REGGE TRAJECTORY FOR THE POSITRONIUM ATOM

Yu. M. MALYUTA

Physics Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor April 23, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1167-1168 (October, 1963)

The approximation scheme for calculating Regge trajectories proposed in a previous paper [5] is employed for determining the positronium energy spectrum.

In connection with the realization of the Chew program [1], prime significance attaches to methods of calculation of Regge trajectories of real physical objects such as atoms, nuclei, and compound particles. In view of the exceeding complexity of the problem, the efforts of the theoreticians have been aimed at finding approximate methods for its solution [2-4]. The purpose of the present paper is to show how the previously proposed approximation scheme for calculation of Regge trajectories [5] can be applied to a description of the energy spectrum of orthopositronium.

1. We write out a system of equations (see [5]) for the amplitude for the scattering of an electron by a positron in the triplet state:1)

$$^1A\ (sz) = {^1A^{II}}\ (sz) - \frac{im^2}{4\pi^3}\sqrt{\frac{p^2}{p^2+m^2}}\int dz_1 \int dz_2 \frac{1}{\sqrt{k\,(zz_1z_2)}}$$
$$\times \ln\frac{z - z_1z_2 + \sqrt{k\,(zz_1z_2)}}{z - z_1z_2 - \sqrt{k\,(zz_1z_2)}}\, {^1A_3^{II}}\ (sz_2)\, {^1A_3}\ (sz_1),$$
$$^1A\ (sz) = \frac{1}{\pi}\int dz_1 \frac{^1A_3\,(sz_1)}{z_1 - z}, \qquad (1)$$

where

$$^1A\ (sz) = \sum_{\lambda_1\lambda_2}\sum_{\lambda_3\lambda_4} C_{1/2\,1/2}\,(11\lambda_4\lambda_3)\, C_{1/2\,1/2}\,(11\lambda_2\lambda_1)$$
$$\times \langle\lambda_4\lambda_3|\,A\ (s + i\varepsilon,\ z)\,|\,\lambda_2\lambda_1\rangle,$$

$$^1A^{II}\ (sz) = \sum_{\lambda_1\lambda_2}\sum_{\lambda_3\lambda_4} C_{1/2\,1/2}\,(11\lambda_4\lambda_3)\, C_{1/2\,1/2}\,(11\lambda_2\lambda_1)$$
$$\times \langle\lambda_4\lambda_3|\,A\ (s - i\varepsilon,\ z)\,|\,\lambda_2\lambda_1\rangle,$$

$$k\ (zz_1z_2) = z^2 + z_1^2 + z_2^2 - 1 - 2zz_1z_2, \quad s = 4\ (p^2 + m^2),$$

$C_{\frac{1}{2}\frac{1}{2}}(11\lambda_i\lambda_j)$ — Clebsch-Gordan coefficient, p — value of three-dimensional momentum of the electron (positron), z — cosine of the scattering angle in the c.m.s., m — mass of the electron (positron), λ_i — helicity of the electron (positron).

We change over in (1) to the pole approximation

$$^1A^{II}\ (sz) = -e^2\bar{v}^2\,(p_2)\,\gamma_\mu\, v^2\,(p_4)\,\bar{u}^1\,(p_3)\,\gamma_\mu u^1(p_1)\,/\,2p^2\,(z - 1),$$
$$^1A_3^{II}\ (sz_2) = \pi e^2\bar{v}^2\,(p_6)\,\gamma_\mu v^2\,(p_4)\,\bar{u}^1\,(p_3)\,\gamma_\mu u^1(p_5)\ \delta\ [2p^2\,(z_2 - 1)],$$

and obtain

$$^1A\ (sz) = -\frac{e^2\,(2p^2 + m^2)\,(z + 1)}{4p^2m^2\,(z - 1)} - \frac{ie^2\,(2p^2 + m^2)}{8\pi^2\sqrt{p^2\,(p^2 + m^2)}}$$
$$\times \int dz_1 \frac{1}{\sqrt{k\,(zz_1z_2)}} \times \ln\frac{z - z_1z_2 + k^{1/2}\,(zz_1z_2)}{z - z_1z_2 - k^{1/2}\,(zz_1z_2)}\, {^1A_3}\ (sz_1),$$
$$^1A(sz) = \frac{1}{\pi}\int dz_1 \frac{^1A_3(sz_1)}{z_1 - z}, \qquad (2)$$

where $z_2 = 1$.

With the aid of the procedure described previously [5], the system (2) admits as $z \to \infty$ of reduction to the following differential equation:

$$zd^1A(sz)/dz = c\ (s) + \alpha\ (s)\ {^1A}\ (sz);$$
$$\alpha\ (s) = -1 + ie^2\ (s - 2m^2)\ /4\pi\sqrt{s\ (s - 4m^2)},$$
$$c\ (s) = -e^2\ (s - 2m^2)\ /2m^2\ (s - 4m^2). \qquad (3)$$

We discard in (3) the term c(s), since it makes no contribution to the asymptotic value governed by the moving singularity. Then the equation

$$zd^1A\ (sz)/dz = \alpha\ (s)\ {^1A}\ (sz)$$

will have a solution

$$^1A\ (sz) = \beta\ (s)\ z^{\alpha(s)}.$$

2. The qualitative features of the Regge trajectory are shown in the figure (not to scale). We solve the equation

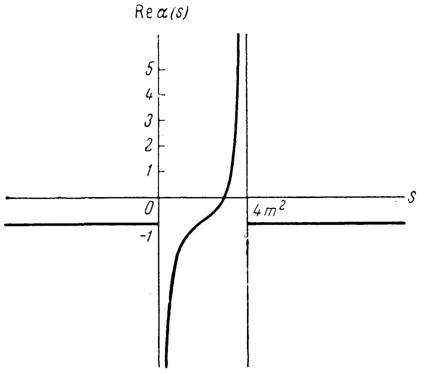
$$\mathrm{Re}\ \alpha\ (s) = l$$

where

$$0 < s < 4m^2, \quad l = 0, 1, 2, \ldots$$

with respect to s (confining ourselves to the fourth-order approximation in the coupling constant):

1)We use here an approximation which does not take into account contributions to the scattering amplitude due to singularities in the invariant u.



$$s = 4m^2 \left[1 - e^4/64\pi^2 (l + 1)^2\right]. \quad (4)$$

We introduce the notation

$$E = \sqrt{s} - 2m. \quad (5)$$

Obviously, $-E$ is the positronium binding energy. Substituting (4) in (5) we get

$$E = -\mu e_0^4/2n^2, \quad (6)$$

where $\mu = m/2$ is the reduced mass, $e_0 = e/\sqrt{4\pi}$ the electron charge in the Gaussian system of units, and $n = l + 1$ is the principal quantum number.

Formula (6) is the Bohr formula for the spectrum of a hydrogenlike atom.

The author is grateful to N. N. Bogolyubov for a discussion of the work.

[1] G. F. Chew, Revs. Modern Phys. **34**, 394 (1962).

[2] B. Lee and R. Sawyer, Phys. Rev. **127**, 2266 (1962).

[3] Amati, Stanghellini, and Fubini, Nuovo cimento **26**, 896 (1962).

[4] A. A. Logunov and A. N. Tavkhelidze, Preprint Joint Inst. Nuc. Res. E-1145, (1962).

[5] Yu. M. Malyuta, JETP **44**, 1317 (1963), Soviet Phys. JETP **17**, 888 (1963).

Translated by A. M. Bincer and J. G. Adashko
190