ELECTROMAGNETIC EMISSION OF ATOMIC ELECTRONS DURING BETA DECAY

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Submitted to JETP editor April 13, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1133-1135 (October, 1963)

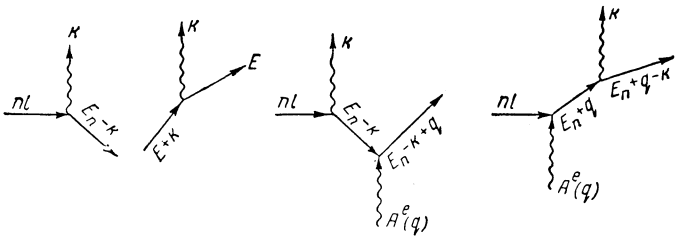
Emission of atomic electrons during β decay, a second order effect with respect to electromagnetic interaction, is considered. Calculations for β decay of S^{35} show that the main contribution is from electron emission of the magnetic type from the outer shell. The theoretical estimates are in good agreement with the experiments.

INTERNAL bremsstrahlung in β decay and K capture was first considered by Knipp and Uhlenbeck[1]. Later Glauber and Martin developed a more consistent theory of radiative capture of orbital electrons[2,3]; Lewis and Ford[4] took account of the contribution from the "virtual intermediate state" to the internal bremsstrahlung in β decay, while Spruch and Gold[5] and Vinh-Mau [6] considered the Coulomb corrections for this case. All these authors consider the emission of the γ quantum by the β particle itself or by the captured orbital electron.

The above-mentioned refinements[4-6] bring theory closer to experiment, but the number of internal bremsstrahlung γ quanta per β decay still remains much larger in experiment than would follow from the theories (see [7,8]). In this connection it is of interest to consider an effect which is of second order in the electromagnetic interaction in β decay, namely the electromagnetic radiation of the atomic electrons, which can also contribute to the internal bremsstrahlung.

The Feynman diagrams (see the figure, cases a and b) correspond to radiative capture of an orbital electron and to radiative β decay, while the two other diagrams (see cases c and d) correspond to the radiation of the atomic electron in a state having a principal quantum number n and an orbital quantum number l, in the case of decay; q is the energy quantum transmitted to the electron by the nucleus when the charge of the nucleus changes as a result of the β decay; following the radiation, the electron is in one of the unoccupied bound states or else in the continuous-spectrum state.

The calculation of the matrix elements was carried out in the nonrelativistic approximation using for the electron in the Coulomb field of the nucleus a Green's function, satisfying a second-



a b c d

order equation (see formulas (2.26)—(2.29) in the paper by Glauber and Martin[2]). The transfer of an energy quantum by the nucleus, upon an "instantaneous" change in its charge, to an electron in a bound or virtual intermediate state, was taken into account by the method proposed by Migdal and Feinberg[9,10] for the calculation of the auto-ionization in β decay.

The formula for the ratio of the probability that an atomic electron will emit during β decay a γ quantum with energy k, referred to a single energy unit, equal to the electron mass m [1)], to the probability of the ordinary β decay, is of the form

$$\frac{w_{\text{rad}}(k)}{w_{\text{ord}}} = \frac{2e^2}{\pi} k \frac{I(W_0 - k)}{I(W_0)} |G(k)|^2. \tag{1}$$

Here G(k) is a function analogous to that introduced by Glauber and Martin [formula (4.3) of [2] and formula (2) fo the present paper]; I(x) is the Fermi integral which arises in the integration over the energy of the β particle (tabulated in the paper by Feenberg and Trigg[11]):

$$I(x) = \int_m^x (x-\varepsilon)^2 \varepsilon \sqrt{\varepsilon^2 - m^2} F(Z, \varepsilon)\, d\varepsilon,$$

where $F(Z, \epsilon)$ — Fermi function, m — electron mass, W_0 — decay energy.

1)We use a system of units in which $\hbar = c = 1$, $e^2 = \alpha = 1/137$.

Since the probability of the emission of the electron from the atom is negligibly small (of the order of Z_{eff}^{-2}, where Z_{eff} is the effective charge of the nucleus for the electron under consideration) compared with the probability of remaining in the bound state, we obtain for $G(k)$

$$G(k) = \frac{1}{2}\sum\left\{\iint \varphi_2^{Z+1\,*}(\mathbf{r}')\, G^Z_{E_{n_1}-k}(\mathbf{r}', \mathbf{r}) \times [\sigma_{\mu\nu}k_\mu e_\nu - 2\nabla \mathbf{e}]\, \varphi_1^Z(\mathbf{r})\, d\mathbf{r}' d\mathbf{r} - \iint \varphi_2^{Z+1\,*}(\mathbf{r}')\, [\sigma_{\mu\nu}k_\mu e_\nu - 2\nabla \mathbf{e}]\, G^Z_{E_{n_1}+k}(\mathbf{r}', \mathbf{r})\, \varphi_1^Z(\mathbf{r})\, d\mathbf{r}' d\mathbf{r}\right\}, \tag{2}$$

where $\varphi_2^{Z+1}(\mathbf{r}')$ and $\varphi_1^Z(\mathbf{r})$ —final and initial nonrelativistic wave functions of the electron with principal quantum numbers n_2 and n_1, orbital momenta l_2 and l_1, total momenta j_2 and j_1, and their projections m_2 and m_1. $\mathbf{e}$ —photon polarization vector, $\sigma_{\mu\nu}$ —spin matrix, G —nonrelativistic Green's function for the electron in the Coulomb field of the nucleus and for the remaining electrons of the atom; the summation is over $\mathbf{e}$, n_2, l_2, j_2, and m_2.

The first term in the square brackets in (2) corresponds to radiation of the magnetic type, and the second to dipole radiation. In the nonrelativistic limit $E = m$ (this occurs approximately at small γ-quantum energies $k < (Z\alpha)^2 m$), and we have for the Green's function

$$G^Z_{E_{n_1}\pm k}(\mathbf{r}', \mathbf{r}) = \frac{1}{2m}\sum_{nl}\frac{\varphi^{Z*}_{nl}(\mathbf{r}')\varphi^Z_{nl}(\mathbf{r})}{E_n - E_{n_1} \mp k}. \tag{3}$$

Glauber and Martin[2] have shown that this form of the Green's function can be used also for "medium" γ-quantum energies $k < (Z\alpha)m$ in the case of magnetic-type radiation.

Using the orthogonality of the functions $\varphi^Z_{nl}(\mathbf{r})$, and also neglecting terms of order Z^{-2} compared with unity, we find by substituting (3) in (2) that the approximate selection rules for the term responsible for the magnetic-type radiation will be $n_2 = n_1$ and $l_2 = l_1$. Thus, only electrons from the outer shell of the atom can contribute to radiation of the magnetic type, provided the outer shell is not filled. The formula for the ratio of the probability of β decay with magnetic type emission of a γ quantum of one energy unit equal to the electron mass m, to the probability of the ordinary β decay, assumes in the approximation of formula (3) the following form:

$$\frac{w_{\text{rad}}(k)}{w_{\text{ord}}} = \frac{e^2}{8\pi}\frac{k}{m^2}\frac{I(W_0 - k)}{I(W_0)}. \tag{4}$$

Calculation by this formula for the case of β^- decay of S^{35} is given in the table, which lists also data obtained by others and the experimental data. In [1,4-6] are given theoretical estimates of the number of γ quanta per β decay, emitted by the β particle itself; we have calculated the number of γ quanta per β decay, emitted by the atomic electrons (we considered only the contribution from magnetic type radiation, since the contribution of the dipole radiation is negligibly small in this case). The seventh column of the table gives the sum of the numbers obtained by us and in [6]; the eighth and ninth columns give the experimental values of the number of γ quanta per β decay.

k/m	$10^5 N_\gamma$, theory					$10^5 N_\gamma$, sum	$10^5 N_\gamma$, expt	
	from[1]	from[4]	[5]	from [6]	this paper		from[7]	from [8]
.05	326	324	—	324	3,66	327.66	325	516
0.10	73	74	—	76	3,66	79,66	92	146
0.15	18.1	20.2	21.4	24.2	2.47	26.67	30,5	50
0.20	4.1	5,2	—	6.5	1,31	7.81	8.1	12
0.25	0.56	0.82	1.06	1,12	0.23	1.35	1.2	—

As can be seen from the table, the contribution made to the internal bremsstrahlung by the radiation of the atomic electrons in β decay is considerable and brings the theoretical estimates much closer to the experimental values.

In conclusion I thank V. P. Sachenko for interest in the work.

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Translated by J. G. Adashko
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