THEORY OF CONVERSION AND SCATTERING OF ELECTROMAGNETIC WAVES IN A NONEQUILIBRIUM PLASMA

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We have investigated conversion and scattering of electromagnetic waves in a nonequilibrium plasma. If the plasma is nearly unstable, the coefficients describing wave scattering and conversion become anomalously large. We consider two cases: a plasma with hot electrons moving through cold ions, and a plasma through which a beam of fast charged particles moves. The spontaneous emission caused by conversion of fluctuating longitudinal waves into transverse waves in a nonequilibrium plasma is considered.

1. INTRODUCTION

It is well known that a plasma can support the propagation of different kinds of weakly damped waves: transverse waves, Langmuir waves, and acoustic waves (in a nonisothermal plasma). Waves are always excited by plasma fluctuations: in thermodynamic equilibrium the amplitudes of these waves are determined by the plasma temperature (we shall call these fluctuation waves). In a nonequilibrium plasma the amplitudes of the fluctuation waves can be found if the distribution function for the plasma particles is known.[1-3] A plasma can also support waves excited by external sources (we shall call these external waves).

The nonlinearity of the plasma equations implies that plasma waves can interact, causing wave scattering and conversion of waves from one kind into another. The scattering and conversion of external waves on plasma fluctuations has been treated widely in the literature (cf. for example [4,2,5]). In all of this work both equilibrium and nonequilibrium plasmas have been considered; however, the plasma was assumed to be in a stable state. It has been recently shown[6,7] that plasma fluctuations increase markedly when an unstable state is approached. The coefficient for scattering of transverse waves into transverse waves increases tremendously;[7] the effect is analogous to critical opalescence in optics. The coefficients describing scattering and conversion of longitudinal waves should also be very large. In the present work we shall be interested, among other things, in scattering of an external longitudinal wave by critical plasma fluctuations and transformation of such a wave into a transverse wave.

Another characteristic feature of a nonequilibrium plasma is the spontaneous emission associated with the conversion of two fluctuating longitudinal waves into a transverse wave. (In the equilibrium case all radiation from the plasma is essentially Rayleigh scattering since the additional effect associated with the interaction between waves must vanish as a consequence of detailed balancing.) We determine the intensity of this spontaneous emission and find that it can be anomalously large when the plasma is close to an unstable state.

Main emphasis is given in the present work to those cases of wave scattering and conversion in which the intensity of the produced radiation is very large by virtue of the existence of critical fluctuations. This situation arises in the interaction between two waves when one has an anomalously large (critical) amplitude, and in the Doppler scattering of a fluctuating wave with an anomalously large amplitude on plasma particles. For this reason, we shall be interested primarily in nonlinear wave interactions; in the analysis of induced scattering of waves by particles we consider only the case of Doppler scattering [Eqs. (24) and (25)].

2. INTENSITY OF THE SECONDARY WAVE

We first determine the amplitude of the secondary wave produced as a result of the interaction between two waves propagating in a plasma. For this purpose we start with the complete system of equations that describe the plasma: the kinetic equation for each particle species and Maxwell's equations.

Assuming that the amplitudes of the interacting waves are small and expanding the particle distribution functions and the electric and magnetic fields in powers of this amplitude, we find that terms of n-th order are given by

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)f_\alpha^{(n)}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}^{(n)}+\frac{1}{c}\left[\mathbf{v}\mathbf{H}^{(n)}\right]\right)\frac{\partial f_\alpha^0}{\partial \mathbf{v}}$$
$$=-\frac{e_\alpha}{m_\alpha}\sum_{n'=1}^{n-1}\left(\mathbf{E}^{(n')}+\frac{1}{c}\left[\mathbf{v}\mathbf{H}^{(n')}\right]\right)\frac{\partial f_\alpha^{(n-n')}}{\partial \mathbf{v}};$$
$$\mathrm{rot}\,\mathbf{H}^{(n)}=\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}^{(n)}+\frac{4\pi}{c}\sum_\alpha e_\alpha\int \mathbf{v}f_\alpha^{(n)}\,d\mathbf{v},\qquad (1)^*$$

$$\mathrm{div}\,\mathbf{E}^{(n)}=4\pi\sum_\alpha e_\alpha\int f_\alpha^{(n)}\,d\mathbf{v},$$
$$\mathrm{rot}\,\mathbf{E}^{(n)}=-\frac{1}{c}\frac{\partial}{\partial t}\mathbf{H}^{(n)},\qquad \mathrm{div}\,\mathbf{H}^{(n)}=0,\qquad (2)$$

where f_α is the distribution function for particles of the α species with mass m_α and charge e_α, **E** is the electric field and **H** is the magnetic field. The nonlinear effect in the interaction between the waves is evidently described by the terms on the right side of the kinetic equation (1).

Solving Eqs. (1) and (2) and limiting ourselves to the case in which both primary waves are longitudinal waves we find the secondary wave

$$E_i^{(2)}(\mathbf{k},\omega)=-iD_{ij}(\mathbf{k},\omega)\int\frac{d\mathbf{k}_1d\omega_1}{(2\pi)^4}C_j(\mathbf{k},\omega_1;\mathbf{k}-\mathbf{k}_1,\omega-\omega_1)$$
$$\times\varphi^{(1)}(\mathbf{k}_1,\omega_1)\varphi^{(1)}(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1),\qquad (3)$$

where $\varphi^{(1)}(\mathbf{k}_1,\omega)=ik^{-2}\mathbf{k}\cdot\mathbf{E}^{(1)}(\mathbf{k},\omega)$ is the potential associated with the primary wave

$$C_i(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2)=-\frac{4\pi}{\omega_1+\omega_2}\sum_\alpha\frac{e_\alpha^3}{m_\alpha^2}$$
$$\times\int\frac{v_i\,d\mathbf{v}}{(\omega_1+\omega_2)-(\mathbf{k}_1+\mathbf{k}_2)\mathbf{v}}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\right)\frac{(\mathbf{k}_2\partial/\partial\mathbf{v})f_\alpha^0}{\omega_2-\mathbf{k}_2\mathbf{v}}\qquad (4)$$

for a transverse secondary wave the tensor D is of the form

$$D_{ij}^t(\mathbf{k},\omega)=(\delta_{ij}-k_ik_j/k^2)(k^2c^2/\omega^2-\varepsilon^t(\mathbf{k},\omega))^{-1},\qquad (5)$$

and for a longitudinal secondary wave this tensor is

$$D_{ij}^l(\mathbf{k},\omega)=-k_ik_jk^{-2}(\varepsilon^l(\mathbf{k},\omega))^{-1}\qquad (6)$$

(ϵ^l and ϵ^t are the longitudinal and transverse dielectric constants of the plasma).

The scattering and conversion intensities are characterized by the change Σ in the square of the amplitude of the secondary wave per unit time

$$\Sigma\equiv\frac{d}{dt}\langle\mathbf{E}^{(2)2}\rangle=\int d\Sigma\qquad (7)$$

(the symbol ⟨...⟩ denotes averaging over the fluctuations).

We first consider the case in which the interaction of two longitudinal waves leads to the formation of a transverse wave. If both colliding waves are fluctuation waves we have from (3) and (5)

$$d\Sigma=U(\mathbf{k})\frac{d\mathbf{k}}{(2\pi)^3},$$
$$U(\mathbf{k})=\frac{\Omega^2+k^2c^2}{8k^2}\int\frac{d\mathbf{k}_1d\omega_1}{(2\pi)^4}[\mathbf{k}\mathbf{C}(\mathbf{k}_1,\omega_1;\mathbf{k}-\mathbf{k}_1,\omega-\omega_1)]^2$$
$$\times\langle\varphi^2\rangle_{\mathbf{k}_1\omega_1}\langle\varphi^2\rangle_{\mathbf{k}-\mathbf{k}_1;\,\omega-\omega_1},\qquad (8)$$

where $\langle\varphi^2\rangle_{\mathbf{k}\omega}$ is the Fourier component of the correlation function for the potential

$$\langle\varphi^2\rangle_{\mathbf{k}\omega}=\int d\mathbf{r}\,dt\,e^{-i\mathbf{k}\mathbf{r}+i\omega t}\langle\varphi(\mathbf{r}+\mathbf{r}_1,t+t_1)\varphi(\mathbf{r}_1,t_1)\rangle$$

while $\Omega^2=4\pi e^2n_0/m$ is the square of the electron plasma frequency.

In deriving Eq. (8) we have neglected higher correlations and written the fourth-order correlation in the form

$$\langle\varphi(\mathbf{k}_1,\omega_1)\varphi(\mathbf{k}_2,\omega_2)\varphi^*(\mathbf{k}_3,\omega_3)\varphi^*(\mathbf{k}_4,\omega_4)\rangle$$
$$=(2\pi)^8\langle\varphi^2\rangle_{\mathbf{k}_1\omega_1}\langle\varphi^2\rangle_{\mathbf{k}_2\omega_2}\{\delta(\mathbf{k}_1-\mathbf{k}_3)$$
$$\times\delta(\omega_1-\omega_3)\delta(\mathbf{k}_2-\mathbf{k}_4)\delta(\omega_2-\omega_4)$$
$$+\delta(\mathbf{k}_1-\mathbf{k}_4)\delta(\omega_1-\omega_4)\delta(\mathbf{k}_2-\mathbf{k}_4)\delta(\omega_2-\omega_3)\}.$$

If one of the colliding waves is an external wave ($\varphi^{(1)}(\mathbf{r},t)=\varphi_0\exp\{i\mathbf{k}_0\cdot\mathbf{r}-i\omega_0t\}$), the conversion coefficient $d\sigma=d\Sigma/|E_0|^2$ is given by

$$d\sigma=\frac{\Omega^2+k^2c^2}{16k^2k_0^2}$$
$$\times[\mathbf{k}\mathbf{C}(\mathbf{k}_0,\omega_0;\mathbf{k}-\mathbf{k}_0,\omega-\omega_0)]^2\langle\varphi^2\rangle_{\mathbf{k}-\mathbf{k}_0,\,\omega-\omega_0}\frac{d\mathbf{k}}{(2\pi)^3}.\qquad (9)$$

We now consider the case in which the secondary wave is a longitudinal wave. If both colliding waves are fluctuation waves we have from Eqs. (3) and (6)

$$d\Sigma=U(\mathbf{k})\frac{d\mathbf{k}}{(2\pi)^3},$$
$$U(\mathbf{k})=\frac{1}{2k^2}\left|\frac{\partial\varepsilon^l(\mathbf{k},\omega)}{\partial\omega}\right|^{-2}\int\frac{d\mathbf{k}_1d\omega_1}{(2\pi)^4}$$
$$\times(\mathbf{k}\mathbf{C}(\mathbf{k}_1,\omega_1;\mathbf{k}-\mathbf{k}_1,\omega-\omega_1))^2\langle\varphi^2\rangle_{\mathbf{k}_1\omega_1}\langle\varphi^2\rangle_{\mathbf{k}-\mathbf{k}_1,\,\omega-\omega_1}.\qquad (10)$$

If one of the colliding waves is an external wave the quantity $d\sigma$ is

$$d\sigma=\frac{1}{4k^2k_0^2}\left|\frac{\partial\varepsilon^l(\mathbf{k},\omega)}{\partial\omega}\right|^{-2}(\mathbf{k}\mathbf{C}(\mathbf{k}_0,\omega_0;\mathbf{k}-\mathbf{k}_0,\omega-\omega_0))^2$$
$$\times\langle\varphi^2\rangle_{\mathbf{k}-\mathbf{k}_0,\,\omega-\omega_0}\frac{d\mathbf{k}_1}{(2\pi)^3}.\qquad (11)$$

In concluding this section we present an expression for the function C in the case in which one of the colliding waves is a Langmuir wave ($\omega_1\sim\Omega$) and the second a low-frequency wave ($\omega_2\ll\Omega$). According to Eq. (4)

$$\mathbf{C}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2)=(e\Omega^2/\omega^2T_e)\mathbf{k}_1\qquad(\omega=\omega_1+\omega_2),\qquad (12)$$

*rot H = curl H; $[\mathbf{v}\mathbf{H}^{(n)}]=\mathbf{v}\times\mathbf{H}^{(n)}$.

where T_e is the electron temperature of the plasma.

3. WAVE CONVERSION AND SCATTERING IN A PLASMA IN THE PRESENCE OF A DIRECTED ELECTRON MOTION

We consider wave scattering and conversion in a nonisothermal plasma in which the electrons move with respect to the ions, which are assumed to be at rest. We shall be especially interested in critical fluctuations; in this case the directed electron velocity u approaches the phase velocity of nonisothermal sound $S = \sqrt{T_e/M}$ (M is the ion mass). The singularity associated with the critical fluctuations arises in the term in the expression for the potential correlation, corresponding to the possibility of the propagation of acoustic oscillations in the plasma:[6,7]

$$\langle \varphi^2 \rangle^S_{\mathbf{q}\omega} = \frac{2(2\pi)^2 T_e^2 S^2 q^2}{\Omega^2 m |\omega - \mathbf{qu}|} \delta(\omega^2 - q^2 S^2). \quad (13)$$

Substituting Eqs. (12) and (13) in Eq. (9) we obtain the following expression for the coefficient describing the conversion of an external Langmuir wave into a transverse wave

$$d\sigma = \frac{\pi^2 e^2 (qS)^2}{2m|\Delta\omega - \mathbf{qu}|} \frac{[\mathbf{k}, \mathbf{k}_0]^2}{k^2 k_0^2} \delta(\Delta\omega^2 - q^2 S^2) \frac{d\mathbf{k}}{(2\pi)^3}, \quad (14)$$

where $\Delta\omega = \sqrt{\Omega^2 + k^2c^2} - \Omega - \mathbf{k}_0 \cdot \mathbf{u} - \frac{3}{2}\Omega(ak_0)^2$ is the change in frequency and $\mathbf{q} = \mathbf{k} - \mathbf{k}_0$ is the change in wave vector characteristic of the conversion (a is the electron Debye radius). When $\mathbf{q} \cdot \mathbf{u} \sim \pm qS$ the coefficient for the δ-function in the expression for $d\sigma$ becomes anomalously large.

Integrating Eq. (14) over the modulus of the vector $\mathbf{k}$ we find the coefficient for conversion of Langmuir waves into transverse waves (per unit solid angle). With $u \approx S$ and $\sin\theta_0 \ll 1$ we have

$$\frac{d\sigma}{do} = \frac{e^2 k_0 \Omega}{16\pi m c^2} f(\theta, \theta_0), \quad (15)$$

where

$$f(\theta, \theta_0) = \frac{\sin^2\theta}{|\cos\theta|}\left[\sqrt{1 + \frac{2c^2k_0}{\Omega S\cos^2\theta}\left(1 + \frac{u}{S}\cos\theta_0\right)} - 1\right]^{-1} \quad (\cos\theta < 0),$$

$$f(\theta, \theta_0) = \frac{\sin^2\theta}{|\cos\theta|}\left\{\left[\left\{1 + \frac{2c^2k_0}{\Omega S\cos^2\theta}\left(1 + \frac{u}{S}\cos\theta_0\right)\right\}^{1/2} - 1\right]^{-1} + \left[1 - \left\{1 - \frac{2c^2k_0}{\Omega S\cos^2\theta}\left(1 - \frac{u}{S}\cos\theta_0\right)\right\}^{1/2}\right]^{-1}\right\} \quad (\cos\theta > 0).$$

(Here and below, θ and θ_0 are the angles formed by the vectors $\mathbf{k}$ and $\mathbf{k}_0$ with the direction of $\mathbf{u}$; the angle between the vectors $\mathbf{k}$ and $\mathbf{k}_0$ is designated by ϑ.)

It is evident that $d\sigma/do$ becomes anomalously large when $1 \pm u\cos\theta_0/S \ll \Omega S\cos^2\theta/c^2k_0$. We note that this condition imposes very stringent limitations on the angle θ_0 and on the quantity u/S.

We now find the growth rate for transverse waves caused by scattering of fluctuation Langmuir oscillations by critical fluctuations with conversion into transverse waves. Assuming that the Langmuir term in the expression for the potential correlation is of the form[1-3]

$$\langle \varphi^2 \rangle^L_{\mathbf{q}\omega} = 2(2\pi)^2 \Omega T_e q^{-2} \delta(\bar{\omega}^2 - \Omega^2 - 3\Omega^2 (aq)^2) \quad (16)$$

($\bar{\omega} = \omega - \mathbf{q} \cdot \mathbf{u}$ is the frequency in the reference system in which the electrons are at rest) and using Eqs. (8), (12) and (13), we have

$$U(\mathbf{k}) = \frac{e^2 T_e \Omega}{m} \int d\mathbf{k}_1 d\omega_1 \frac{[\mathbf{kk}_1]^2}{k^2 k_1^2} \frac{(qS)^2}{|\Delta\omega - \mathbf{qu}|} \delta(\Delta\omega^2 - q^2 S^2) \times \delta([\omega_1 - \mathbf{k}_1\mathbf{u}]^2 - \Omega^2), \quad (17)$$

where $\Delta\omega = \sqrt{\Omega^2 + c^2k^2} - \omega_1$ and $\mathbf{q} = \mathbf{k} - \mathbf{k}_1$.

It is evident that the function $U(\mathbf{k})$ becomes infinite when $u \approx S$ if $\cos\theta \to c^2k/2\Omega u$. We can estimate the coefficient of the resonance denominator $|c^2k^2/2\Omega - \mathbf{k} \cdot \mathbf{u}| - 1$ by integrating Eq. (17) with respect to k_1 up to $k_1 = \tilde{a}^{-1}$, where $\tilde{a}$ is equal to several Debye lengths. We thus find

$$U(\mathbf{k}) \sim \frac{e^2 T_e \sin^2\theta}{m\tilde{a}^3} \left|\frac{c^2k^2}{2\Omega} - \mathbf{ku}\right|^{-1}. \quad (18)$$

According to Eqs. (15) and (18) the radiation from a plasma due to conversion of fluctuations or external Langmuir waves into transverse waves is anomalously large only in the long wavelength region, where $k \sim \Omega u/c^2$.

We now consider the scattering of Langmuir oscillations. Using Eqs. (11) and (13) for the scattering coefficient we have

$$d\sigma = \frac{(\pi e)^2}{2m} \frac{(qS)^2}{|\Delta\omega - \mathbf{qu}|} \cos^2\vartheta\, \delta(\Delta\omega^2 - q^2S^2) \frac{d\mathbf{k}}{(2\pi)^3}, \quad (19)$$

where $\Delta\omega = \frac{3}{2}\Omega^2(k^2 - k_0^2) + \mathbf{q} \cdot \mathbf{u}$ is the change in frequency caused by scattering.

Integrating $d\sigma$ with respect to the modulus of the vector $\mathbf{k}$, it is easily shown that if

$$(\cos\theta - \cos\theta_0)^2 = (2S/u)^2 \sin^2(\vartheta/2) \quad (20)$$

the expression $d\sigma/do$ becomes infinite. Near directions determined by Eq. (20) $d\sigma/do$ is of the form

$$\frac{d\sigma}{do} = \frac{e^2k_0^2}{16\pi m} \frac{\sin(\vartheta/2)\cos^2\vartheta\,|u\cos\theta_0 + 3\Omega a^2 k_0|^{-1}}{|(u/S)(\cos\theta_0 - \cos\theta) \pm 2\sin(\vartheta/2)|}. \quad (21)$$

4. WAVE CONVERSION IN A PLASMA IN THE PRESENCE OF A BEAM

We now consider wave conversion in a plasma in the presence of a beam. It is of special interest to investigate the case in which the beam velocity is close to the critical velocity at which the Langmuir oscillations become unstable, assuming that the wave vector is equal to the change in wave vector due to scattering.

Under these conditions the correlation function for the potential contains the term[7] 1)

$$\langle\varphi^2\rangle^L_{q\omega} = \frac{2(2\pi)^2\Omega^2 T_1}{q^2|\omega - \mathbf{qu}|}\delta(\omega^2 - \Omega_q^2), \tag{22}$$

which can be anomalously large. Here

$$\Omega_q^2 = \Omega^2 + 3\Omega^2(aq)^2 - (\Omega\Omega_1)^2 m/q^2 T_1,$$

$\Omega_1^2 = 4\pi e^2 n_1/m$; u, T_1 and n_1 are the mean velocity, temperature and density of the beam (we assume that the beam is hot).

We first consider the emission from a plasma due to the conversion of two fluctuating longitudinal waves into a transverse wave. This emission is due to processes of two kinds: scattering of two Langmuir waves on each other and scattering of Langmuir waves on low-frequency fluctuations.

In the low-frequency region ($\omega \ll q\sqrt{T/M}$) the correlation function for the potential fluctuations is[1-3]

$$\langle\varphi^2\rangle^D_{q\omega} = \frac{T^2}{4e^2 n_0 q}\sqrt{\frac{2\pi M}{T}}, \tag{23}$$

where T is the plasma temperature. Substituting this expression in Eq. (8) and using Eq. (12) we have

$$U^D(\mathbf{k}) \sim 0.4\frac{e^2\Omega^2 T_1}{m}\int d\mathbf{k}_1 \frac{[\mathbf{kk}_1]^2}{k^2 k_1^2}\frac{\delta(\omega^2 - \Omega_{k_1}^2)}{\Omega_{k_1} - \mathbf{k}_1\mathbf{u}}, \tag{24}$$

where the integration is carried out over the region $|\mathbf{k}\cdot\mathbf{u}| < \Omega_{k_1}(1 - 1/\Delta)$; Δ is a large parameter that characterizes the maximum possible amplitude of the critical fluctuations (this cutoff parameter can only be found exactly from a nonlinear theory).

It is evident from Eq. (24) that the transverse radiation produced as a result of conversion is longwave radiation $k \ll \Omega/c$. To be definite we assume that $ck/\Omega \gg (n_1 T/k_0 T_1)^{1/4}$ and the conversion coefficient if found to be

$$U^D(\mathbf{k}) \sim 0{,}3\,(e^2 T_1/mua^2)\ln\Delta. \tag{25}$$

We now consider the contribution to $d\Sigma$ from scattering of Langmuir waves by Langmuir waves. In this case, in accordance with Eq. (4) the function C is

$$\mathbf{C}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2) = \frac{e\Omega^2}{m\omega^2}\frac{\mathbf{k}_1}{\omega_2}\left(\frac{\mathbf{k}_2^2}{\omega_2} - \frac{\mathbf{k}_1^2 - \mathbf{k}_2^2}{\omega_1 + \omega_2}\right) \quad (\omega = \omega_1 + \omega_2). \tag{26}$$

1)In neglecting nonlinear effects of the interaction between fluctuations we can use Eq. (22) in the range of frequencies ω and wave vectors q in which the plasma oscillations are nongrowing.

Substituting Eqs. (22) and (26) in Eq. (8), we have

$$U^l(\mathbf{k}) = \frac{e^2\Omega^8 T_1^2}{2m^2\omega^2}\int\frac{d\mathbf{k}_1 d\omega_1[\mathbf{kk}_1]^2}{(\mathbf{k}-\mathbf{k}_1)^2(\omega-\omega_1)^2 k^2 k_1^2}$$
$$\times\left[\frac{(\mathbf{k}-\mathbf{k}_1)^2}{\omega-\omega_1} - \frac{2\mathbf{kk}_1 - k^2}{\omega}\right]^2\frac{\delta(\omega_1^2 - \Omega_{k_1}^2)\delta([\omega-\omega_1]^2 - \Omega_{\mathbf{k}-\mathbf{k}_1}^2)}{|\omega_1 - \mathbf{uk}_1|\,|(\omega-\omega_1) - \mathbf{u}(\mathbf{k}-\mathbf{k}_1)|}, \tag{27}$$

where the integration is carried out over the region

$$\tilde{a}k_1 < 1, \qquad |\mathbf{u}\,\mathbf{k}_1| < \Omega_{\mathbf{k}_1}(1 - 1/\Delta),$$
$$|\mathbf{u}(\mathbf{k}-\mathbf{k}_1)| < \Omega_{\mathbf{k}-\mathbf{k}_1}(1 - 1/\Delta).$$

It is easily shown that the quantities U^L and U^D do not have poles due to the existence of critical fluctuations. The quantity U^L is of order

$$U^L(\mathbf{k}) \sim 0.1\,(e^2 T_1^2/m^2 u\tilde{a}^4)\,\delta(c^2k^2 - 3\Omega^2)\ln\Delta. \tag{28}$$

We now find the coefficient for the conversion of an external Langmuir wave into a transverse wave. Using Eqs. (9), (22) and (26) we have

$$d\sigma = \frac{\pi^2 e^2(\Omega^2 + k^2c^2)}{128 m^2\Omega^2}T_1\frac{(3q^2 - k_0^2)^2}{|\Delta\omega - \mathbf{qu}|q^2}\sin^2\vartheta\,\delta(\Delta\omega^2 - \Omega_q^2)\frac{d\mathbf{k}}{(2\pi)^3}. \tag{29}$$

It is evident that as $|\mathbf{q}\cdot\mathbf{u}| \to \Omega_q$ the coefficient of the δ-function in this expression grows without limit.

Integrating Eq. (29) with respect to the modulus of the vector **k** we have

$$\frac{d\sigma}{do} = \frac{\sqrt{3}e^2\Omega T_1}{64\pi m^2c^3}\frac{k_0^2\sin^2\vartheta}{\Omega + \mathbf{k}_0\mathbf{u}}. \tag{30}$$

If the projection of the wave vector of the external Langmuir wave in the direction of u is approximately $(-\Omega/u)$ the quantity $d\sigma/do$ characterizing the growth rate of the transverse waves propagating in a given direction is found to be anomalously large.

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