DECAYS OF THE HYPERNUCLEUS Σ^+p

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The probabilities for the decay of the hypernucleus Σ^+p into $d + \pi^+$, $2p + \pi^0$, and $p + n + \pi^+$ are calculated. The probability for decay into $d + \pi^+$ depends strongly on the binding energy and the spin of Σ^+p. Measurement of the rate of this decay can be employed to determine whether the π^+ decay of free Σ hyperons proceeds via the s or the p state. The spectrum of the π mesons in the three particle decays is discussed.

1. INTRODUCTION

THE existence of Σ hypernuclei is at present not yet convincingly established. Several attempts have been made to observe the bound systems Σ^+p and Σ^-n in the interaction of K^- with the nuclei of photoemulsions,[1-4] but these have not led to any definite results. Neither has the formation of Σ^-n hypernuclei in the reaction

$$K^- + d \to \Sigma^- n + \pi^+ \tag{1}$$

in the deuterium bubble chamber been observed.[5]

Notwithstanding the negative outcome of these experiments, it is at present still impossible definitely to rule out the existence of Σ hypernuclei. The mechanism of the formation of Σ hypernuclei in the capture of K^- by the nuclei of emulsions may be quite different from the mechanism of the formation of Λ hypernuclei. Σ hyperons formed in the nucleus are very likely to transform into Λ hyperons through the strong interaction with the nucleons. Hence the conditions in photoemulsions are much less favorable for the formation of Σ hyperfragments than for the formation of Λ hyperfragments. In the capture of K^- by the nuclei of deuterium the formation of Σ^-n will be forbidden if the spin of Σ^-n is equal to $I = 0$ and the parity of $K\Sigma$ negative.[6] Thus the problem of the existence of Σ hypernuclei remains open.[7]

The discovery of Σ hypernuclei would be of great interest for the study of the properties of the Σ-nucleon forces as well as for the determination of the characteristics of the decay interactions of the Σ hyperons. Of most interest from this point of view would be the observation of the hypernucleus Σ^+p, which can decay into the following channels:

$$\Sigma^+p \to d + \pi^+, \tag{2a}$$
$$\to p + n + \pi^+, \tag{2b}$$
$$\to p + p + \pi^0. \tag{2c}$$
$$\to p + p. \tag{2d}$$

The hypernucleus Σ^-n has only one, hard to identify, decay mode: $\Sigma^-n \to n + n + \pi^-$. This decay has been considered in the work of Lyul'ka[8] and Common.[9] In the present paper we restrict ourselves to the discussion of the pionic decays of Σ^+p of the type (2a) to (2c). We shall calculate the decay rates of (2a) to (2c) as functions of the spin and the binding energy of Σ^+p.

2. THE DECAY OF THE FREE Σ^+ HYPERON

The decays

$$\Sigma^+ \to p + \pi^0, \tag{3a}$$
$$\to n + \pi^+ \tag{3b}$$

have the following properties:[10]

1) the partial rates are approximately equal: $w(\pi^0) \approx w(\pi^+)$;

2) the asymmetry is maximal in the decay (3a) and is absent in the decay $\alpha_0 \approx 1$, $\alpha_+ \approx 0$.

It follows from these facts that the decay (3b) goes either only through the s or only through the p state. Both states contribute to the decay (3a). If we confine our attention to the decays of free Σ hyperons, we can determine the orbital angular momentum of the emitted π meson in the decay (3b) simply by studying the polarization of the neutrons in the decay of polarized Σ's. Such an experiment has been proposed by Chou Kuang-chao and others,[11,12] but has not yet been carried out.

The decays (3) will be described by the amplitude

$$M = G\int\left[\bar{\psi}_N (a + b\gamma_5)\gamma_\mu \psi_\Sigma \frac{\partial\varphi}{\partial x_\mu}\right] dx. \quad (4)$$

The coefficients a and b give the magnitude of the decay amplitude in the s and p states, respectively.

In the nonrelativistic approximation, the matrix element (4) for the decay of the Σ hyperon takes the form

$$M = G\int\Big\{a\left[\psi_N^*\psi_\Sigma\dot{\varphi} + \frac{i}{M_N}(\nabla\psi_N^*)\psi_\Sigma\nabla\varphi + \frac{i}{2M_N}\psi_N^*\psi_\Sigma\nabla^2\varphi\right]$$
$$+ b\left[\psi_N^*\sigma\psi_\Sigma\nabla\varphi + \frac{i}{M_N}(\nabla\psi_N^*)\sigma\psi_\Sigma\dot{\varphi}\right.$$
$$\left.+ \frac{i}{2M_N}\psi_N^*\sigma\psi_\Sigma\nabla\dot{\varphi}\right]\Big\} dx, \quad (5)$$

where M_N is the mass of the nucleon.

From (5) we obtain for the decay rate of the Σ hyperon at rest and the asymmetry coefficients:

$$w = (G^2E_N/2\pi M_\Sigma)\, k\, (a^2A^2 + b^2B^2),$$
$$\alpha = 2abAB/(a^2A^2 + b^2B^2); \quad (6)$$
$$A = \omega + k^2/2M_N, \qquad B = (1 + \omega/2M_N)\, k.$$

Here ω and k are the energy and the momentum of the π meson, and E_N is the total energy of the recoiling nucleon.

Taking into account the properties of the Σ^+ decays, we find two sets of values for the coefficients a and b which are in agreement with these properties:

$$\text{A.}\quad a_0^2 = 1,\quad b_0^2 = A^2/B^2 = 1.39;$$
$$a_+^2 = 0,\quad b_+^2 = 2A^2/B^2 = 2.77. \quad (7)$$
$$\text{B.}\quad a_0^2 = 1,\quad b_0^2 = A^2/B^2 = 1.39;\qquad a_+^2 = 2,\quad b_+^2 = 0.$$

Here the indices 0 and + denote the charges of the π mesons in the decays (3).

3. THE DECAY $\Sigma^+p \to d + \pi^+$

The matrix element for the decay of the hypernucleus can be obtained from the matrix element for the decay of the free Σ hyperon (5) by the method of Dalitz and Liu.[13] For the decay (2a), this matrix element has the form

$$M = iG(2\Omega\omega)^{-1/2}\,\delta(\mathbf{k}_d + \mathbf{k})\,\{a\chi_{IM}^\Sigma(1,2)\,\chi_{1M'}^d(1,2)\,[\omega F(k)$$
$$+ k^2G(k)/2M_N] - b\chi_{IM}^\Sigma(1,2)\,\sigma_1\chi_{1M'}^d(1,2)\,\mathbf{k}\,[F(k)$$
$$+ \omega G(k)/2M_N]; \quad (8)$$

$$F(k) = \int\psi_d(\xi)\,\psi_\Sigma(\xi)\,e^{-i\xi\mathbf{k}/2}\,d\xi,$$
$$G(k) = \frac{2}{k}\int\frac{\xi\mathbf{k}}{\xi}\,\psi_\Sigma(\xi)\,e^{-i\xi\mathbf{k}/2}\,\frac{\partial\psi_d(\xi)}{\partial\xi}\,d\xi, \quad (9)$$

where χ^Σ, ψ_Σ; χ^d, ψ_d are the spin and space parts of the wave functions of Σ^+p and the deuteron.

As spatial wave functions we have a Hulthen function for Σ^+p as well as for the deuteron:

$$\psi(\xi) = \frac{1}{\sqrt{4\pi}}\sqrt{\frac{2\alpha\beta(\alpha+\beta)}{(\alpha-\beta)^2}}\,\frac{e^{-\alpha\xi} - e^{-\beta\xi}}{\xi}. \quad (10)$$

The parameters α and β in (9) are related to the binding energy and the effective range in the following way:

$$\alpha^2 = 2M^*B, \qquad \beta = 3/r_0,$$

where M^* is the reduced mass of the system of two particles.

Figures 1 and 2 show the dependence of the functions F(k) and G(k) on the binding energy B_Σ of the hypernucleus Σ^+p and the effective range r_0 of the Σ^+-p interaction calculated with the help of the wave functions (10). As seen from the figures, the dependence of these functions on the binding energy is strong, whereas the dependence on the effective range is weak. Therefore, all the data below will correspond to the case $r_0 = 0$ ($\beta = \infty$).

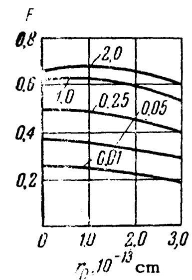

FIG. 1. Dependence of the function F(k) on the effective range of the Σp interaction r_0 for various binding energies B_Σ of the hypernucleus Σ^+p. The numbers near the curves are the values of B_Σ in MeV.

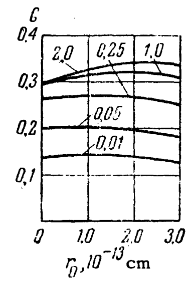

FIG. 2. Dependence of the function G(k) on the effective range of the Σp interaction r_0 for various binding energies B_Σ. The numbers near the curves are the values of B_Σ in MeV.

For the probability of the decay (2a) we obtain from (8) after summation and averaging over the spins:

$$w_d = (G^2E_d/2\pi M_{\Sigma p})\, k\, [s_0a^2(\omega F + k^2G/2M_N)^2$$
$$+ s_1b^2k^2(F + \omega G/2M_N)^2], \quad (11)$$

where $s_0 = 0$, $s_1 = 1$ for I = 0 and $s_0 = 1$, $s_1 = 2/3$ for I = 1 (I is the spin of Σ^+p); E_d is the total energy of the deuteron in the decay (2a); $M_{\Sigma p}$ is the mass of the decaying hypernucleus.

Figure 3 shows the dependence of the rate of the deuteron decay of Σ^+p on the binding energy B_Σ for different values of the spin of Σ^+p and the two variants A and B of (7). One concludes from the non-occurrence of Σ^-n in the capture of K in deuterium that, if Σ^-n exists, its spin must be $I = 0$. The same spin should be expected for Σ^+p by charge symmetry. It follows from (7) and (11) that for $I = 0$ only the variant A leads to a decay into a deuteron. Thus the observation of the decay of Σ^+p into a deuteron would indicate that the π^+ decay of the free Σ hyperon goes through the p state.

4. THE THREE-PARTICLE DECAYS $\Sigma^+p \to p + p + \pi^0$ AND $\Sigma^+p \to n + p + \pi^+$

In the discussion of these decays we shall not take into account the interaction in the final state. In this case the wave function of the nucleons in the final state with account of the identity of the particles is

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \Omega^{-1}(1+\varepsilon^2)^{-1/2}\,[e^{i(\mathbf{k}_1\mathbf{r}_1+\mathbf{k}_2\mathbf{r}_2)} + \varepsilon e^{i(\mathbf{k}_1\mathbf{r}_2+\mathbf{k}_2\mathbf{r}_1)}]\,\chi^N_{IM}(1,2), \tag{12}$$

where $\mathbf{k}_1$, $\mathbf{k}_2$ are the momenta of the nucleons and χ^N_{IM} is the spin wave function; $\epsilon = \pm 1$ for a transition into the singlet and triplet states of the two protons, respectively, $\epsilon = 0$ for the decay (2b).

The matrix element for the three particle decay of the hypernucleus can be obtained from the matrix element of the decay of the free Σ hyperon (5) in the same way as in the case of the two-particle decay. The three-particle matrix element has the form

$$\begin{aligned} M_\varepsilon = &-i\frac{G}{\sqrt{\omega\Omega}}\,\delta(\mathbf{k}'+\mathbf{k})\Big\{\chi^\Sigma_{IM}(1,2)\,\chi^N_{I'M'}(1,2) \\ &\times\Big[\Big(\omega-\frac{\mathbf{k}\mathbf{k}_0}{M_N}\Big)\int e^{i\xi(\mathbf{k}/2+\mathbf{k}_0)}\psi_\Sigma(\xi)\,d\xi + \varepsilon\Big(\omega+\frac{\mathbf{k}\mathbf{k}_0}{M_N}\Big) \\ &\times\int e^{i\xi(\mathbf{k}/2-\mathbf{k}_0)}\psi_\Sigma(\xi)\,d\xi\Big] - b\chi^\Sigma_{IM}(1,2)\,\boldsymbol{\sigma}_1\chi^N_{I'M'}(1,2) \\ &\times\Big[\Big(\mathbf{k}-\frac{\omega}{M_N}\mathbf{k}_0\Big)\int e^{i\xi(\mathbf{k}/2+\mathbf{k}_0)}\psi_\Sigma(\xi)\,d\xi \\ &+\varepsilon\Big(\mathbf{k}+\frac{\omega}{M_N}\mathbf{k}_0\Big)\int e^{i\xi(\mathbf{k}/2-\mathbf{k}_0)}\psi_\Sigma(\xi)\,d\xi; \end{aligned} \tag{13}$$

$$\mathbf{k}' = \mathbf{k}_1 + \mathbf{k}_2, \qquad \mathbf{k}_0 = \tfrac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2).$$

The index ϵ has the values indicated above and determines the matrix elements for the transitions into the various spin and charge states of the two nucleons.

After summation and averaging over the spins we find for the three-particle decay rate as functions of the energy of the emitted π meson, using (10) and (13),

$$dw = (G^2/16\pi^3)\,M_N\,\omega\,[a^2(s_0^sT_0^s + s_0^tT_0^t) + b^2(s_1^sT_1^s + s_1^tT_1^t)]\,d\omega; \tag{14}$$

$$T_0(\varepsilon) = \omega^2 f(k) - 2\omega k k_0 g(k)/M_N + \varepsilon\omega^2 q(k),$$

$$T_1(\varepsilon) = (k^2 + \omega^2k_0^2/M_N^2)\,f(k) - 2\omega k k_0 g(k)/M_N + \varepsilon\,(k^2 + \omega^2 k_0^2/M_N^2)\,q(k); \tag{15}$$

$$k_0^2 = (M_\Sigma - M_N)\,M_N - k^2/4 - \omega M_N - M_N B_\Sigma, \tag{16}$$

where $T^s = T(1)$, $T^t = T(-1)$ for the decays (2c) and $T^s = T^t = T(0)$ for the decays (2b).

For zero effective range of the Σp interaction

$$f(k) = 16\pi\alpha k k_0/[\alpha^2 + (k_0 + k/2)^2]\,[\alpha^2 + (k_0 - k/2)^2];$$

$$q(k) = \frac{8\pi\alpha}{\alpha^2 + k_0^2 + k^2/4}\ln\frac{\alpha^2 + (k_0 + k/2)^2}{\alpha^2 + (k_0 - k/2)^2};$$

$$g(k) = [q(k) - f(k)]\,(\alpha^2 + k_0^2 + k^2/4)/k_0 k. \tag{17}$$

The values of the coefficients s are given in the table.

Spin of Σ^+p	s_0^s	s_0^t	s_1^s	s_1^t
0	1	0	0	1
1	0	1	$1/3$	$2/3$

For $k_0 = k/2$ the function $f(k)$ and hence the spectrum of the mesons have a maximum. The maximum lies close to the upper limit of the spectrum. As the binding energy B_Σ is decreased, the width of the maximum decreases proportionally, and its height increases in inverse proportion to $\sqrt{B_\Sigma}$. For small B_Σ the main contribution to (14) will come from the terms with $f(k)$ and the spectrum of the mesons near the maximum can be described by the resonance formula

$$T_0(\varepsilon) \approx 4\pi^2\Big(\omega_r^2 + \frac{\omega_r k_r^2}{M_N}\Big)\frac{k_r}{M_\Sigma}\frac{\Gamma}{\pi}\frac{1}{\Gamma^2/4 + (\omega-\omega_r)^2},$$

$$T_1(\varepsilon) \approx 4\pi^2\Big(k_r + \frac{\omega_r k_r}{2M_N}\Big)^2\frac{k_r}{M_\Sigma}\frac{\Gamma}{\pi}\frac{1}{\Gamma^2/4 + (\omega-\omega_r)^2}; \tag{18}$$

$$\Gamma \approx 2\alpha k_r/M_\Sigma = 9.8\sqrt{B_\Sigma}\ \text{MeV} \tag{19}$$

(ω_r is the energy of the π meson in the decay of the free Σ hyperon).

Figure 3 shows the dependence of the three-particle decay rates on the assumed binding energy B_Σ. For $B_\Sigma \to 0$ the three-particle decay rates go over into the partial pionic decay rates of the free Σ hyperon.

Figure 4 shows the spectra of the π mesons for the two values of the binding energy. If the binding energy is small, the π mesons are practically monoenergetic and have an energy equal to the energy of the meson in the decay of the free Σ. In this case the outward appearance of the decay of the hypernucleus does not differ from that of the decay of the free Σ hyperon and it is difficult to identify the hypernucleus by its decay.

The two-particle decays of Σ^+p, whose obser-

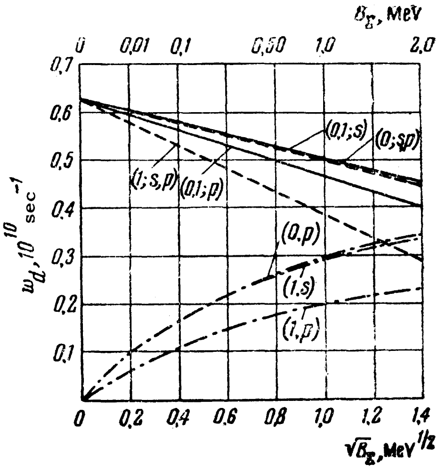

FIG. 3. Dependence of the decay rates w_d of the hypernucleus Σ^+p on the binding energy B_Σ. The labels near the curves have the following meaning: (0,1; s) – spin of the hypernucleus I = 0 or I = 1, the decay of Σ goes through the s state; (0; s, p) – spin I = 0, the decay of Σ goes through the s or the p state (π^0 decay); (0, 1; p) – I = 0 or I = 1, the decay of Σ goes through the p state, etc. The solid curve: $\Sigma^+p \to n + p + \pi^+$, dotted curve: $\Sigma^+p \to p + p + \pi^0$, dash-dotted curve: $\Sigma^+p \to d + \pi^+$.

vation would be unequivocal proof of the existence of Σ^+p, have small rates at small values of B_Σ. It is therefore impossible at present to exclude the possibility that the weakly bound system Σ^+p exists but has not yet been observed on account of the small probability of formation and the difficulty of its identification.

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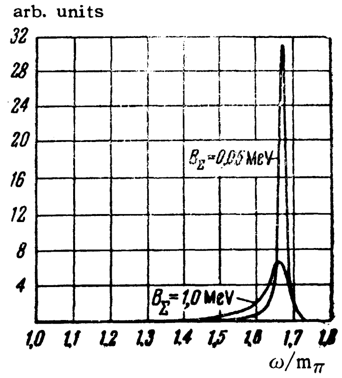

FIG. 4. Spectrum of the π^+ mesons in the decay $\Sigma^+p \to n + p + \pi^+$ for the two binding energies B_Σ = 1 MeV and B_Σ = 0.05 MeV.

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