EFFECT OF ANNIHILATION ON PERIPHERAL NUCLEON-ANTINUCLEON INTERACTIONS

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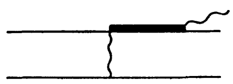
We consider why the cross sections for peripheral interactions between nucleons or between a nucleon and antinucleon at ~ 2 BeV differ by several times whereas according to the single-meson approximation they should be the same. A possible cause is that in the case of nucleon-antinucleon interaction intense annihilation occurs and suppresses all other processes in virtue of the unitarity condition. The effect of the unitarity condition is estimated using the scattering matrix and the optical model apparatus. The estimate shows that the unitarity condition has a small effect in the case of nucleon-nucleon interaction, but is large in the case of nucleon-antinucleon interaction and may cause the disagreement.

1. It is known that inelastic nucleon interactions at $E_{l.s.} \sim 2$ BeV are well described theoretically within the framework of the one-pion exchange (OPE) approximation [1]. At this energy, the main contribution to the inelastic interaction is made by the nucleon-pion isobar production process, shown in the figure. The cross section for the interaction



of a proton with a proton, calculated in the OPE approximation [1], yields $\sigma_{pp}^{OPE} = 22$ mb, constituting the bulk of the experimental value [2] $\sigma_{pp}^{exp} = 23$ mb. Other characteristics of the process (angular and energy distributions etc) also agree well with the calculations. The experimentally obtained cross section for the proton-neutron interaction at the same energies is of the same order [3], $\sigma_{pn}^{exp} = 21 \pm 4$ mb. Calculations of the cross section of proton-neutron interactions within the OPE framework were not made with the same accuracy as for the pp interaction, but it follows from general considerations that σ_{pp}^{OPE} and σ_{pn}^{OPE} cannot differ greatly.

The cross section for the interaction between a proton and an antiproton ($p\bar{p}$), calculated in the OPE approximation, should coincide with σ_{pn}^{OPE}. Indeed, the OPE expression for the cross section does not include the amplitudes of the direct nucleon interaction, and contains the cross sections for the interaction of pions with nucleons in a state with total isospin T (see, for example, [4]). The coefficients for the cross sections are determined by the isospin of the two-nucleon system. However, in virtue of the invariance against charge conjugation, the cross section for the interaction of the pion with the antinucleon is equal to the cross section of the interaction of the pion with the nucleon (if the isospins of the states coincide). Therefore the OPE cross section will depend only on the total isospin of the system of two nucleons (or nucleon and antinucleon). A system consisting of a proton and antiproton is in this respect equivalent to a system consisting of a proton and a neutron, and therefore the cross sections calculated in the OPE approximation should coincide: $\sigma_{p\bar{p}}^{OPE} = \sigma_{pn}^{OPE}$. On the other hand, from the experimental data on the $p\bar{p}$ interaction, it follows that:

a) The total cross section for inelastic pp interaction $\sigma_{p\bar{p}}^{in}$ at $E_{l.s.} = 2$ BeV is much larger than the cross section for the inelastic pn interaction $\sigma_{pn}^{in} = 50$ mb [5]. The bulk of it is made up of the annihilation cross section.

b) The cross section for the inelastic peripheral processes in $p\bar{p}$ interactions [6], that is, processes of the type

$$\begin{aligned} p+\bar{p} &\to p+\bar{p}+\pi^0 \\ &\to n+\bar{n}+\pi^0 \\ &\to n+\bar{p}+\pi^+ \\ &\to p+\bar{n}+\pi^- \end{aligned} \tag{1}$$

amounts to $\sigma_{p\bar{p}}^{per} \approx 5-6$ mb, that is, one-third or one-fourth the total cross section of the analogous processes in pn interaction.

This raises the question of whether the calculations made by Ferrari and Selleri [1] are correct, and how is such an appreciable difference between the theoretical and experimental values of the cross sections for the peripheral processes to be explained (this was first pointed out by M. I. Podgoretskiĭ). One of the possible and most natural causes of this is the following: the annihilation process (which occurs in the $p\bar{p}$ and is forbidden in pn) competes with processes of the type (1) and suppresses them. In order to estimate the suppression, it is necessary to take into account the condition of unitarity in the direct channel[1)] (see below).

At the present time there is no way of accurately taking into account the unitarity condition in inelastic processes. For an approximate account it is possible to use the scattering matrix (R matrix) formalism[7] and the optical model[8]. The present paper is devoted to this question.

The R-matrix formalism is applicable only to binary reactions. The rank of the R matrix is determined by the number of possible channels. Processes in which there are more than two particles in the final state can be reduced to binary by gathering the matrices in two groups. It is then possible to use the formalism of the high-rank R matrix (strictly speaking, its rank tends to infinity in this case). The R-matrix formalism then becomes complicated and is not very effective.

An exception is the case when the amplitude of the elastic scattering is pure imaginary. The condition for the vanishing of the real part of the elastic-scattering amplitude greatly simplifies the analysis but, on the other hand, it is fully justified (at sufficiently high energy).

In the appendix we consider a multichannel process. The results coincide with the simpler model with a small number of channels. Therefore we shall consider in the text model processes (two- and three-channel reactions), and assume that the real part of the elastic amplitude vanishes.

2. Let us consider pn and $p\bar{p}$ interactions. We assume that the inelastic pn interaction results in the final state in a nucleon and an isobar (which then decays into a pion and nucleon). We shall describe this process by a two-channel matrix with elements proportional to the "uncorrected" partial transition amplitudes (with momentum l). Namely, R_{11} and R_{22} are the "uncorrected" partial amplitudes of the "potential" scattering of a nucleon by a nucleon and of a nucleon by an isobar; R_{12} is the "uncorrected" amplitude of the transition from two nucleons into a nucleon with isobar, calculated in the OPE approximation.

We shall describe the $p\bar{p}$ interaction by a third-rank matrix $\bar{R}$, since three channels are possible here: elastic scattering (its potential part is described by the element $\bar{R}_{11}$), peripheral interaction, that is, production of a nucleon and an anti-isobar (its "uncorrected" amplitude is $\bar{R}_{12}$, and in accordance with the foregoing we have in the OPE approximation $R_{12} = \bar{R}_{13}$), and annihilation. In the latter case we shall assume that two pion resonances (which then break up into two pions) are formed. The "uncorrected" amplitude of this process is $\bar{R}_{13}$.

The unitary ("uncorrected") partial amplitudes of the transition will be elements of the T-matrix, connected with the R matrix by the relation (see, for example, [7])

$$T = 2\pi\sqrt{\rho}\,\frac{R}{1 - 2\pi i R\rho}\sqrt{\rho}. \tag{2}$$

The statistical-weight matrix is $\rho_{ij} = \delta_{ij}k_i/\omega$; k_i and ω are the momenta of the secondary particles and the total c.m.s. energy.

The elements of the T matrix are simply related with the partial elastic and inelastic interaction cross sections:

$$\sigma_l^{el} = 4\,|T_{11}|^2, \qquad \sigma_l^{in} = 4\sum_{j\geqslant 2}|T_{1j}|^2. \tag{3}$$

The total cross sections will be

$$\sigma = \frac{\pi}{k^2}\sum_l (2l+1)\,\sigma_l. \tag{4}$$

Here k — momentum of the initial particles in the c.m.s.

We assume that the amplitudes of elastic scattering are pure imaginary. Then from the condition $\mathrm{Re}\,T_{11} = \mathrm{Re}\,T_{22} = 0$ it follows that in the pn interaction $R_{11} = R_{22} = 0$ (that is, there is no potential scattering), and the T-matrix elements have the simple form

$$T_{11} = i\,\frac{a^2}{1+a^2}, \qquad T_{22} = i\,\frac{a^2}{1+a^2},$$
$$T_{12} = \frac{a}{1+a^2}, \qquad a = 2\pi\sqrt{\rho_1\rho_2}\,R_{12}. \tag{5}$$

For $a^2 \ll 1$ we have $T_{12} \approx a$, that is, the "corrected" amplitude coincides with the "uncorrected" one. An account of the unitarity condition introduces corrections of the order of a^2.

In the case of $p\bar{p}$ interaction, the condition that the amplitudes must be imaginary leads to analogous simplifications: there should be no "potential" scattering, $\bar{R}_{11} = \bar{R}_{22} = \bar{R}_{33} = 0$; in addition, the term $\bar{R}_{23} = 0$ must also vanish. Then the unitary

1)The point is that no account was taken in the calculations of [1] of the unitarity condition in the direct channel, and the calculated amplitudes of the processes [1] are in Mandelstam's terminology [9] "uncorrected."

"corrected" amplitudes $\overline{T}_{1j}$, describing the $p\bar{p}$ interaction, assumes the form

$$\overline{T}_{11} = i\frac{a^2+b^2}{1+a^2+b^2}, \qquad \overline{T}_{12} = \frac{a}{1+a^2+b^2}, \qquad \overline{T}_{13} = \frac{b}{1+a^2+b^2}, \tag{6}$$

where $b = 2\pi\sqrt{\rho_1\rho_3}\,R_{13}$.

Using (5) and (6), we can write the partial cross sections of the pn interaction (σ) and the $p\bar{p}$ interaction ($\bar{\sigma}$) in the form

$$\begin{aligned} \sigma_l^{el} &= 4|T_{11}|^2 = 4\left(\frac{a^2}{1+a^2}\right)^2, \\ \sigma_l^{in} &= 4|T_{12}|^2 = 4\frac{a^2}{(1+a^2)^2}, \\ \bar{\sigma}_l^{el} &= 4|\overline{T}_{11}|^2 = 4\left(\frac{a^2+b^2}{1+a^2+b^2}\right)^2, \\ \bar{\sigma}_l^{\rm per} &= 4|\overline{T}_{12}|^2 = 4\frac{a^2}{(1+a^2+b^2)^2}, \\ \bar{\sigma}_l^{\rm ann} &= 4|\overline{T}_{13}|^2 = 4\frac{b^2}{(1+a^2+b^2)^2}, \\ \bar{\sigma}_l^{in} &= \bar{\sigma}_l^{\rm per} + \bar{\sigma}_l^{\rm ann} = 4\frac{a^2+b^2}{(1+a^2+b^2)^2}. \end{aligned} \tag{7}$$

We see from [9] that the annihilation process (that is, the quantity b), can influence the single-meson process, that is, the quantity $\bar{\sigma}_l^{\rm per}$ decreasing the cross section of the latter. The larger b^2, the larger the suppression. As $b^2 \to \infty$, the amplitude of the single-meson process can be completely suppressed, but in this case the amplitude of the annihilation process itself and the partial elastic cross sections also tend to zero. This situation does not correspond to reality, since it follows from experiment that the elastic scattering cross section is smaller than the inelastic cross section. If $\bar{\sigma}_l^{el} \le \bar{\sigma}_l^{in}$, the maximum possible value is $a^2 + b^2 = 1$ (then $\bar{\sigma}_l^{el} = \bar{\sigma}_l^{in}$). According to (6) the "corrected" amplitude $\overline{T}_{12}$ is then half that of the uncorrected one (that is, of a), and the partial cross section is one-quarter that obtained without account of unitarity.

In the more general case the ratio of the cross sections of the peripheral interaction for pn and $p\bar{p}$ collisions (this ratio can be called the suppression factor η_l) is equal to

$$\eta_l = \left(\frac{1+a^2+b^2}{1+a^2}\right)^2. \tag{8}$$

Another important characteristic is the ratio of the "corrected" inelastic cross sections to the "uncorrected" ones. This quantity is a correction coefficient κ_l, which takes into account the unitarity condition; it is different in pn and $p\bar{p}$ interactions:

$$\varkappa_{l,pn} = |T_{12}|^2 a^{-2} = (1+a^2)^{-2}, \qquad \varkappa_{l,p\bar{p}} = (|\overline{T}_{12}|^2 + |\overline{T}_{13}|^2)/(a^2+b^2) = (1+a^2+b^2)^{-2}. \tag{9}$$

To estimate η_l and κ_l for $E_{l.s.} \approx 2$ BeV, we use the optical model, since the experimental data are as a rule interpreted on its basis [5,10].

The connection between the parameters of the optical model and the partial cross sections is in the form

$$\sigma_l^{el} = 4|T_{11}|^2 = |1 - A_l|^2, \qquad \sigma_l^{in} = 4\sum_{j\geqslant 2}|T_{1j}|^2 = 1 - |A_l|^2, \tag{10}$$

where $A_l = A(r)$ for $r = (2l+1)/2k$; $A(r)$ is the transparency for the impact parameter r used in the optical model.

If the elastic scattering amplitude is imaginary the transparency is real [7]. In this case, using (5), (6), and (10) we obtain

$$a^2 = (1-A)/(1+A), \qquad a^2+b^2 = (1-\overline{A})/(1+\overline{A}), \tag{11}$$

where A and $\overline{A}$ are the transparencies for the pn and $p\bar{p}$ interactions.

The suppression factor η and the coefficient κ will on the basis of (8), (9), and (11) be equal to

$$\eta = [(1+A)/(1+\overline{A})]^2, \qquad \varkappa = \tfrac{1}{4}(1+A)^2. \tag{12}$$

The cross section $\sigma_{p\bar{p}}^{\rm per}$ in the optical model is of the form

$$\bar{\sigma}_{p\bar{p}}^{\rm per} = \frac{\pi}{k^2}\sum_l (2l+1)(1-A^2)\left(\frac{1+\overline{A}}{1+A}\right)^2 = 2\pi\int r\,dr\,\frac{(1-A^2(r))}{\eta}. \tag{13}$$

The transparency $\overline{A}(r)$ for $E_{l.s.} \sim 2$ BeV was determined by Armenteros et al [5]. In the interval $0.5 \le r \le 1$, which makes the main contribution to the integral (13) (such values of r correspond to momenta $l \sim 15-30$), $\overline{A} \approx 0.15-0.3$. The value of A for pn interactions has never been determined directly anywhere. There exist, however, many data on the transparency in pp interactions, which is much larger than $\overline{A}$ in the interval $0.5 \le r \le 1$, namely [10], $\overline{A}_{pp} \approx 0.7-0.8$. If we assume that $A \sim A_{pp}$, then in the region making the main contribution the suppression factor will be $\eta = 2.5$.

It must be borne in mind that this quantity has been obtained on the basis of rather crude estimates (the crudeness is connected principally with the ambiguity in the definitions of A and $\overline{A}$). It follows, however, that the mechanism considered, which is connected with the unitarity condition, can explain the difference in the cross sections of the peripheral processes for pn and $p\bar{p}$ interactions. We note also that the $p\bar{p}$-interaction angular distribution of the secondary particles in these processes can differ from that calculated in the OPE approximation, and from that observed in pn interactions. This is connected with the fact the suppression factor is different for different r and consequently its influence on the different partial cross sections (with different l) is not the same. However, there should be no difference in the isotopic relations (charge distributions) in peripheral pn and $p\bar{p}$ processes.

The quantity A_{pp} can be used also to estimate κ. For $A_{pp} \sim 0.8$ we have $\kappa \approx 0.8$, that is, it differs little from unity (by approximately 20%). At higher energies the transparency A_{pp} increases and approaches unity. Thus, the influence of the unitarity condition in nucleon-nucleon interactions is small.

Finally, we must discuss the character of elastic pp interaction in the energy region 2—10 BeV, that is, in the region where the annihilation processes play a noticeable role in inelastic interactions. According to Diddens et al[11], the pp interactions have even in this region a "Regge" behavior, that is, they agree with the Regge and Gribov[12] predictions based on the hypothesis of the predominant role of one vacuum pole. On the other hand, this region cannot be called "asymptotic," since the condition $\sigma_{pp} = \bar{\sigma}_{pp}$ is not satisfied in it.

From our point of view, the "Regge" behavior of elastic interaction is explained by the fact that the inelastic pp interaction has an OPE character[13]. From this point of view, $p\bar{p}$ scattering should not have a "Regge" behavior in this region, since inelastic $p\bar{p}$ interaction is not a one-pion interaction. In particular, the diffraction peak of $p\bar{p}$ scattering should not "twist" as is the case with pp interaction.

In conclusion, I consider it my pleasant duty to express gratitude to M. I. Podgoretskiĭ and E. L. Faĭnberg for very useful discussions of the work.

APPENDIX

We consider a multichannel process of binary reactions at high energy. Here $\rho_{ij} = \delta_{ij}$ and

$$T = r/(1 - ir), \qquad r = 2\pi R. \tag{A.1}$$

We assume that the number of channels n, determining the rank of the matrix, is large, $n \gg 1$ (actually, to describe many-particle states in terms of the R matrix it is necessary to consider $n \to \infty$), and that the elements of the matrix are small, $|r_{ij}| \ll 1$, and tend to zero as $n \to \infty$ (the quantities r_{ij} will be defined more accurately below). We shall use expansion in r_{ij}.

Then the T-matrix elements will be

$$T_{ij} = \left(\frac{r}{1-ir}\right)_{ij} = i\left\{\delta_{ij} - \left(\frac{1}{1-ir}\right)_{ij}\right\} = i\delta_{ij} - i\,\frac{M_{ij}}{\det}, \tag{A.2}$$

where $\det = \|1 - ir\|$, and M_{ij} is the minor of this determinant, obtained by crossing out the i-th line and the i-th column. The amplitude of elastic scattering is

$$T_{11} = i - iM_{11}/\det. \tag{A.3}$$

The partial cross section of the inelastic processes is

$$\sigma_l^{in} = 4\sum_{j\geqslant 2}^{n} |T_{1j}|^2 = \frac{4}{\det}\sum_{j\geqslant 2}^{n} |M_{1j}|^2. \tag{A.4}$$

Accurate to terms $(r_{ij})^2$ we have

$$M_{11} = 1 + \sum_{i\neq j\geqslant 2}^{n} r_{ij}^2, \qquad \det = 1 + \sum_{i\neq j\geqslant 2}^{n} r_{ij}^2,$$
$$M_{1j} = r_{1j}. \tag{A.5}$$

In order for the partial cross sections of the elastic and inelastic processes to be of the same order, it is necessary to have

$$\sum_{j\geqslant 2}^{n} |M_{1j}|^2 = \sum_{j\geqslant 2}^{n} r_{1j}^2 \sim 1, \quad r_{1j}^2 \sim \frac{1}{n}, \qquad r_{1j} \sim \frac{1}{\sqrt{n}}. \tag{A.6}$$

We stipulate that the real part of the amplitude vanish. To this end it is sufficient to set equal to zero the imaginary parts of the determinant and of the minor. The imaginary parts contain odd powers of r and are equal to (we take into account the terms r and r^3)

$$\begin{aligned} \operatorname{Im}\det &= -\sum_{j=1}^{n} r_{jj} - \sum_{i\neq j\neq k\geqslant 1} r_{ij}r_{jk}r_{ki}, \\ \operatorname{Im} M_{11} &= -\sum_{i\neq j\neq k\geqslant 2}^{n} r_{ij}r_{jk}r_{ki}. \end{aligned} \tag{A.7}$$

If we disregard the special case when the terms in the sums of (A.7) cancel each other completely, it follows from the condition $\operatorname{Re} T_{11} \ll \operatorname{Im} T_{11}$ that

$$\sum_{j=1}^{n} r_{jj} \ll 1, \qquad \sum_{i\neq j\neq k\geqslant 1}^{n} r_{ij}r_{jk}r_{ki} \ll 1.$$

These conditions are satisfied if

$$|r_{jj}| \ll 1/n, \qquad |r_{ij}|_{i\neq j\geqslant 2} \ll 1/n. \tag{A.8}$$

Thus, the elements of the R matrix in the first row (and accordingly in the first column, except for the element r_{11}) are much larger than the remainder. We can represent the r-matrix approximately in the form

$$r = \begin{pmatrix} 0 & r_{12} & r_{13} \cdots r_{1i} \\ r_{21} & & \\ r_{31} & 0 & \\ \vdots & & \\ r_{i1} & & \end{pmatrix}. \quad \text{(A.9)}$$

The physical meaning of the conditions (A.8) consists in the following. The first condition of (A.8) signifies simply that there is no "potential" scattering. This condition always obtains at high energies. The second of the conditions (A.8) can be interpreted in the following manner: the "direct" interaction between two states, each of which consists of several particles, is small compared with the interaction of two particles.

We shall now show that the expressions (12) (obtained in the case of the two- and three-channel processes) are valid also if the r-matrix has the form (A.9).

Let the elements r_{1i} for $2 \leq i \leq n'$ describe processes of the first type (for example, one-pion processes) and let r_{1i} describe for $n' < i \leq n$ processes of the second type (for example, annihilation).

When the processes of the second type are turned off we have

$$\det = 1 + a^2, \qquad M_{1i} = r_{1i},$$

$$\sigma_l^{in} = 4\frac{1}{(\det)^2}\sum_{i\geqslant 2}^{n'} |M_{1i}|^2 = 1 - A_l^2 = 4\frac{a^2}{1+a^2}, \quad \text{(A.10)}$$

where

$$a^2 = \sum_{i\geqslant 2}^{n'} r_{1i}^2.$$

When the processes of the second type are turned on, the partial cross section of all the inelastic processes is

$$\bar{\sigma}_l^{in} = 4\frac{a^2+b^2}{(1+a^2+b^2)^2} = 1 - \bar{A}_l^2, \qquad b^2 = \sum_{i\geqslant n'}^{n} r_{1i}^2. \quad \text{(A.11)}$$

The partial cross section of the processes of the first type will be

$$\bar{\sigma}_l^{\text{per}} = 4a^2(1+a^2+b^2)^{-2}. \quad \text{(A.12)}$$

It follows from (A.10) and (A.11) that

$$a^2 = (1-A)(1+A)^{-1}, \qquad a^2 + b^2 = (1-\bar{A})/(1+\bar{A}),$$

after which we obtain

$$\bar{\sigma}_l^{\text{per}} = (1-A^2)(1+\bar{A})^2(1+A)^{-2},$$

which is equivalent to the factor η given in (12). We similarly obtain the expression for κ.

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