	NS_{123}	I_{right}	I_{left}	R
Carbon target, measurement at the maximum; $\theta_1=72°15'$, $E_1=160$ MeV, $E_1'=156$ MeV	6 280 000	2 087	2 110	0.99 ± 0.03
Carbon target, background measurement; $\theta_1=75°40'$, $E_1'=100$ MeV	4 080 000	681	741	0.92 ± 0.05
Carbon target, measurements at the maximum; $\theta_1=73°15'$, $E_1=140$ MeV, $E'_1=135$ MeV	9 600 000	4 301	4 216	1.02 ± 0.02
Carbon target, background measurements; $\theta_1=74°55'$, $E_1'=110$ MeV	7 720 000	1 601	1 517	1.06 ± 0.036
Lead target, measurement at the maximum; $\theta_1=71°35'$, $E_1'=167$ MeV	4 920 000	21 125	20 832	1.01 ± 0.01
Lead target, background measurements; $\theta_1=74°55'$	3 500 000	3 724	3 624	1.025 ± 0.023

E_1 is the kinetic energy of the recoil proton, determined from its range; E_1' is the kinetic energy of the recoil protons determined from the angle θ_1.

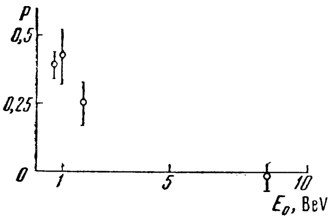

FIG. 4. Polarization of protons moving forwards in the c.m.s. in pp scattering at $t = -0.28$ (BeV/c)^2 and E_0 equal to 0.66,[4] 0.97,[5] 1.74,[6] and 8.5 BeV.

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DETERMINATION OF THE REAL PART OF THE SCATTERING AMPLITUDE FOR AN ASYMPTOTIC POWER LAW BEHAVIOR OF THE IMAGINARY PART

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THE experimental data on the total cross sections in the scattering of particles and antiparticles at high energies can evidently be well approximated by power series in the energy. In this connection the problem arises of calculating the real part of the forward scattering amplitude for large energy by means of the dispersion relations (d.r.) with a power-law approach of $\sigma_t(E)$ to its limiting values. In such an investigation one usually employs the Sommerfeld-Watson-Regge representation which yields for the asymptotic behavior of the real parts of the amplitude $D^{\pm}(E)$ (D^+ is symmetric and D^- is antisymmetric in the total laboratory energy E of the incoming particles) the relations

$$D^+(E) = D(E) + \widetilde{D}(E) = -\sum \beta_i^+ E^{\alpha_i^+} \operatorname{ctg}(\pi\alpha_i^+/2),$$
$$D^-(E) = D(E) - \widetilde{D}(E) = \sum \beta_i^- E^{\alpha_i^-} \operatorname{tg}(\pi\alpha_i^-/2), \qquad (1)*$$

respectively for the asymptotic sum and the difference of the cross sections for particles and for antiparticles [5]

$$A^{\pm}(E) = [\sigma_t(E) \pm \widetilde{\sigma}_t(E)]\, E/4\pi = \sum \beta_i^{\pm} E^{\alpha_i^{\pm}}. \qquad (2^{\pm})$$

However, such a representation of the real parts can in practice be unsatisfactory because it does not take into account the minor singularities in the l-plane, which can still provide large contributions at energies ~ 10 GeV.

We start from dispersion relations with one subtraction[1] for the forward scattering amplitude, averaged over the spins. The imaginary part of such an amplitude is connected with the total cross section by the optical theorem. We assume that from a certain energy on, say, $E' \gtrsim \epsilon$, the variation of the total cross sections can be approximated by (2). Then accurate to terms with $(\epsilon/E)^2$ one can represent the functions $D^{\pm}(E)$ in the form

$$D^{\pm}(E) = C^{\pm}\begin{Bmatrix}1\\E\end{Bmatrix} + D^{\pm}(m)\begin{Bmatrix}1\\E/m\end{Bmatrix} - \frac{1}{4\pi^2}\int_m^{\varepsilon}\begin{Bmatrix}E'\\E\end{Bmatrix}\frac{\sigma_t \pm \tilde{\sigma}_t}{k'}dE'$$
$$+ \sum \beta_i^{\pm}\frac{E^2}{4\pi^2}\int_{\varepsilon}^{\infty}\begin{Bmatrix}E'\\E\end{Bmatrix}\frac{E'^{\alpha_i^{\pm}-1}dE'}{E'(E'^2-E^2)}. \qquad (3^{\pm})$$

The upper and the lower factors in the braces are associated with D^+ and D^-, respectively; $E^2 = m^2 + k^2$; m is the mass of the incoming particle. For $(\pi^{\pm}p)$ scattering we have

$$C^{\pm}\begin{Bmatrix}1\\E\end{Bmatrix} = \frac{2f^2}{m^2(1-m^2/4M^2)}\begin{Bmatrix}m^2/2M\\E\end{Bmatrix},$$

$f^2 = 0.08$; M is the nucleon mass. For the case of $(p-p, \tilde{p})$ scattering the quantities $C^{\pm}$ contain in addition to pole terms contributions from the integration in the unphysical region $E' \leq M$ which compensates for the divergence of the first integral ($\sigma_t(p\tilde{p}) \sim 1/k$ as $k \to 0$).

If we disregard the question of the contribution of the unphysical region to the d.r. for $(p-p, \tilde{p})$ then the evaluation of $D^{\pm}(E)$ reduces to the evaluation of principal value integrals of the type $E^2\int_{\epsilon}^{\infty} E'^{\alpha-1}(E'^2-E^2)^{-1}dE'$. For $E^2 > \epsilon^2$ this can be accomplished by the substitution

$$E^2\int_{\varepsilon}^{\infty}\frac{E'^{\alpha-1}dE'}{E'^2-E^2} \to E^2\int_0^{\infty}\exp\left(-\frac{\varepsilon^2}{E'^2}\right)\frac{E'^{\alpha-1}dE'}{E'^2-E^2} = I(-E^2, \alpha).$$

The integral with the limits 0 to ∞ is taken by means of the Lebesgue-Stieltjes transform[2] and the result is of the form

$$I(-E, \alpha) = E^{\alpha}e^{-\varepsilon^2/E^2}\left\{-\frac{\pi}{2}\operatorname{ctg}\frac{\pi\alpha}{2} + \Gamma\left(\frac{2-\alpha}{2}\right)\left[\frac{1}{0!\,\alpha}\left(\frac{\varepsilon}{E}\right)^{\alpha} + \frac{1}{1!\,(\alpha+2)}\left(\frac{\varepsilon}{E}\right)^{\alpha+2} + \ldots\right]\right\}. \qquad (4)$$

It follows from (4) that the contribution of the term with $\alpha^+ = 0$ to the imaginary part of the symmetric amplitude (2^+) will not lead to divergence in the integral $\int_{\epsilon}^{\infty}$ of (3^+). Therefore the question arises: which terms of the d.r. (3^+) diverge as $\alpha^+ \to 0$ in the case when the term with $\alpha^+ = 0$ in (2) represents the imaginary part of Gell-Mann's "ghost"? It turns out that the divergence appears in the subtraction term [here $D^+(m)$]. This result is obvious if one represents the amplitude as a sum of pole terms of the type $[P_\alpha(E) + P_\alpha(-E)]/\sin\pi\alpha$ and writes for them d.r. in E with one subtraction. Such d.r. with a subtraction at $E = 0$ can be found in the paper[3] by Igi. It is essential that the presence of terms with $\alpha = 0$ in $A^+(E)$ is not necessarily connected with singularities in the real part. As an example we mention $A^+(E) = E^{+\alpha} + E^{-\alpha}$ $(\alpha \to 0)$. Here $D^+(E) \approx \ln E$. The contribution of the terms with $\alpha^+ \approx 0$ is observable at $E \gtrsim 5$ BeV in $(p\tilde{p})$ scattering: $\sigma_t(p\tilde{p}) = (40 + 170/E)$ mb (energy in BeV).[4] Therefore the measurement of D(E) together with a more accurate measurement of $\sigma_t(p\tilde{p})$ is of particular interest for the determination of the character of the singularities in the l-plane which determine the asymptotic behavior of the $(p-p, \tilde{p})$ interaction.

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*tg = tan, ctg = cot.

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