ON THE IRROTATIONAL REGION IN ROTATING He II

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It is shown that an irrotational region is formed not only when helium rotates between two cylinders but also in an ordinary vessel, where the irrotational region surrounds the central vortex.

It was shown by Bendt and Oliphant [1] and by Khalatnikov and one of the authors [2] that when He II is rotated between two coaxial cylinders a vortex-free region which does not contain Onsager-Feynman vortex filaments is formed around the internal cylinder. In this region the superfluid executes potential rotation and the circulation around the internal cylinder is several orders of magnitude larger than the circulation quantum $\kappa = 2\pi\hbar/m$ (m — mass of the helium atom).

It is natural to assume that the formation of an internal vortex-free region may be energetically convenient not only in the presence of an internal cylinder. One cannot exclude the possibility that such a region surrounds a central vortex situated along the rotation axis, the circulation Γ of which can be also very large compared with the circulations κ of the remaining vortices and the core of which is a natural internal cylinder.

In order to check on this hypothesis, we carried out calculations by a procedure which does not differ in principle from that used in [1,2] (minimization of the free energy at T = 0°K). The main difference lies in the following: (1) The expression for the free energy has been supplemented by a term $2\pi r_1\sigma$, where r_1 is the radius of the core of the central vortex and σ is the surface tension coefficient; (2) the free energy is minimized not only with respect to Γ and r_i (r_i is the radius of the irrotational region), but also with respect to r_1.

The solution of the equations resulting from the minimization leads to an implicit dependence of the sought values of r_1, r_i, and Γ on the angular velocity ω_0, via the parameter x which determines these four quantities. The results of the numerical estimates are listed in Table I. It was found that, at all rotational velocities usually encountered in the experiments on rotating He II, the circulation of the central vortex was very close to κ, and the radius of its core was smaller than any of the minimal estimates of the radius a_0 of the Onsager-Feynman vortex core. This means that the quantities r_1, r_i, and Γ given in Table I, which ensure the minimum of the free energy, cannot be realized in practice.

Table I

x	ω_0, sec^{-1}	$10^9 r_1$, cm	$10^2 r_i$, cm	Γ/κ
10^{15}	0.12	7.2	23	1.17
10^{14}	1.1	7.4	7.4	1.19
10^{13}	9.6	7.7	2.4	1.22
$5\cdot10^{12}$	18	7.8	1.8	1.22
$3\cdot10^{12}$	31	7.9	1.4	1.23

We have therefore revised the estimate of r_i, minimizing the free energy only with respect to this quantity and putting $\Gamma = \kappa$ and $r_1 = a_0 = 2.7 \times 10^{-8}$ cm. The results of this calculation are listed in Table II. It also shows the ratio r_i/b, where $b = (\kappa/2\pi\omega_0)^{1/2}$ is a quantity on the order of half the distance between the Onsager-Feynman vortices outside the irrotational region.

It is found thus that the rotation of helium in a vessel without an internal cylinder does not lead to very large values of the circulation about the axis of rotation. On the axis there is the usual quantized Onsager-Feynman vortex. This vortex, however, is singled out among the remaining vortices in that the distances between the remaining

Table II

ω_0, sec^{-1}	$10^2 r_i$, cm	r_i/b
0,01	82	6.6
0,1	25	6.4
1	7.8	6.2
5·	3.4	6.0
10	2.4	6.0
20	1.7	5.9
30	1.4	5.9

vortices and the central one are somewhat larger than the distances between the remaining vortices themselves, so that there is a small irrotational region around the axis. Although this picture does not correspond to the results of the complete minimization of the free energy, it is nevertheless energetically more favored than the case of uniform distribution of vortices.

According to the experimental data obtained by Tsakadze[3], the radius of the irrotational region for $\omega_0 = 1\ \text{sec}^{-1}$ and $T = 1.38°K$ is of the order of 1 mm, which is close to our estimate 0.8 mm at $T = 0°K$ (Table II).

[1] P. J. Bendt and T. A. Oliphant, Phys. Rev. Lett. **6**, 213 (1961).

[2] M. P. Kemoklidze and I. M. Khalatnikov, in press.

[3] Dzh. S. Tsakadze, JETP, in press.

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