THE ABSORPTION OF ELECTROMAGNETIC WAVES BY NEUTRINOS

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It is shown that a process of absorption of electromagnetic waves by neutrinos is possible in matter. The absorption cross section shows a resonance behavior near the natural frequencies of the medium. Possibilities for observing the absorption of electromagnetic waves by neutrinos are analyzed.

1. At present there is great interest in the question of the possible experimental proof that there is a weak (ee)($\nu\nu$) interaction.[1)] We would like here to call attention to the possibilities connected with the study of the interaction of electromagnetic waves (in particular, photons) with neutrinos. We note that the character of the interaction is decidedly altered if the process occurs in a medium with $\epsilon^t(\omega, k) \neq 1$ [$\epsilon^t(\omega, k)$ is the dielectric constant of the medium], in which the process can occur in a lower order of perturbation theory, since the medium takes up part of the momentum. For example, a collisionless process is possible in which a photon (or, also, a plasmon) can produce electron-positron pairs.[9] According to the author of [9], the region of frequencies ω and wave numbers k ($\hbar = c = 1$) in which such a process is possible is the region shaded with vertical lines in Fig. 1. There is an energy threshold (curve A). We note that there would be no threshold if the mass of the particles were zero. This means that in a medium with $\epsilon^t < 1$ (for example, a plasma) a photon can produce a neutrino-antineutrino pair.

Following the work in [9], and to a considerable extent using its results, Adams, Ruderman, and Woo[10] have made a calculation of the process in the approximation corresponding to the diagram of Fig. 2. According to [10], in stellar plasma the production of neutrino-antineutrino pairs by photons and plasmons at $\rho \gtrsim 10^5$ g/cm^3 and $T \gtrsim 10^8$ °K may be the main process in energy loss from a star, and at certain stages may determine the time scale of its evolution.

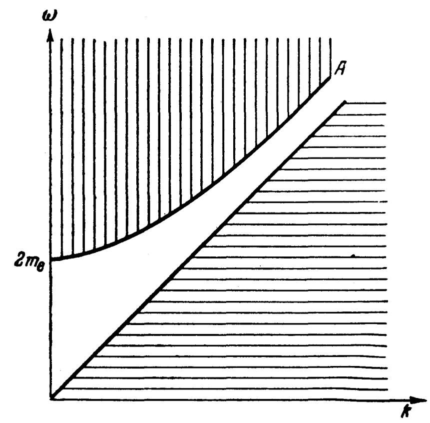

FIG. 1

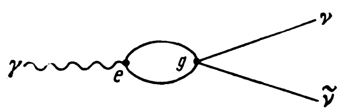

FIG. 2

2. Let us consider the process of absorption of a photon by a neutrino which is shown in first-order perturbation theory by the diagram of Fig. 3. This process is analogous to the absorption of plasma waves treated by Landau.[11] The essential point is that the process can occur between transverse electromagnetic waves and neutrinos, and it is important that the absorption depends on the concentration of neutrinos. This is not true of the process shown in Fig. 2. The previous analysis[9] of the conservation laws holds also for this

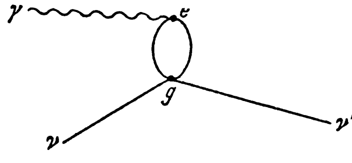

FIG. 3

1)In particular, it has been shown in papers by Pontecorvo[1] and others[2-8] that various mechanisms for neutrino emission in weak interactions may considerably affect the rate of evolution of stars.

case if in [9] we set $m_e = 0$. In Fig. 1 the region of ω and k allowed by the conservation laws for the process 3 is indicated by horizontal shading lines. By the relation $\mathbf{k}^2 = \epsilon^t(\omega, \mathbf{k})\omega^2$ the condition $k > \omega$ means $\epsilon^t > 1$. In the region of transparency, in which we shall be interested, the operator for a transverse electromagnetic wave can be written in the form[10,12]

$$A_\mu(x) = \sum_{\mathbf{k},\lambda} \left[\frac{\partial}{\partial\omega}\omega^2\varepsilon^t\right]^{-1/2} e_\mu^{(\lambda)} (c_{\mathbf{k}\lambda}e^{ikx} + c_{\mathbf{k}\lambda}^{+}e^{-ikx}) \sqrt{4\pi}. \quad (1)$$

The normalization volume is $\Omega = 1$, and $e_\mu^{(\lambda)}$ are the polarization unit vectors, $e_\mu^{(\lambda)} = \{\mathbf{e}^{(\lambda)}, 0\}$, $\mathbf{e}^{(1)}\mathbf{e}^{(2)} = 0$, $\mathbf{e}^{(\lambda)}\mathbf{k} = 0$. The interaction Lagrangian $L = L_e + L_I + L_{II}$ is given by

$$L_e = e\bar{\psi}A_\mu\gamma_\mu\psi, \quad (2)$$

$$L_I = -g\bar{\psi}\gamma_\mu\psi\bar{\varphi}\gamma_\mu(1+\gamma_5)\varphi + \text{comp. conj.}$$
$$L_{II} = -g\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\varphi}\gamma_\mu(1+\gamma_5)\varphi + \text{comp. conj.} \quad (3)$$

where g is the weak-interaction constant, ψ is the operator for electrons, and φ is that for neutrinos. Using only L_e and L_I, we get[2)] the matrix element M_I for the process in question:

$$M_I = 2ge\left\langle \int \mathrm{Sp}\,[G(x,x')\gamma_\mu G(x',x)\gamma_\nu]\bar{\varphi}(x')\gamma_\nu(1+\gamma_5) \times \varphi(x')\,dx\,dx' \right\rangle. \quad (4)$$

Let us express M_I in terms of the polarization operator $\Pi_{\mu\nu}(x, x') = -ie^2\mathrm{Sp}\,G(x, x')\gamma_\mu G(x', x)\gamma_\nu$; for a system with translational symmetry the Fourier components of this operator are connected with the dielectric constant $\epsilon^t(\omega, \mathbf{k})$ by the relations [9]

$$\Pi_{i4}^t = \Pi_{4i}^t = \Pi_{44}^t = 0,$$

$$4\pi\Pi_{ij}^t = \omega^2(\varepsilon_t(\omega,\mathbf{k}) - 1)(\delta_{ij} - k_ik_j\mathbf{k}^{-2}),$$

where i, j = 1, 2, 3.[3)] We then get from Eq. (4) an expression for the probability (per unit time) for absorption of a photon of frequency ω and momentum $\mathbf{k}$ by a neutrino of momentum $\mathbf{p}$:

$$w_{\mathbf{p}}(\omega,\mathbf{k}) = \frac{g^2}{4\pi e^2}\frac{\omega^4(\varepsilon^t-1)^2}{\partial(\omega^2\varepsilon^t)/\partial\omega}\left\{\frac{2(\mathbf{e}^{(\lambda)}\mathbf{p})^2}{|\mathbf{p}||\mathbf{p}+\mathbf{k}|} + \frac{|\mathbf{p}|\omega - (\mathbf{pk})}{|\mathbf{p}||\mathbf{p}+\mathbf{k}|}\right\} \times \delta(\omega + |\mathbf{p}| - |\mathbf{p}+\mathbf{k}|). \quad (5)$$

Here $\omega^2\epsilon^t = \mathbf{k}^2$.

The result (5) is obtained by using only the translational properties of the polarization operator, but not $G(x, x')$, and holds for arbitrary media homogeneous in space and unchanging in time. The logarithmic decrement $\gamma_{\mathbf{k}}$ of the electromagnetic waves can be found from the relation

$$\frac{\partial N_{\mathbf{k}}}{\partial t} = -2\gamma_{\mathbf{k}}N_{\mathbf{k}} = -\int\{w_{\mathbf{p}}(\omega,\mathbf{k}) - w_{\mathbf{p}-\mathbf{k}}(\omega,\mathbf{k})\}\,n_\nu(\mathbf{p})\,d\mathbf{p}N_{\mathbf{k}}, \quad (6)$$

where $N_{\mathbf{k}}$ is the number of quanta of the electromagnetic field and $n_\nu(\mathbf{p})$ is the momentum distribution function of the neutrinos. In the case of isotropic $n_\nu(\mathbf{p})$ the result averaged over polarizations is

$$\mathrm{Im}\,\varepsilon_\nu^t = \gamma_{\mathbf{k}}\omega^{-2}\frac{\partial}{\partial\omega}\omega^2\,\mathrm{Re}\,\varepsilon^t = \frac{g^2}{4\pi e^2}\frac{\omega^2(\varepsilon^t-1)^3}{(\varepsilon^t)^{3/2}}\int\frac{n_\nu(\mathbf{p})\,dp}{p}. \quad (7)$$

Here $\epsilon^t > 1$. For the equilibrium distribution $n_\nu(p) = (2\pi)^{-3}\cdot(e^{p/T_\nu} + 1)^{-1}$ we have

$$\mathrm{Im}\,\varepsilon_\nu^t = \frac{g^2}{4\pi e^2}\frac{\omega^2(\varepsilon^t-1)^3}{(\varepsilon^t)^{3/2}}\frac{T_\nu^2}{32}. \quad (8)$$

It is not hard to indicate the result that is obtained if we take L_{II} into account. The matrix element M_{II} differs from Eq. (4) only by the fact that instead of $\Pi_{\mu\nu}$ there is the quantity

$$\Pi_{\mu\nu}^{\gamma_5} = -ie^2\,\mathrm{Sp}\,G(x,x')\gamma_\mu\gamma_5G(x',x)\gamma_\nu.$$

Consequently in the expression (7) for the absorption we must replace the factor $(\epsilon^t - 1)^2$ by $(\epsilon_{\gamma_5}^t - 1)^2$, where $\epsilon_{\gamma_5}^t$ differs from ϵ^t by having the matrix elements of the type

$$\langle n_0|\gamma_ie^{i\mathbf{kr}}|n\rangle\langle n|\gamma_je^{-i\mathbf{kr}}|n_0\rangle,$$

which occur in ϵ^t replaced by

$$\langle n_0|\gamma_i\gamma_5e^{i\mathbf{kr}}|n\rangle\langle n|\gamma_je^{-i\mathbf{kr}}|n_0\rangle.$$

For $kr \ll 1$ the first matrix element $\sim\langle n_0|\Sigma_i|n\rangle$ (Σ_i is the spin matrix) is not zero for magnetic-dipole transitions, and consequently the second matrix element must correspond to quadrupole transitions. Thus L_{II} contributes only near lines for which magnetic-dipole (spin) and quadrupole transitions are simultaneously allowed. The corresponding "oscillator strength" in $\epsilon_{\gamma_5}^t$ differs from the oscillator strength for dipole transitions at roughly the same frequency by a factor of the order of magnitude $kr/v \sim \epsilon^t\omega r/v \sim \epsilon^t$. But the cross section is further proportional to the quantity $\epsilon^t - 1$, which in this case becomes $\epsilon_{\gamma_5}^t - 1$, smaller by a factor $(\mathbf{k}\cdot\mathbf{r})^2$.

3. The absorption effect is very small. An important fact is that the absorption is of a sharp

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2) In most cases the effect given by L_{II} is small.

3) Here we have dropped the vacuum part of the polarization operator and are taking into account only the main part associated with $\epsilon^t \neq 1$.

resonance character (near the natural frequencies of ϵ^t, where $\epsilon^t - 1$ is large). For this reason the absorption line of the medium should be as narrow as possible, and the frequency should be highly monochromatic. Obviously there is no need to measure the absolute absorption effect, which may be mainly not due to neutrinos, since in principle one can distinguish the neutrino absorption by a relative comparison of intensities with and without neutrinos present, and by the asymmetry of the effect. The condition $\epsilon^t > 1$ means that neutrino absorption occurs only in the low-frequency half of the line. If there is appreciable absorption the effect will be a violent shift of the absorption line.

We can write the free path of the radiation relative to neutrino absorption, $x_\nu = 2(\epsilon^{t\,1/2}/\omega)\cdot \mathrm{Im}\,\epsilon^t$ in terms of the cross section, $x_\nu = n_\nu^{-1}\sigma_\nu^{-1}$:

$$\sigma_\nu = \frac{g^2}{4\pi e^2}\frac{\omega^3(\varepsilon^t - 1)^3}{(\varepsilon^t)^2}\left\langle\frac{1}{p_\nu}\right\rangle,$$

$$\left\langle\frac{1}{p_\nu}\right\rangle = \int p^{-1} n_\nu(\mathbf{p})\,d\mathbf{p} \Big/ \int n_\nu(\mathbf{p})\,d\mathbf{p}, \qquad (9)$$

where n is the number of neutrinos per unit volume. For an optimal estimate of the possible effect we shall suppose that the absorption not due to neutrinos is compensated, for example by subsequent amplification, and that the effect can be increased, for example by repeated passages of the radiation through the region with neutrino absorption and the region of amplification.

We note that, unlike well known methods for detecting neutrinos, which have energy thresholds 10—50 keV, neutrino absorption has no threshold, and on the contrary is more effective at low energies [Eq. (9)], for which it is possible to accumulate neutrinos.[13] Obviously, however, the maximum accumulation corresponds to filling up to the Fermi level, $n_{\nu,\,max} \sim p_{\nu F}^3$, and with decreasing $p_{\nu F}$ the free path x_ν increases as $1/p_{\nu F}^2$ ($p_{\nu F}$ is the Fermi momentum). Assuming $\langle 1/p_\nu^{-1}\rangle \sim 50^{-1}$ eV, $\omega_0^2/m_e^2 = 4\pi Ne^2/m_e^2 \sim 10^{-6}$, and that the width of the resonance line is of the order of the natural width, we get an estimate of the cross section for resonance absorption:

$$\sigma_{\nu,\,max} \sim 10^{-11}\left(\frac{\hbar\omega_0}{m_e c^2}\right)^2 \hbar c \left\langle\frac{1}{p_\nu}\right\rangle\left(\frac{\hbar}{m_e c}\right)^2 \mathrm{cm}^2 \sim 10^{-41}\,\mathrm{cm}^2. \quad (10)$$

For $n_\nu \sim 10^{16}\,\mathrm{cm}^{-3}$ we have $x_\nu \sim 10^{25}$ cm. The fraction of the intensity absorbed in a path x is $J^{-1}\Delta J_{\nu\,max} = x/x_\nu$.

The quantity (10) does not depend on the frequency ω_s of the electromagnetic waves. The effect is possible for radio frequencies $\omega_s \sim 10^{10}\,\mathrm{sec}^{-1}$, optical frequencies $\omega_s \sim 10^{15}\,\mathrm{sec}^{-1}$, and nuclear resonance absorption $\omega_s \sim 10^{20}\,\mathrm{sec}^{-1}$. The possible path length x decreases with increase of ω_s,[4)] but it becomes possible to register individual quanta. Whereas for radio frequencies with $x \sim 10^{15}$ cm ($t \sim 1$ day) we have $J_\nu^{-1}\Delta J_\nu \sim 10^{-10}$, for visible light with $x \sim 10^8$ cm ($t \sim 0.01$ sec) we have $J_\nu^{-1}\Delta J_\nu \sim 10^{-17}$. The measurement of such small changes $J_\nu^{-1}\Delta J_\nu$ presents great difficulties.

It would be possible to use compensated null methods and, for example, to single out components of the photocurrent corresponding to a modulation of the radiation by a periodically varying neutrino absorption. It must be remembered that the estimates given correspond to the optimal case, since we have abstracted from the non-neutrino absorption, and compensating it would present very severe demands in the way of accuracy.

4. We shall touch on possible astrophysical applications. There are no grounds for supposing that the Fermi momentum $p_{\nu F}$ of the world background of neutrinos[14] is larger than $10^{-2}-10^{-3}$ eV, i.e., $\sigma_{\nu\,max} \sim 10^{-38}\,\mathrm{cm}^2$, but x_ν is larger than in the example just given by a factor 10^6-10^7. There is, however, a possibility in principle of detecting neutrinos of these low energies. Furthermore it is not excluded that the concentration of neutrinos in some small regions of cosmic space is at present anomalously large. The shape of absorption lines could give information about this.

We note that the absorption increases with increase of the density and the frequency ω. Therefore γ-ray astronomy of discrete nuclear resonance lines could give definite information about the neutrinos of superdense stars.

In conclusion we call attention to the interest in effects which arise from the scattering of electromagnetic waves by neutrinos.

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Note added in proof (October 2, 1963). In the paper by Adams, Ruderman and Woo[10] (ARW) there is an error in the normalization of the potential of the longitudinal plasma waves, the result of which is that in the expression obtained by ARW for the differential probability for production of a neutrino-antineutrino pair by a plasmon there is a superfluous factor $(1 - k^2/\omega^2)$. For the process of Fig. 2 we have $k < \omega$, and a calculation shows that the expression of ARW for the integrated neutrino emission from unit volume of plasma must be multiplied by a factor 3/2. This correction does not have much effect on the qualitative conclusions drawn by ARW.

4)Already in the optical region one needs very exact mirror surfaces $\Delta r \sim 10^{-7}$ cm for $x \sim 10^8$ cm.

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193