estimates show that this discrepancy must be ascribed to the influence of the negative distortion of the employed lens.

The radiation angles of the extraordinary waves agreed better with the calculated values, since they differed little from the radiation angle in the lens, so that the influence of the lens distortion was smaller.

The experimental results for the radiation angles $\theta_{e.\text{exp}}$ are listed in Table II (for wavelengths close to $\lambda = 5893\,\text{Å}$), and are in good agreement with theory. The accuracy with which $\theta_e(\pi/2)$ is determined is higher than that for $\theta_e(0°)$. The reason for it is that when $\varphi = 0$ the intensity of the extraordinary waves is equal to zero and it is difficult to determine k.

The polarization properties of the Cerenkov radiation excited by particles moving perpendicular to the optical axis was checked also with the setup of Fig. 1, but with a polaroid added between the lens and the photographic plate. The directions of the electrical vectors of both types of waves, predicted by the theory, are best determined with the aid of the formula of Musicar[10].

The components of the electric vector of the ordinary waves along the axes x, y, and z are given by

$$\mathbf{e}_o = (-n_2, n_3, 0)\,/\,n_o^2\sqrt{1 - n_1^2}, \tag{10}$$

where

$$n_1 = -\sin\theta\sin\varphi, \qquad n_2 = \cos\theta, \qquad n_3 = \sin\theta\cos\varphi. \tag{11}$$

Equations (10) and (11) show that, independently of the angle φ the electric vector $\mathbf{e}_o$ lies in the plane (x, y) perpendicular to the optical axis, and when $\varphi = \pi/2$ its direction coincides with the x axis (but the intensity is equal to zero).

The components of the electric vector of the extraordinary waves are of the following form[10]:

$$\mathbf{e}_e = \left(-\frac{n_1 n_3}{n_o^2},\ -\frac{n_1 n_2}{n_o^2},\ \frac{1 - n_1^2}{n_e^2}\right)\frac{1}{\sqrt{1 - n_1^2}}, \tag{12}$$

where n_1, n_2, and n_3 are determined by the same equations (11).

The directions $\mathbf{e}_o$ and $\mathbf{e}_e$, calculated for the case of the motion of a particle with $\beta = 0.81$ perpendicular to the optical axis of the calcite crystal, are shown in Fig. 9.

The polarization properties of the radiation excited by the particles moving perpendicular to the optical axis were checked in two experiments.

In the first experiment a polyvinyl polaroid was

Table II

φ	$\theta_{e,\,\mathrm{th}}$	k_{th}	k_{exp}	$\theta_{e,\,\mathrm{exp}}$
0°	33°51′	0.929	0.92±0.03	33°36′±40′
90°	36°48′	1.095	1.09±0.01	36°40′±14′

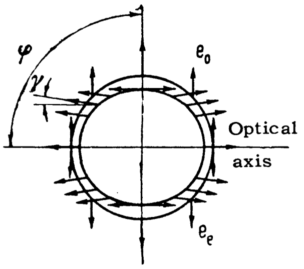

FIG. 9. Directions of electric vectors of the extraordinary and ordinary waves of Cerenkov radiation, calculated by formulas (10)–(12) for the case of particle motion with $\beta = 0.81$ perpendicular to the optical axis.

placed between the lens and the photographic plate and oriented in such a way as to transmit radiation with an electric vector perpendicular to the optical axis. In this case the ordinary waves should be transmitted completely (disregarding the absorption of the polaroid), and the extraordinary waves should be almost completely blocked. The photograph obtained in this experiment (Fig. 10) shows the ordinary waves (outer arc) and the arcs from the isotropic medium (inner arcs). In addition, the polaroid transmitted also an insignificant part of the extraordinary waves (which can be distinguished only in the negative), essentially at angles $\varphi \approx 70$, 110, 250, and 290° (see the explanations below). At $\varphi = 90°$ the intensity of the transmitted extraordinary waves was equal to zero, as it should be.

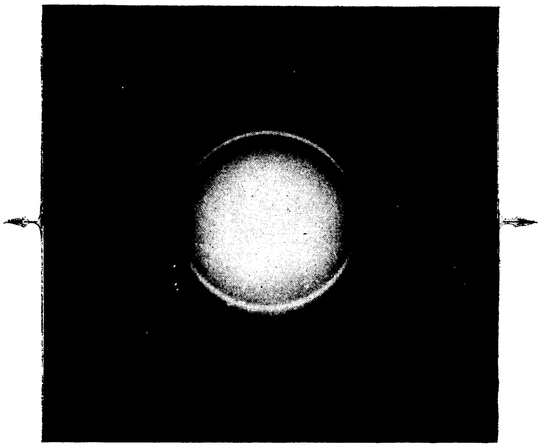

FIG. 10. Photograph of Cerenkov radiation excited by particles moving perpendicular to the optical axis, but passing through a polaroid with an axis oriented perpendicular to the optical axis (the arrow indicates the direction of the crystal optical axis).

In the second experiment the polaroid was rotated through 90°. In this case the ordinary waves on the photograph should disappear, and the extraordinary waves should pass almost completely. However, on the photograph (Fig. 11) obtained in this experiment, the ordinary waves have disappeared completely only at $\varphi = 0°$. At $\varphi = 45$, 135, 225, and 315°, the ordinary waves pass through the polaroid and are obtained in the form of four rather weak arcs, which could not be reproduced on Fig. 11.

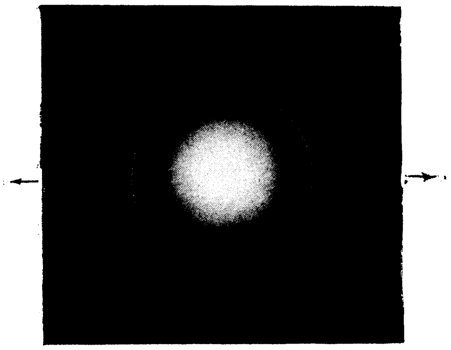

FIG. 11. The same as in Fig. 10, but the polaroid axis is parallel to the crystal axis.

Such a disparity between the experiment and the predictions of the theory is only illusory and is connected with two effects. One is that when the radiation leaves the crystal and goes into the isotropic medium the polarization properties of the radiation may change if the electric vector of the wave in the crystal does not coincide with the plane of incidence. (The electric vector of the ordinary wave lies in the plane of incidence only for $\varphi = 0$.)

Rotation of the electric vector on passing through the separation boundary is determined by the formula

$$\operatorname{tg}\delta_2 = \cos(\theta_2 - \theta_1)\operatorname{tg}\delta_1, \tag{13}$$

where δ_1 is the angle between the plane of incidence of the ray in the first medium and the plane in which lies the electric vector in the same medium (in other words, δ_1 is the azimuth of the polarization), δ_2 is the polarization azimuth in the second medium, and θ_1 and θ_2 are the angles between the ray and the normal to the separation plane in the first and second media, respectively.

The direction of rotation is determined by the fact that the plane of polarization always tends to approach the plane of incidence of the ray. Allowance for this circumstance under the conditions of the second experiment (Fig. 11) explains only qualitatively the presence past the polaroid of arcs of extraordinary waves, making angles of 45° with one another, since calculation calls for the intensity of these arcs to be $\sim 4 \times 10^{-6}$ of W_0.

The second and main effect consists in the following. It is known[13] that although the polarizing ability of polyvinyl polaroids is maintained up to large aperture angles ($2\theta \sim 60°$), it begins to decrease rapidly above these angles. It is typical here that the change in the polarizing ability is anisotropic in the angle φ, and has maxima every 90° relative to this angle, that is, the pattern agrees with our observation. At $\theta \approx 50°$, the transmitting ability of the polaroid at these values of φ decreases by several per cent, and this is enough to explain the effect observed in our case.

The threshold properties of the Cerenkov radiation for a particle moving perpendicular to the optical axis were verified also with a deuteron beam of velocity $\beta_d = 0.5652$ below the threshold excitation velocity of the ordinary and, all the more, the extraordinary waves. As predicted by the theory, no radiation was observed.

Thus, the described set of experiments, aimed at verifying the properties (directivity, polarization, threshold, distribution of intensity relative to the angle φ) of Cerenkov radiation excited by protons with $\beta = 0.81$ in a calcite crystal, enables us to state that the theory and experiment are in complete agreement.

In conclusion I take this opportunity to thank I. M. Frank for a discussion and interest in the work, L. M. Belyaev and A. B. Gil'varg for the calcite crystals, and Professor H. Barwich and H. Junglausson for help in acquiring the Agfacolor negative color film.

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[1] V. L. Ginzburg, JETP **10**, 608 (1940).

[2] I. E. Tamm and I. M. Frank, DAN SSSR **14**, 107 (1937).

[3] A. Kolomenskii, DAN SSSR **86**, 1097 (1952).

[4] N. I. Kaganov, JETP **23**, 507 (1953).

[5] A. G. Sitenko and M. I. Kaganov, DAN SSSR **100**, 681 (1955).

[6] V. E. Pafomov, Tr. FIAN SSSR (Transactions, Physics Institute, Academy of Sciences) **16**, 94 (1961).

[7] K. Tanaka, Phys. Rev. **93**, 459 (1954).

[8] B. M. Bolotovskiĭ, UFN **62**, 201 (1957).

[9] I. M. Frank, UFN **68**, 387 (1959).

[10] C. Muzicar, JETP **39**, 163 (1960), Soviet Phys. JETP **12**, 117 (1961).

[11] I. M. Vasilevskiĭ and Yu. D. Prokoshkin, Atomnaya énergiya **7**, 225 (1959).

[12] M. Born, Optics (Russian translation) ONTI, 1937.

[13] M. Haase, Z. Techn. Physik **18**, No. 3, 1937.

[14] M. V. Leikin and Yu. N. Denisyuk, Optiko-mekhanicheskaya promyshlennost, (Optical and Mechanical Industry) **9**, 27 (1959).

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