K_{e5} *DECAY*

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Submitted to JETP editor June 11, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 2009-2014 (December, 1963)

The K_{e5} decay rates are calculated for the cases of direct interaction and interaction via an intermediate η meson. The isotopic relations for various charge channels of the reaction are considered on the basis of the $\Delta T = 1/2$ rule.

1. INTRODUCTION

K meson decays are at present the subject of much attention. Of particular importance for a test of the Sakata model [1] are the leptonic decays of the K mesons, K_l:

$$K \to n\pi + l + \nu, \tag{1.1}$$

where l stands for the electron or the muon.

The K_{l3} and K_{l4} decays have been considered in the literature. [2-4] Energy conservation still allows for the K_{e5} decay, whereas decays with larger numbers of π mesons and the $K_{\mu5}$ decay are energetically impossible. In the present paper we calculate the K_{e5} decay rate.

Assuming, according to the Sakata model, that the leptonic decays of strange particles are due to the interaction $(\bar{p}\Lambda)(\bar{e}\nu) + (\bar{\Lambda}p)(\bar{\nu}e)$, it can be shown that the K_l decays satisfy the selection rules [5]

$$\Delta Q = \Delta S = \pm 1, \quad \Delta T = 1/2, \tag{1.2}$$

where ΔQ, ΔS, and ΔT are, respectively, the change of the charge, the strangeness, and the isotopic spin of the strongly interacting particles.

2. MATRIX ELEMENT

In first order in the weak interaction the K_{e5} decay is described by a single Feynman graph (Fig. 1), where the bubble A represents the interaction of the strongly interacting particles. The matrix element for the K_{e5} decay can be written in the form of a product of the lepton current $j_l = \nu O e$ and the current of the strongly interacting particles $j_s = \varphi_K V \varphi_1^+ \varphi_2^+ \varphi_3^+$:

$$M = G\, 2^{-1/2} j_s^+ j_l, \tag{2.1}$$

where $G = 1.01 \times 10^{-5}/m^2$ (m is the nucleon mass) is the weak interaction constant, and φ_K, φ_1, φ_2, φ_3, ν, and e are the wave functions of the K meson, the three π mesons, the neutrino, and the electron, respectively.

According to the V-A theory we have $O = \gamma_\alpha(1 + \gamma_5)$. As there exists no theory of strong interactions, the vector cannot be determined; only its most general form satisfying the requirements of relativistic invariance can be given:

$$V_\alpha = f_1 q_{1\alpha} + f_2 q_{2\alpha} + f_3 q_{3\alpha} + f_4 Q_\alpha + f_5 \varepsilon_{\alpha\mu\nu\sigma} q_{1\mu} q_{2\nu} q_{3\sigma} + \varepsilon_{\alpha\mu\nu\sigma} Q_\mu (f_6 q_{1\nu} q_{2\sigma} + f_7 q_{1\nu} q_{3\sigma} + f_8 q_{2\nu} q_{3\sigma}), \tag{2.2}$$

where Q, q_1, q_2, and q_3 are the four-momenta of the K meson and the π mesons; the functions f depend on the various invariants formed from the momenta of the strongly interacting particles. In a phenomenological theory they are unknown and are, as usual, considered to be constants.

Since the Q value of the reaction is small, the orbital angular momenta of the π mesons will be zero, and hence the matrix element must be symmetric under the interchange of an arbitrary pair of π mesons. Therefore, we must discard in (2.2) all terms except the first four; we assume further that $f_1 = f_2 = f_3 = f_4 = f$. Thus we have for the final form of the vector V_α

$$V_\alpha = f(q_{1\alpha} + q_{2\alpha} + q_{3\alpha}). \tag{2.3}$$

3. THE K_{e5} DECAY RATE

For generality, let us consider the decay of a K meson into n π mesons and a lepton pair. The differential probability for such a decay is

$$dw_n = \frac{(2\pi)^4 \delta^{(4)}(Q - q - q_e - q_\nu)}{2M(2\pi)^{3n+6} 2^{n+2}} \frac{d^3q_e d^3q_\nu}{E_e E_\nu} \prod_{i=1}^{n} \frac{d^3 q_i}{E_i} \sum_{e,\nu} |M|^2, \tag{3.1}$$

where

$$q = q_1 + \ldots + q_n,$$

$$q_i^2 = m^2, \quad q_e^2 = m_e^2, \quad q_\nu^2 = 0, \quad Q^2 = M^2.$$

Summing over the electron and neutrino polarizations, we obtain

$$\sum_{e,\nu} |M|^2 = 4G^2f^2\,[2\,(q_eq)\,(q_\nu q) - (q_eq_\nu)\,(q^2)].$$

Using the method developed by one of the authors[6] we can easily reduce the total decay rate to a single integral (see the Appendix):

$$W_n = \frac{G^2f^2M^{2n+3}\,i^n\beta^3\alpha^{2n+1}}{2^{3n+2}\,\pi^{n-1}}\,\frac{\partial}{\partial\beta}\left(\alpha\frac{\partial^2}{\partial\alpha^2} - 3\frac{\partial}{\partial\alpha}\right)$$
$$\times \int \frac{dz}{z^{n+5}}\,\frac{H_1^{(1)}(z)\,H_1^{(2)}(\beta z)}{\beta}\left[\frac{H_1^{(2)}(\alpha z)}{\alpha}\right]^n, \qquad (3.2)$$

where $\beta = m_e/M$, $\alpha = m/M$, and the contour of integration goes around the negative real axis in the z plane.

The integral (3.2) is not expressible in terms of elementary functions. However, its value can be obtained in the relativistic ($m_e \approx 0$, $m \approx 0$) and nonrelativistic ($m_e \approx 0$, $nm \approx M$) limits:

$$W_n^r = \frac{G^2f^2nM^{2n+3}}{\pi^{2n+1}\,2^{4n+2}\,\Gamma(n+4)\,\Gamma(n+2)}, \qquad (3.3)$$

$$W_n^{nr} = \frac{G^2f^2n^2M^{(n-1)/2}\,(M - nm)^{(3n+7)/2}}{2^{(5n-3)/2}\pi^{3(n+1)/2}\,n^{n/2}\,\Gamma[(3n+9)/2]}. \qquad (3.4)$$

We further find for the ratio of the K_{e4} and K_{e5} decay rates

$$\frac{W_3^{nr}}{W_2^{nr}} = \left(\frac{Mf_5}{f_4}\right)^2 \frac{\Gamma(7{,}5)\,(1 - 3m/M)^8}{6\pi\sqrt{6\pi}\,\Gamma(9)\,(1-2m/M)^{6,5}} \approx 2.5\cdot 10^{-8}. \qquad (3.5)$$

Since the interaction constants f are made dimensionless with the help of a mass of order M, it is clear that the K_{e5} is a very rare phenomenon.

4. K_{e5} DECAY VIA THE η MESON

The phenomenological treatment of the graph of Fig. 1 given in the preceding section led to a very small K_{e5} decay rate. However, it might be expected that the probability for such a decay via the η meson, whose mass is close to 3m, will be somewhat larger on account of the smallness of the denominator of the propagation function. Let us, therefore, consider the K meson decay illustrated by the graph of Fig. 2.

The matrix element for this decay has the form

$$M = f\,\frac{\bar{\nu}\hat{q}\,(1+\gamma_5)\,e}{q^2 - \mu^2}\,F, \qquad (4.1)$$

where q and μ are the momentum and the mass of the η resonance. The denominator in the matrix element can be considered constant, since $q^2 \approx (3m)^2 \approx M^2$ in the nonrelativistic approximation.

The decay rate of the K meson can in this case be expressed in terms of $W_3^{nr}(K_e)$:

$$W_\eta(K_{e5})/W_3^{nr}(K_{e5}) = f^2F^2/f_5^2\,(\mu^2 - M^2)^2. \qquad (4.2)$$

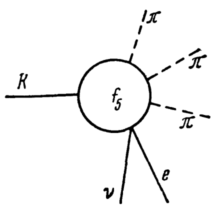

FIG. 1

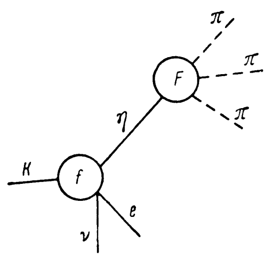

FIG. 2

If we assume that unitary symmetry is not violated strongly, we can estimate the values of the constants f and F. According to the work of Kobzarev and Okun',[7] the relation $f = f_3\sqrt{3}$ must hold between the constants for the K decay into a lepton pair and η and π mesons, respectively. For an estimate of F we use the relation between the partial widths Γ[8] for $\eta \to 2\gamma$ and $\eta \to \pi^+\pi^0\pi^-$, $\Gamma(\eta \to 2\gamma)/\Gamma(\eta \to \pi^+\pi^0\pi^-) = 1.32$, and the connection between the decay constants for $\eta \to 2\gamma$ and $\pi \to 2\gamma$ from unitary symmetry, $f_\pi = f_\eta\sqrt{3}$.

Finally, we obtain for the ratio (4.2)

$$\frac{W_\eta}{W_3^{nr}} = \left(\frac{f_3\sqrt{3}}{f_5M^2}\right)^2 \frac{\pi^2 2^7 3\sqrt{3}}{(\mu^2/M^2 - 1)^2\,(1 - 3m/\mu)^2}$$
$$\times \left(\frac{f_\pi}{f_\eta}\right)^2 \left(\frac{\mu}{m}\right)^3 \frac{\Gamma(\pi \to 2\gamma)}{\mu} \approx 10^{-3}. \qquad (4.3)$$

Thus the decay of the K meson through the η resonance cannot increase the probability for the K_{e5} decay obtained in the preceding section.

5. ISOTOPIC RELATIONS

By charge conservation and the selection rules (1.2), the following decay channels are allowed for the K^+ and K^0 mesons:

$$K^+ \to \pi^0 + \pi^0 + \pi^0 + e^+ + \nu, \qquad (5.1)$$
$$K^+ \to \pi^0 + \pi^+ + \pi^- + e^+ + \nu, \qquad (5.2)$$
$$K^0 \to \pi^0 + \pi^0 + \pi^- + e^+ + \nu, \qquad (5.3)$$
$$K^0 \to \pi^+ + \pi^- + \pi^- + e^+ + \nu. \qquad (5.4)$$

The decay rates for these reactions will be denoted by W_1, W_2, W_3, and W_4, respectively. The decays of the $\bar{K}^0$ and K^- mesons are obtained from (5.1) to (5.4) by charge conjugation.

The derivation of the isotopic relations is in our case the same as in the well known τ decay.[9-13] Since the isotopic spin of the K meson is ½, the π mesons can, according to the $\Delta T = 1/2$ rule, be in the states with total isotopic spin T = 0 and T = 1. For a derivation of the isotopic relations it is convenient to introduce the unphysical "spurion" S with $T = 1/2$ and $T_3 = -1/2$.[2]

The reactions (5.1) to (5.4) can be regarded as processes $K + S \to 3\pi$ which conserve isotopic spin.

By the rules of vector addition, the wave functions of the initial state can be written in the form

$$|K^+\rangle \equiv |K^+S\rangle = 2^{-1/2}(|1,0\rangle - |0,0\rangle),$$
$$|K^0\rangle = |K^0S\rangle = |1,-1\rangle, \qquad (5.5)$$

where $|T, T_3\rangle$ denotes the eigenfunction of the operator of the total isotopic spin T^2 and its projection T_3 with the eigenvalues $T(T+1)$ and T, respectively. The wave functions of the final states can be expressed in terms of the amplitudes $a(0)$ and $a(1)$ for the transitions of the system between the states with given isotopic spin T:

$$2^{-1/2}(|1,0\rangle - |0,0\rangle) \to 2^{-1/2}(a(1)\,|1,0\rangle - a(0)\,|0,0\rangle),$$
$$|1,-1\rangle \to a(1)\,|1,-1\rangle. \qquad (5.6)$$

Following the work of Berestetskiĭ,[14] let us consider now the charge distribution of the π mesons in the states with the isotopic spin 0 and 1. The wave function of each of the π mesons is a product of the space and spinor parts. The isotopic part is a vector in isotopic space which will be denoted by π in the following. The projection of this vector on the coordinate axis corresponds to the various charge states of the π meson.

There is only one wave function for the three π mesons in the state with $T = 0$, $\Phi = (\pi_1, [\pi_2, \pi_3])\Psi$, where Ψ is the space part of the wave function, since it is impossible to construct any other scalar from the vectors π_1, π_2, and π_3. This wave function corresponds to the single charge distribution $\pi^+\pi^0\pi^-$.

The state with $T = 0$ is, according to group theory, described by two wave functions, corresponding to the two different irreducible representations of the commutation group:

$$\Phi_A = \Psi_A\,[\pi_1(\pi_2,\pi_3) + \pi_2(\pi_3,\pi_1) + \pi_3(\pi_1,\pi_2)],$$
$$\Phi_B = \Psi_{1B}e_1 + \Psi_{2B}e_2 + \Psi_{3B}e_3, \qquad (5.7)$$

where

$$e_1 = 2\pi_1(\pi_2\pi_3) - \pi_2(\pi_1\pi_3) - \pi_3(\pi_1\pi_2),$$
$$e_2 = 2\pi_2(\pi_1\pi_3) - \pi_1(\pi_3\pi_2) - \pi_3(\pi_1\pi_2),$$
$$e_3 = 2\pi_3(\pi_2\pi_1) - \pi_2(\pi_3\pi_1) - \pi_1(\pi_2\pi_3)$$

and Ψ_A and Ψ_B are the space parts of the wave functions which have the symmetries given by the Young schemes A and B of Fig. 3. It follows from this that the wave function of the π mesons in the state with $T = 1$ can be represented as $\Phi = \alpha\Phi_A + \beta\Phi_B$, with $\alpha^2 + \beta^2 = 1$.

Writing the scalar products in (5.7) explicitly, we obtain the following charge distributions:

$$|1,1\rangle = \alpha\,[5^{-1/2}2\,(\pi^+\pi^+\pi^-) + 5^{-1/2}(\pi^+\pi^0\pi^0)]$$
$$+\beta\,[2^{-1/2}(\pi^+\pi^+\pi^-) + 2^{-1/2}(\pi^+\pi^0\pi^0)],$$
$$|1,0\rangle = \alpha\,[\sqrt{2/5}\,(\pi^+\pi^-\pi^0) + \sqrt{3/5}\,(\pi^0\pi^0\pi^0)] + \beta\,[\pi^+\pi^-\pi^0],$$
$$|1,-1\rangle = \alpha\,[5^{-1/2}\,2\,(\pi^-\pi^-\pi^+) + 5^{-1/2}(\pi^-\pi^0\pi^0)]$$
$$+\beta\,[2^{-1/2}(\pi^-\pi^-\pi^+) + 2^{-1/2}(\pi^-\pi^0\pi^0)],$$
$$|0,0\rangle = \pi^+\pi^0\pi^-. \qquad (5.8)$$

Substituting (5.8) in (5.6), we find for the reaction amplitudes

$$A_1 = \alpha a(1)\sqrt{3/10}, \quad A_3 = (\alpha/\sqrt{10} + \beta/2)a(1),$$
$$A_2 = (\alpha/\sqrt{5} + \beta/\sqrt{2})a(1)$$
$$- a(0)/\sqrt{2}, \quad A_4 = (\alpha\sqrt{2/5} + \beta/2)a(1). \qquad (5.9)$$

The magnitude of the coefficients α and β cannot be determined. But since the kinetic energy is small, we may assume that all three π mesons are in the S state, i.e., that the space part of the wave function is symmetric, so that the isotopic spin part must also be symmetric. The function Φ_B does not have this property, therefore the coefficient β must be zero. Furthermore, the state with $T = 0$ is also antisymmetric, so that we must take $a(0) = 0$. From (5.9) we obtain the following relations between the reaction probabilities:

$$W_1 : W_2 : W_3 : W_4 = 3:2:1:4. \qquad (5.10)$$

In conclusion we express our gratitude to L. B. Okun' and I. Yu. Kobzarev for suggesting this problem and constant interest in this work.

APPENDIX

In calculating the total decay rates we encounter the following types of integrals:

$$J = \int \frac{d^3q}{E} e^{-iqx}, \quad J_\alpha = \int \frac{d^3q}{E} e^{-iqx} q_\alpha, \quad J_{\alpha\beta} = \int \frac{d^3q}{E} e^{-iqx} q_\alpha q_\beta. \qquad (A.1)$$

Clearly, the following relation exists between them:

$$J_\alpha = i\partial J/\partial x_\alpha, \quad J_{\alpha\beta} = i^2\partial^2 J/\partial x_\alpha \partial x_\beta. \qquad (A.2)$$

The value of the integral J is well known:[6]

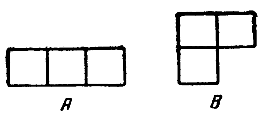

FIG. 3

$$J = 2\pi^2 i m^2 H_1^{(2)}(m\varkappa)/m\varkappa,$$

where $\kappa = \sqrt{x^2}$. The differentiation with respect to the coordinates in (A.2) can easily be replaced by a differentiation with respect to the masses. For example, we obtain for J_α

$$J_\alpha = -2\pi^2 m^2 \frac{\partial}{\partial x_\alpha}\left[\frac{H_1^{(2)}(m\varkappa)}{m\varkappa}\right] = -2\pi^2 m^2 \frac{\partial}{\partial m\varkappa}\left[\frac{H_1^{(2)}(m\varkappa)}{m\varkappa}\right]\frac{\partial m\varkappa}{\partial x_\alpha}$$

$$= -\frac{2\pi^2 m^3 x_\alpha}{\varkappa^3}\frac{\partial}{\partial m}\left[\frac{H_1^{(2)}(m\varkappa)}{m}\right]; \quad \frac{\partial}{\partial m\varkappa} = \frac{1}{\varkappa}\frac{\partial}{\partial m}.$$

The integral $J_{\alpha\beta}$ is computed in a similar way. By this method, the coefficients of the vector integrals are much faster and more simply evaluated than by the usual Dalitz method of invariant integration.

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Translated by R. Lipperheide
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