REGGE SHRINKAGE OF THE DIFFRACTION PEAK AND THE ROLE OF MULTIMESON INTERACTIONS

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Elastic pp and πp high energy interactions are considered. A definite regularity in the experimental results can be discerned. If the inelastic process is mainly of a peripheral nature (pp interaction) then elastic scattering does exhibit the Regge behavior. However, if nonperipheral processes (πp interaction) play a significant role then such a behavior is absent. A mixed model is considered in which the scattering amplitude contains a Regge term as well as an ordinary diffraction term. Estimates of the relative contributions of these terms based on experimental data on inelastic interactions qualitatively agree with the observed elastic scattering. Some consequences of such a model are discussed.

As was already noted earlier[1] the Regge behavior of the elastic-scattering amplitude of strongly-interacting particles[2] can be understood as being due to one-meson ("peripheral") inelastic interactions between these particles, while the many-particle contribution to the amplitude does not have a Regge behavior[3]. In such an interpretation, the exchange of the vacuum Reggion in elastic scattering is equivalent to the exchange of a pion pair.

This has naturally raised the question of the universality of the Regge method. In particular, if the Regge method does not take into account many-meson and in general many-particle exchanges, then it is possible that with increasing energy the elastic scattering will not disappear completely, but only its Regge part will disappear, while the many-particle contribution may not disappear. Recent experiments on elastic pp scattering[4], which have confirmed the prediction of the Regge-pole theory, and on πp scattering (see [5] and also Ramsay's report at the Physics Institute of the Academy of Sciences seminar concering the data of S. Lindenbaum and L. Yuan), which did not confirm these predictions, have increased the interest in this question.

We wish to call attention in the present communication to independent (albeit indirect) experimental data on inelastic pp and πp interactions, which offer evidence that the role of many-particle exchanges is precisely the one that can lead to the observed difference between the diffraction peaks of elastic pp and πp scattering.

We first estimate, on the basis of the aforementioned experiments, the contribution of the many-particle exchanges to inelastic pp and πp interactions.

Since we are interested here in estimates only, we can break up quite roughly the diagrams of the inelastic interactions into two classes: one-meson diagrams, which are considered in the theory of peripheral collisions[6,7] (that is, diagrams which can be cut up into two parts, in such a way that only one meson line is cut open), and many-particle diagrams, which can be interpreted for the sake of clarity and brevity as the interaction between cores of particles ("central collisions," core-core interactions[8]). The corresponding inelastic processes can be regarded, in the case when a large number of particles is created, as being incoherent, so that the cross sections of the peripheral (one-meson) interactions σ_{in}^{P} and "central" interactions (collisions with core) σ_{in}^{C} can be added together:

$$\sigma_{in} = \sigma_{in}^{P} + \sigma_{in}^{C}. \tag{1}$$

A crude estimate of their relative contribution to pp interactions on the one hand and to πp interactions on the other can be obtained from two classes of experiments.

A. Experiments at 7–9 BeV. Experiments on pp scattering were analyzed in [9] at a laboratory energy E_{lab} = 9 BeV and experiments on πp scattering were analyzed in [10] at E_{lab} = 7 BeV, with various measured characteristics (distributions over the angles, energies, recoil momenta of the target nucleons, etc.) compared with the deductions

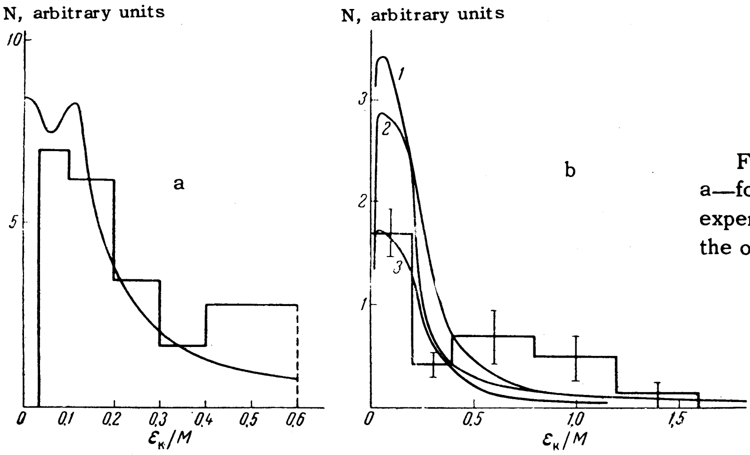

FIG. 1. Distribution of the recoil nucleon kinetic energies: a—for pp interactions, b—for πp interactions. Histograms—experimental data, smooth curves—results of calculation in the one-meson approximation.

of the theory of peripheral collisions (inelastic one-meson interaction). The general conclusion has been essentially that in the case of pp interactions the agreement between the theoretical and experimental results is good, while for πp interactions the agreement is much worse.

Figure 1 shows the distributions of the recoil nucleon kinetic energies ϵ_k, obtained experimentally and calculated in the one-meson approximation. In the case of πp interaction the calculation includes the insufficiently well-known cross section of ππ interaction. It has been assumed for curve 1 that the ππ interaction manifests itself only in the state of the ρ resonance, while for curve 2 the interaction occurs at all energies, $\sigma_{\pi\pi}$ being constant (as can be seen, the curves do not differ much). The areas under these curves have been normalized to the total number of observed events. Finally, curve 3 was obtained from curve 2 by reducing the ordinates by a factor 1.7 to make it describe well the part of the experiment corresponding to small ϵ_k. The curve for pp interaction was normalized in the same way.

It is easy to see that in the case of pp interaction the curve fails to include (in the case of large ϵ_k) approximately 15% of all the events, while in the case of πp interaction the number is approximately 45%. Large ϵ_k correspond to small impact parameters, so that they can be related to many-particle interactions. Thus, we can conclude that

$$\sigma^{C}_{pp,\,in}/\sigma^{P}_{pp,\,in}\,|_{E_{\rm lab}\approx 9\,\rm BeV} \approx \frac{15}{85} \sim 0{,}2\,; \qquad (2a)$$

$$\sigma^{C}_{\pi p,in}/\sigma^{P}_{\pi p,in}\,|_{E_{\rm lab}=7\,\rm BeV} \approx \frac{45}{55} \sim 0{,}8. \qquad (2b)$$

A remark that the non-peripheral interactions play a considerable role in πp collisions at 7 BeV, and that they play a relatively small role in pp interactions (in the interval 10^{10}—10^{11} eV) is contained already in the paper by Birger and Smorodin [11].

B. Experiments at 300 BeV. Dobrotin and Slavatinskii [12] investigated in cosmic rays inelastic nucleon-nucleon interactions in a cloud chamber at a known (simultaneously measured) energy $E_{\rm lab} \sim 300$ BeV. Studying the angular distribution of the created particles in the c.m.s. and the inelasticity coefficients of both the incoming nucleon and of the target nucleon (K and $K_{\rm mir}$) in a system in which the incoming nucleon was at rest, Dobrotin and Slavatinskii observed a clear-cut correlation between the angular distributions and the values of $K_{\rm lab}$ and $K_{\rm mir}$, which can be clearly seen from Fig. 2. Each interaction event is represented on the diagram by a separate symbol. The total number of showers was 46. The observed correlation, in accordance with Dobrotin and Slavatinskii [12], can be interpreted in the following manner.

There are four types of interactions:

a) $K_{\rm lab} \sim K_{\rm mir} \gtrsim 0.35$, that is, $K_{\rm lab}$ and $K_{\rm mir}$

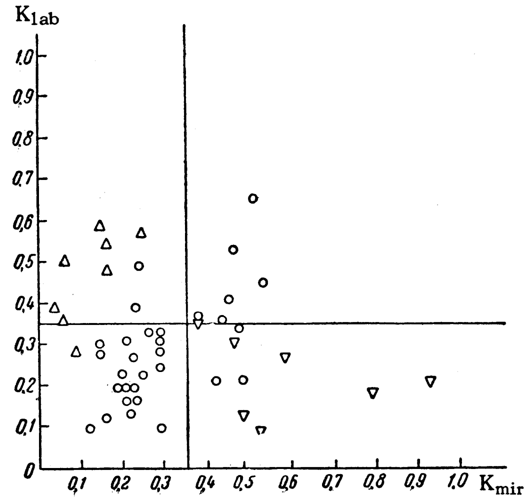

FIG. 2. $K_{\rm lab}$ and $K_{\rm mir}$ for individual showers. △—showers in which the particles were emitted predominantly forward in the c.m.s., ▽—backward, o—symmetrical forward-backward scattering.

are large—these cases corresponding to collisions of the nucleon cores. This type of interaction constitutes 15% of the cases;

b) $K_{lab} \sim K_{mir} \lesssim 0.35$, that is, K_{lab} and K_{mir} are small—collision between the virtual pion of the incoming nucleon and the virtual pion of the target nucleon. This includes 45% of the events;

c) $K_{lab} \lesssim 0.35$ is small while $K_{mir} \gtrsim 0.35$ is large—collision between virtual pion of incoming nucleon and the core of the target nucleons. This includes 20% of the events;

d) $K_{lab} \gtrsim 0.35$ is large, $K_{mir} \lesssim 0.35$ is small —collision between virtual pion of the target nucleon and the core of the incoming nucleon. This type corresponds to 20% of the cases.

The critical value 0.35 was chosen by us arbitarily, to prevent cases with asymmetrically scattered outgoing particles from being included in case a).

With such an interpretation, the fraction of the events a) gives the relative cross section for the interaction between two nucleon cores, b) gives the relative cross section of the $\pi\pi$ interaction, while cases c) plus d) give the cross section for the interaction between the pion and the core of the nucleon (for a pion with energy on the order of 45 BeV); b) plus c) plus d) correspond to $\sigma^{P}_{pp,in}$, etc. From this we get

$$\sigma^{C}_{pp,\ in}/\sigma^{P}_{pp,\ in}\Big|_{E_{lab}\approx 300\,\mathrm{BeV}} \approx \frac{15}{85} \sim 0.2\,; \tag{3a}$$

$$\sigma^{C}_{\pi p,\ in}/\sigma^{P}_{\pi p,\ in}\Big|_{E_{lab}\approx 45\,\mathrm{BeV}} \approx \frac{20+20}{45} \sim 0.9. \tag{3b}$$

Of course, it would be naive to attach serious significance to the exact figures. However, from a comparison of these data it is quite reasonable to conclude that the core (that is, the many-particle interactions) plays a major role in πp interactions, whereas in pp interactions its role is insignificant.

This conclusion from inelastic-interaction experiments allows us to attempt to explain qualitatively the different behaviors of elastic pp and πp interactions. Namely, we can estimate phenomenologically the influence of the many-particle interaction on elastic scattering. Attempting as before to obtain a quasi qualitative estimate, we regard the elastic-scattering amplitude F as consisting of two parts, one peripheral and of the Regge type, F^P, and the other of the many-particle and central type, F^C:

$$F = F^P + F^C,$$

$$d\sigma_{el}/dt = \pi p^{-2}\,|F|^2\ (F = 4ps^{-1}A\ (s,\,t)), \tag{4}$$

where p is the momentum, s the square of the total energy in the c.m.s., and t the square of the momentum transfer. For F^P and F^C we assume

$$F^P\,(t) = \frac{ip}{4\pi}\,\sigma^P \exp\left\{(l_0\,(t) - 1)\ln\frac{s}{s_0} + A_0 t\right\},\ F^C = \frac{ip}{4\pi}\,\sigma^C f\,(t). \tag{5}$$

Here σ^P —total cross section for peripheral interaction, and σ^C —total cross section for many-particle interaction. Experiment usually yields the quantity

$$\Omega = \frac{d\sigma_{el}/dt}{(d\sigma_{el}/dt)_{t=0}} = \frac{1}{(1+\sigma^C/\sigma^P)^2}\left(\exp\left\{(l_0\,(t)-1)\ln\frac{s}{s_0} + A_0 t\right\} + \frac{\sigma^C}{\sigma^P} f\,(t)\right)^2 = e^{At}. \tag{6}$$

We have assumed here that f(t) is real, that is, that the many-particle scattering is of the pure diffraction type.

From a reduction of the experimental data on the basis of the Regge pole theory [2], that is, by taking into account only the first term in (6) ($\sigma^C \to 0$), it was found that $l'(0) = \epsilon = 1/M^2 = 2/s_0$; $A_0 \approx 1.6\,M^{-2}$ for $-t \lesssim 0.5-1.0\,M^2$. (In our case these parameters would, strictly speaking, have to be determined anew). Obviously, inasmuch as $\sigma^C_{in}/\sigma^P_{in} \sim 0.2$ for pp interaction and ~ 0.9 for πp interaction, the contribution of the Regge term to the elastic πp scattering will not be overwhelming and will decrease with increasing $\ln(s/s_0)$.

Thus, in such an approach we must expect that in πp scattering, unlike pp scattering, the shrinkage of the diffraction peak can be noticed only for small $\ln(s/s_0)$, for which perhaps the conditions under which all poles except the vacuum pole can be neglected are not yet realized [13]. In this case there may not be any shrinkage of the peak for πp scattering.

In order to illustrate how this difference can come about, we present the following example.

Let F^C be determined by scattering on an absorbing disc, the density of which depends on the radius like

$$D\,(r) = D_0 e^{-\alpha r^2},\quad 1/\alpha = \overline{r^2} \equiv R^2, \tag{7}$$

where R^2 is the mean square of the radius. Then we obtain from the usual Kirchoff approximation formulas (see, for example [14]) the cross sections for the elastic, inelastic, and total interactions:

$$\sigma^C_{el} = D_0^2\pi/2\alpha, \tag{8a}$$

$$\sigma^C_{in} = 2\pi\alpha^{-1}D_0\left(1 - \tfrac{1}{4}D_0\right), \tag{8b}$$

$$\sigma^C = 2\pi\alpha^{-1}D_0, \tag{8c}$$

$$f\,(t) = e^{t/4\alpha} = e^{R^2 t/4}. \tag{8d}$$

The model depends on two parameters (D_0 and α) and is therefore, of course, quite arbitrary. We assume for $\sqrt{R^2}$ a value $1/2\mu$ simply because this is the characteristic distance at which one can expect a two-meson contribution to the interaction:

$$1/\alpha = 1/4\mu^2 \approx 11/M^2. \tag{8e}$$

For Regge scattering, substituting $l_0(t) - 1 \approx t/M^2$, we obtain

$$\sigma_{el}^P = \frac{(\sigma^P)^2}{2\gamma}, \quad 2\gamma = \frac{32\pi}{M^2}\left(\ln\frac{s}{s_0} + A_0\right) \approx 45\left(1.6 + \ln\frac{s}{s_0}\right)\text{mb} \tag{9}$$

Using the experimental values for σ_{in}, σ_{el}, and σ, and breaking up σ_{in} into σ_{in}^P and σ_{in}^C in accordance with (1), (3a), and (3b), determining D_0 in accordance with (8b) from σ_{in}^C and (8e), determining then σ_{el}^P and σ_{el}^C in accordance with (9) ($\sigma^P = \sigma_{el}^P + \sigma_{in}^P$), (8b), and (8a) (where we take $\ln(s/s_0) = 2.4$), and calculating finally from this σ^P and $\sigma^C = \sigma_{in}^C + \sigma_{el}^C$, we obtain the following table (all cross sections in millibarns):

	Experiment			Theory						
	σ	σ_{in}	σ_{el}	σ_{in}^P	σ_{in}^C	σ_{el}^P	σ_{el}^C	σ^P	σ^C	σ^C/σ^P
pp	42	33	9	28	5	6	0.2	34	5.2	0.15
πp	25	21	4	11	10	0.8	1.0	12	11	0.9

Thus, for $-t \ll M^2$ we can write

$$\Omega_{pp} = \frac{1}{1.44}\left(\exp\left\{\left(\ln\frac{s}{s_0} + 1.6\right)\frac{t}{M^2}\right\} + 0.15\, e^{2.8t/M^2}\right)^2, \tag{10}$$

$$\Omega_{\pi p} = \frac{1}{3.60}\left(\exp\left\{\left(\ln\frac{s}{s_0} + 1.6\right)\frac{t}{M^2}\right\} + 0.9\, e^{2.8t/M^2}\right)^2. \tag{11}$$

Consequently, so long as we can assume that $l_0(t) - 1 \approx tM^{-2}$, scattering by the core does by itself decrease more slowly with t than the Regge term. It becomes predominant if

$$\left(\varepsilon \ln\frac{s}{s_0} + A_0 - \frac{R^2}{4}\right)|t| > \ln\frac{\sigma^P}{\sigma^C}, \tag{12}$$

that is, for our choice of parameters s_0, ϵ, R^2, and σ^P/σ^C for pp scattering, if

$$\ln\frac{s}{s_0} \gtrsim 1.3 + \frac{2M^2}{|t|},$$

$$E_{\text{lab}} > 3.7 \exp\left\{\frac{M^2}{|t|}\ln\frac{\sigma^P}{\sigma^C}\right\} \approx 3.7\, e^{2M^2/|t|}\ \text{BeV} \tag{13a}$$

and for πp scattering if

$$\ln\frac{s}{s_0} \gtrsim 1.3, \quad E_{\text{lab}} > 3.7 \exp\left\{\frac{M^2}{|t|}\ln\frac{\sigma^P}{\sigma^C}\right\} \approx 3.7\ \text{BeV}. \tag{13b}$$

It is obvious that for sufficiently small $-t$, the Regge behavior should occur for pp scattering in a wide region, up to very high energies. With increasing $|t|$, this energy region decreases and when $-t \sim 3M^2/4$ it has a limit $E_{lab} \sim 50$ BeV.

For πp scattering the Regge behavior of the scattering amplitude can be encountered only at very low energies. However, the influence of many poles is possibly still strong here, and there may be no region of applicability for the Regge behavior.

All the foregoing has pertained to the case $|t| \ll M^2$, where $l_0(t)$ is more or less known from experiment. For large $|t|$, the experimental data are still very unreliable. They definitely indicate nevertheless a slower course of the $l_0(t)$ curve. If, as can be expected from theoretical considerations, the function $l_0(t)$ tends to the constant value $l_0(t) = -1$, then

$$\Omega \to \frac{1}{3.60}\left(\exp\left\{-2\ln\frac{s}{s_0} + A_0 t\right\} + \exp\left\{\frac{R^2 t}{4} - \ln\frac{\sigma^P}{\sigma^C}\right\}\right) \tag{14}$$

and Regge behavior of the amplitude does not take place if

$$\ln\frac{s}{s_0} > \frac{1}{2}\left[\left(\frac{R^2}{4} - A_0\right)|t| + \ln\frac{\sigma^P}{\sigma^C}\right]. \tag{15}$$

In this case the term A_0t becomes quite essential. There are no theoretical predictions for it. If we assume here, in accordance with the experimental indications [4], that $-A_0t \ll 1$, then the Regge behavior of the amplitude can again come into play. Thus, for example, when $-t = 2M^2$ we obtain from (6) that the Regge shrinkage of the peak occurs up to an energy $E_{lab} \sim 50$ BeV, in the case of pp scattering, and to $E_{lab} \sim 20$ BeV in the case of πp scattering.

Thus, the conditions for the appearance of a Regge behavior of the amplitude are particularly favorable in two regions: at very low $-t$ (small scattering angles, when the peripheral interaction should be particularly pronounced), and at very large $-t$ (when the Regge method for $-A_0t \ll 1$ prescribes a weak dependence of the amplitude on t).

It must be emphasized once more that the estimates connected with formulas (7)—(15) are presented exclusively in order to illustrate the possible influence of the many-particle interactions, and must under no circumstances be taken literally.

The presence of a non-Regge component F^C will naturally influence many predictions of the moving-pole method. The cross section for elastic interaction will not decrease without limit with increasing energy, but will tend to a constant value σ_{el}^C. For our form factor it is, to be sure, exceedingly small (see the table). The mean square of the momentum transfer will likewise not decrease without limit, but will tend to a constant value, in our example to $|\bar{t}| = 8\mu^2$.

The effective interaction radius is determined by the peripheral (Regge) term and will therefore behave as predicted previously, that is, it will increase with the energy. The ratios between the interaction cross sections of different particles [15] should, generally speaking, not be maintained. They remain in force only for cross sections of peripheral interactions.

This raises the question: how can the component F^C be described within the framework of the method of complex orbital angular momenta? The quantity F^C adds to the invariant scattering amplitude $A(s,t)$ an additional term $A^C(s,t) = (s/4p)F^C \sim sf(t)$. It can be treated in two ways.

a) If the product form of this term is regarded not as approximate but as exact, then in the partial amplitude $f(l,t)$ there should correspond to it a singularity of the type $1/(l-1)$ ("fixed" pole at $l = 1$). This is not compatible with the main premises of the moving-pole method (see [2]), for it leads either to violation of the analyticity of the function $f(l,t)$ or to the impossibility of analytic continuation of the unitarity condition into the l plane.

In the former case the function $f(l,t)$ has in the l-plane a line of singularities situated to the right of the point $l = 1$. This situation contradicts the Regge hypothesis, but does not contradict the Mandelstam representation. In the latter case (no continuation of the unitarity relation), the Mandelstam representation is violated.

b) It can be assumed that the form $sf(t)$ is an approximation to a more complicated function, an approximation valid in a limited region of values of t and s. We can then attempt to describe it within the framework of the moving-pole method as being the contribution from the pole of the function $f(l)$ whose trajectory $l_2(t)$ has a much smaller slope at $t = 0$ than the trajectory of the principal pole ($|\epsilon_2| \ll |\epsilon_1|$). Both trajectories should cross in this case at the point $t = 0$, $l = 1$.

We hope to consider these possibilities in greater detail in a separate paper. We can state in advance, however, that in any case there will appear many new arbitrary parameters in the method of complex orbital angular momenta. Of course, the method loses much of its elegance in this case. At any rate, the simplest variant of the method considered in [2,15], can no longer be regarded as all-inclusive.

Note added in proof (September 14, 1963). According to the already published complete text of the paper by Foley, Lindenbaum, Love, Osaki, Russel, and Yuan (Phys. Rev. Lett., 10, 376, 1963), the diffraction πp curves at different energies coincide within the limits of rather small experimental errors. To reconcile Formula (6) with these data, the coefficient of the second term must be larger $[(\sigma^C/\sigma^P)_{\pi p} \sim 2]$. This, of course, does not contradict the foregoing crude estimates.

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