ON PROCESSES DETERMINED BY FERMION REGGE POLES

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πN-scattering at high energies and angles close to 180° is considered. Scattering in these conditions is assumed to be described by fermion Regge poles. A simple proof is presented that the amplitude is determined by pairs of complex conjugate Regge poles. Polarization phenomena are investigated for one and two pole pairs. Three types of relations (crossing, factorization and isotopic) between cross sections of various processes governed by the same Regge poles are considered.

1. INTRODUCTION

THE investigation of large-angle scattering of mesons by nucleons at high energies is of considerable interest from the point of view of checking on the Regge pole hypothesis in strong interactions. It has been discussed from this point of view in a whole series of theoretical papers [1-4]. In addition to checking on the hypothesis, the investigation of large-angle scattering makes it possible to ascertain whether the known baryons are elementary particles or members of a Regge family.

It was observed earlier [4] that the fermion Regge poles, which determine the asymptotic behavior of the scattering at large angles, have properties substantially different from boson Regge poles. The poles corresponding to states with opposite parity turn out to be complex conjugate in the physical region of scattering. Therefore the asymptotic behavior of the large-angle scattering is determined not by individual Regge poles, but by pairs of complex-conjugate poles. This phenomenon was discussed in several papers [5].

In the present paper we present a simple proof of the above-indicated properties of fermion poles, without leaning (in contrast with [4,5]) on rather cumbersome expansions in partial amplitudes, but in the spirit of the correspondence principle, regarding the asymptotic amplitudes corresponding to the Regge poles as being "Reggeized" pole Feynman amplitudes. We furthermore investigate the cross section and the polarization correlations in the cases when the asymptotic behavior is determined by one or two pairs of complex-conjugate poles, and consider the relations between different processes brought about by the same pair of Regge poles, relations due to the fact that the poles have a definite signature and isotopic spin, and residues that factorize.

2. REGGEIZATION OF THE FERMION POLE

Let us consider πN scattering, due to a fermion Regge pole. We denote the 4-momenta of the colliding pion and nucleon by k_1 and p_1, and those of the outgoing particles by k_2 and p_2; the 4-momentum of the Reggion is denoted by q (see Fig. 1).

The πN-scattering amplitude, which is usually written in the form [1)]

$$M = \bar{u}_2 (A + B\hat{k}) u_1, \tag{1}$$

where $k = \frac{1}{2}(k_1 + k_2)$, is best rewritten in the form

$$M = \bar{u}_2 (a + b\hat{q}) u_1. \tag{2}$$

From the relation $q = p - k$, where $p = \frac{1}{2}(p_1 + p_2)$, it follows that $a = A + mB$ and $b = -B$.

We note that the scalar functions a and b, like A and B, depend only on the invariants s, t, and u:

$$s = (p_1 + k_1)^2, \quad t = (p_1 - p_2)^2, \quad u = (p_1 - k_2)^2 = q^2. \tag{3}$$

It was shown in a paper by one of the authors [4] that the fermion poles form complex-conjugate pairs, corresponding to states with given angular momentum j and opposite parity. This proof used the helicity representation and was rather cumbersome. The expression obtained in [4] for the contribution to the amplitude of one pole with definite parity (plus or minus) can be written in the form

[1)]We use units $\hbar = c = 1$ and the Feynman notation $\hat{q} = q_4\gamma_4 - \mathbf{q}\cdot\boldsymbol{\gamma}$, where $\gamma_4 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, $\boldsymbol{\gamma} = \begin{pmatrix}0 & \sigma\\ -\sigma & 0\end{pmatrix}$, $\gamma_5 = -\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$.

$$M_\pm = f_\pm \bar{u}_2(p_2)\,[\hat{q} \pm \sqrt{u}]\,u_1(p_1),$$

$$f_+ = f(\sqrt{u}, s), \quad f_- = f(-\sqrt{u}, s). \tag{4}$$

Expression (4) is very natural from the point of view of going over from the usual poles of the Feynman perturbation theory (see Fig. 1) to the Regge poles. In this transition m in the numerator of the expression $(\hat{q} + m)/(q^2 - m^2)$ is replaced by the Reggion mass $-\sqrt{u}$, while the denominator which does not become infinite for unphysical j, is included in $f_\pm$.

The expression $(\hat{q} \pm \sqrt{u})$, which is contained in (4), has a simple physical meaning. By acting on the state of the πN system in the c.m.s. of the u-channel, this expression picks out states with definite parity. Indeed, in the c.m.s. of the u-channel we have

$$(\hat{q} + \sqrt{u}) = \sqrt{u}\,(\gamma_4 + 1) = 2\sqrt{u}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

and therefore the amplitudes M_+ and M_- are written in the two-component form as

$$M_+ \sim f_+\sqrt{u}\,(1\cdot 1), \tag{5}$$

$$M_- \sim f_-\sqrt{u}\left(\frac{\sigma p_2}{E+m}\cdot\frac{\sigma p_1}{E+m}\right). \tag{6}$$

The amplitude (6) differs from (5) in that the initial and final states have in it a different parity, owing to the pseudoscalar factors $\boldsymbol{\sigma}\cdot\mathbf{p}_1$ and $\boldsymbol{\sigma}\cdot\mathbf{p}_2$. Since by assumption f_+ and f_- are such that M_+ and M_- contain contributions from states having the same definite angular momentum, the parities of these states are opposite. We emphasize that unlike the expression $(\hat{q} \pm \sqrt{u})$, the Feynman numerators $(\hat{q} + m)$ and $(\hat{q} - m) = -\gamma_5(q + m)\gamma_5$ in the usual pole terms, although each other's "parity mirrors," do not have any definite parity, and yield, for example, both S and P scattering.

We have thus shown that the contribution of one fermion pole with definite parity has the form (4). However, the expression for one pole for arbitrary f(u, s) has a singularity at u = 0, owing to the presence of $\sqrt{u}$. The existence of such a singularity contradicts the Mandelstam analytic properties of the amplitudes a and b, appearing in (2). Therefore the amplitude M cannot be defined by one pole only, and must have the form

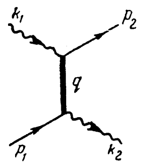

FIG. 1

$$M = M_+ + M_- = f_q\hat{q} + f_u\sqrt{u}, \tag{7}$$

$$f_q = (f_+ + f_-), \quad f_u = (f_+ - f_-). \tag{8}$$

At that it is necessary here that either $f_+ = f_-$, or each of them contains a root singularity at u = 0 in such a way that

$$f_+ = f(\sqrt{u}, s), \quad f_- = f(-\sqrt{u}, s). \tag{9}$$

The first possibility, which we shall not consider, corresponds to degeneracy in parity. If the second possibility takes place, then f_u should be proportional to $u^{-1/2}$ as $u \to 0$. A behavior of the $u^{1/2}$ type would imply the vanishing of M as $u \to 0$, for which there are no grounds whatever.

Let us write $f_\pm$ in the usual manner in terms of the residues $(r_\pm(u))$ and the position of the Regge poles $(j_\pm(u))$:

$$f_+^{e,o} = \frac{1 \pm e^{-i\pi\alpha_+}}{\sin\pi\alpha_+}\left(\frac{s}{s_0}\right)^{\alpha_+} r_+, \tag{10}$$

$$f_-^{e,o} = \frac{1 \pm e^{-i\pi\alpha_-}}{\sin\pi\alpha_-}\left(\frac{s}{s_0}\right)^{\alpha_-} r_-, \tag{11}$$

where $\alpha_\pm = j_\pm - 1/2$, and the indices e and o denote respectively even and odd signatures.

From (7)—(9) and (10) and (11) it follows that

$$\alpha_+ = \alpha(\sqrt{u}), \quad \alpha_- = \alpha(-\sqrt{u}), \tag{12}$$

$$r_+ = r(\sqrt{u}), \quad r_- = r(-\sqrt{u}). \tag{13}$$

Since the absorptive parts of the amplitudes a and b in expression (2), which are equal to

$$a_1 = [(s/s_0)^{\alpha_+} r_+ - (s/s_0)^{\alpha_-} r_-]\sqrt{u}, \tag{14}$$

$$b_1 = [(s/s_0)^{\alpha_+} r_+ + (s/s_0)^{\alpha_-} r_-], \tag{15}$$

should be real when $u < (m + \mu)^2$, we get for u < 0

$$\alpha_+ = \alpha_-^*, \quad r_+ = r_-^*. \tag{16}$$

Thus, these simple considerations lead to the conclusion that there exist complex-conjugate poles and that the asymptotic πN-scattering amplitude oscillates with increasing energy s as shown in [4].

3. CROSS SECTION AND SPIN CORRELATIONS IN πN SCATTERING AT HIGH ENERGIES

To calculate the cross section and the polarization effects it is convenient to write the quantities $f_\pm^{e,o}$ in the form

$$f_\pm^e = R^e \exp\{\pi(\pm\nu - i\mu \pm i\lambda)/2\},$$

$$f_\pm^o = iR^o \exp\{\pi(\pm\nu - i\mu \pm i\lambda)/2\}. \tag{17}$$

Here μ and ν are respectively the real and imaginary parts of the complex orbital angular momentum α:

$$\alpha_\pm = \mu \pm i\nu. \tag{18}$$

The form of R for respectively positive and negative signature is

$$R^{e,o} = \sqrt{2}\, r (s/s_0)^{\mu} (\operatorname{ch} \pi\nu \mp \cos \pi\mu)^{-1/2}, \quad (19)^*$$

where r is the absolute value of the complex-conjugate residues $r_{\pm}$:

$$r_{\pm} = r e^{\pm i\varphi}. \quad (20)$$

By virtue of the fact that $\alpha_{\pm}$ and $r_{\pm}$ are complex conjugate, the concept of the parity of the Regge pole becomes meaningless, and it is necessary to agree which of the amplitudes will be called f_+ and which f_-, and also what is α_+, and what is α_-. Since the quantities $f_{\pm}$ and $\alpha_{\pm}$ are well defined for $u > 0$, we shall define their values for $u < 0$ by means of the condition that the singular point $u = 0$ is passed from above $u \to u + i\epsilon$ ($\epsilon > 0$), so that $\sqrt{u} = i\sqrt{-u}$.

The quantity λ contained in (17) is, unlike μ, ν, r, and φ, a function of not only u but also of s, and is equal to

$$\lambda(s, u) = 2\pi^{-1} [\nu \ln (s/s_0) + \varphi - \psi], \quad (21)$$

where the value of ψ, which depends on the signature, is defined by the relations

$$\operatorname{tg} \psi^e = \operatorname{th} \frac{\pi\nu}{2} \operatorname{ctg} \frac{\pi\mu}{2}, \quad \operatorname{tg} \psi^o = -\operatorname{th} \frac{\pi\nu}{2} \operatorname{tg} \frac{\pi\mu}{2}. \quad (22)\dagger$$

Let us write down the amplitude M(7) for $s \gg u, m^2$ in the c.m.s. of the s-channel in the two-component form:

$$M = \bar{u}_2 (f_q \hat{q} + f_u \sqrt{u})\, u_1 \approx 2\sqrt{u}\, \varphi_2^* [f_q \sigma \mathbf{n} + f_u]\, \varphi_1. \quad (23)\ddagger$$

Here

$$u_1 = \begin{pmatrix} 1 \\ \sigma \mathbf{n}_1 \end{pmatrix} \varphi_1, \quad u_2 = \begin{pmatrix} 1 \\ \sigma \mathbf{n}_2 \end{pmatrix} \varphi_2, \quad \mathbf{n} = \frac{[\mathbf{n}_1 \mathbf{n}_2]}{|[\mathbf{n}_1 \mathbf{n}_2]|}.$$

We fix the nucleon polarization before and after the scattering in the direction of the unit vectors ζ_1 and ζ_2, respectively; then

$$\varphi_{1\rho} \varphi_{1\tau}^* = \tfrac{1}{2} (1 + \zeta_1 \sigma)_{\rho\tau}, \qquad \varphi_{2\rho} \varphi_{2\tau}^* = \tfrac{1}{2} (1 + \zeta_2 \sigma)_{\rho\tau}. \quad (24)$$

For the differential scattering cross section for fixed polarizations we have, using the standard Feynman normalization

$$d\sigma = -\frac{u}{4\pi} \Big\{ F_1 [1 + (\zeta_1 \mathbf{n})(\zeta_2 \mathbf{n})] + F_2 [(\zeta_1 \mathbf{n}) + (\zeta_2 \mathbf{n})] + F_3 [(\zeta_1 \mathbf{n})(\zeta_2 \mathbf{n}) - \zeta_1 \zeta_2] + F_4 \mathbf{n} [\zeta_1 \zeta_2] \Big\} \frac{do}{4\pi}; \quad (25)$$

$$F_1 = \tfrac{1}{4} (f_q f_q^* + f_u f_u^*) = \tfrac{1}{2} (f_+ f_+^* + f_- f_-^*) = R^2 \operatorname{ch} \pi\nu, \quad (26)$$

$$F_2 = \tfrac{1}{4} (f_q f_u^* + f_u f_q^*) = \tfrac{1}{2} (f_+ f_+^* - f_- f_-^*) = R^2 \operatorname{sh} \pi\nu, \quad (27)^*$$

$$F_3 = \tfrac{1}{4} (f_q f_q^* - f_u f_u^*) = \tfrac{1}{2} (f_+ f_-^* + f_- f_+^*) = R^2 \cos \pi\lambda, \quad (28)$$

$$F_4 = \tfrac{i}{4} (f_q f_u^* - f_u f_q^*) = \tfrac{i}{2} (f_- f_+ - f_+ f_-^*) = R^2 \sin \pi\lambda. \quad (29)$$

If we average (25) over the spin states of the initial nucleons and sum over the spin states of the final nucleons, then we obtain an expression for the differential cross section:

$$d\bar{\sigma} = -\frac{u}{2\pi} F_1 \frac{do}{4\pi} = -\frac{ur^2}{\pi} \frac{(s/s_0)^{2\mu} \operatorname{ch} \pi\nu}{\operatorname{ch} \pi\nu \mp \cos \pi\mu} \frac{do}{4\pi}. \quad (30)$$

It must be emphasized that the cross section (30) varies monotonically with the energy s.

The quantity F_2 determines the polarization of the recoil nucleon upon scattering of a pion by an unpolarized target. The degree of polarization is

$$P = F_2/F_1 = \operatorname{th} \pi\nu. \quad (31)$$

This expression likewise does not oscillate with increasing energy [2)].

The quantities F_3 and F_4 determine the polarization correlations of the initial and final nucleons, and oscillate with a variation of the energy and of the momentum transfer, since $\lambda = \lambda(s, u)$ in accordance with (21).

For not too high energies, such as are feasible in modern accelerators, the contribution of the complex conjugate pole pairs with smaller values of Re j than in the principal pair may not be negligibly small. It is therefore meaningful to ascertain the behavior of the cross section with account of two pole pairs.

In the case of two pole pairs with the same signature we have

$$f_{\pm} = R_1 \exp \{\pi (\pm \nu_1 - i\mu_1 \pm i\lambda_1)/2\} + R_2 \exp \{\pi (\pm \nu_2 - i\mu_2 \pm \lambda_2)/2\}, \quad (32)$$

and consequently the quantities F_i entering in (25) are equal to, in accordance with (26)–(29),

$$F_1 = R_1^2 \operatorname{ch} \pi\nu_1 + R_2^2 \operatorname{ch} \pi\nu_2 + R_1 R_2 \Big[e^{\pi(\nu_1+\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 - \lambda_2 - \mu_1 + \mu_2) + e^{-\pi(\nu_1+\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 - \lambda_2 + \mu_1 - \mu_2)\Big], \quad (33)$$

$$F_2 = R_1^2 \operatorname{sh} \pi\nu_1 + R_2^2 \operatorname{sh} \pi\nu_2 + R_1 R_2 \Big[e^{\pi(\nu_1+\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 - \lambda_2 - \mu_1 + \mu_2) + e^{-\pi(\nu_1+\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 - \lambda_2 + \mu_1 - \mu_2)\Big], \quad (34)$$

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*ch = cosh.

†tg = tan, ctg = cot, th = tanh.

‡$[\mathbf{n}_1 \mathbf{n}_2] = \mathbf{n}_1 \times \mathbf{n}_2$.

2)In the course of calculating the polarization in [4], an error has crept in, which was pointed out to the authors by M. P. Rekalo, who considered the question of fermion poles in pion photoproduction.

*sh = sinh.

$$F_3 = R_1^2 \cos \pi\lambda_1 + R_2^2 \cos \pi\lambda_2 + R_1 R_2 \left[e^{\pi(\nu_1-\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 + \lambda_2 - \mu_1 + \mu_2) + e^{-\pi(\nu_1-\nu_2)/2} \cos \frac{\pi}{2} (\lambda_1 + \lambda_2 + \mu_1 - \mu_2) \right], \quad (35)$$

$$F_4 = R_1^2 \sin \pi\lambda_1 + R_2^2 \sin \pi\lambda_2 + R_1 R_2 \left[e^{\pi(\nu_1-\nu_2)/2} \sin \frac{\pi}{2} (\lambda_1 + \lambda_2 - \mu_1 + \mu_2) + e^{-\pi(\nu_1-\nu_2)/2} \sin \frac{\pi}{2} (\lambda_1 + \lambda_2 + \mu_1 - \mu_2) \right]. \quad (36)$$

Owing to the presence of interference terms proportional to R_1R_2, the characteristic oscillations with energy and momentum transfer are experienced not only by the polarization correlations F_3 and F_4, but by the differential cross section itself (F_1) and by the polarization (F_2). This is a very important circumstance, for in experiment it is simpler to observe oscillations in the differential cross section than in the polarization correlation.

From the form of R and λ [see (19)—(22)], and also from the fact that there is no possibility of determining independently by experiment the values of r and φ [see (26)—(29)] it follows that by measuring the πN scattering it is impossible to determine the signature of the individual pair of fermion poles.

Let us consider now the question of the possibility of determining the sign of the signature in the case when there are two pairs of poles. Expressions (33)—(36) were derived under the assumption that the signatures of both pole pairs were the same. In the case when the signature of the first pair of poles is positive and that of the second is negative, it is necessary, in accordance with (17) to make in (33)—(36) the substitution $\mu_2 \to \mu_2 - 1$. If, to the contrary, the signature of the first pair is negative and that of the second is positive, then $\mu_1 \to \mu_1 - 1$. It is easy to see that in these two cases opposite signs are obtained for the interference terms R_1R_2 (the arguments of both the cosine and sine differ by π). However, since the signs of R_1 and R_2 cannot be determined independently, these two cases are actually indistinguishable from each other. It is obvious, however, that both cases differ from the two other cases, in which the signatures of the two pairs of poles are the same. (The transition from identical to opposite signatures leads to the substitution sin $\rightleftarrows$ cos in expressions (33)—(36).)

We thus arrive at the conclusion that it is possible to determine by experiment the relative signature of two pairs of fermion poles, and that the determination of the absolute signature of one pair of fermion poles is impossible. In this respect, fermion poles are perfectly analogous to boson poles. The only exception are poles (of the type P′, ω^0, ρ^0) whose signature is determined relative to the signature of the pole P, and the latter is determined independently by the optical theorem, by virtue of which it should be positive.

To conclude this section we note that the expressions given above can be valid only for such u as correspond to values of μ larger than -1. In the vicinity of $\alpha = -1$ there should occur an accumulation of the Regge poles, fully analogous to that considered earlier[6]. This causes, in accordance with (30) the cross section dσ/do not to be able to decrease with increasing s faster than $1/s^2$ for fixed u. The presently available experimental data[7] pertain to pion energies ≤ 5 BeV.

4. VARIOUS RELATIONS BETWEEN DIFFERENT PROCESSES

Just as in the case of boson poles, the hypothesis of fermion Regge poles leads to a large number of relations between the cross sections of different processes. The simplest example of such relations are the relations between the amplitude of the direct and crossed reactions, for example, πN scattering and 2-meson annihilation of a nucleon-antinucleon pair. These reactions should be compared under conditions when the πN scattering occurs at angles close to 180°, and in the annihilation the angles between the nucleon and the meson that is secondary in the scattering are small (small angles between particles 2 and 4 on Fig. 2). Under these conditions the annihilation and scattering amplitudes are equal for poles with even signature and have opposite signs for poles with odd signature (R^e does not reverse sign, R^o reverses sign). The expression for the two-meson annihilation cross section has the same form as the scattering cross section, and is described by expressions (25)—(29) and (33)—(36), the only difference being that the interference terms R_1R_2 in (33)—(36) reverse sign if poles 1 and 2 have different signatures.

Analogous equalities between the cross sections occur for the reactions listed in Table I ($\sigma_1 = \sigma_{1'}$, $\sigma_2 = \sigma_{2'}, \ldots$). The particles in the first column of

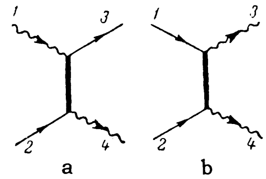


FIG. 2

Table I

Pairs of conjugate reactions		Type of Regge poles*
1. $\pi+N\to N+\pi$	1′. $\bar{N}+N\to\pi+\pi$	N or Δ
2. $\bar{K}+N\to\Sigma+\pi$	2′. $\bar{\Sigma}+N\to K+\pi$	N or Δ
3. $\pi+N\to N+\eta$	3′. $\bar{N}+N\to\pi+\eta$	N
4. $\bar{K}+N\to\Lambda+\pi$	4′. $\bar{\Lambda}+N\to K+\pi$	N
5. $\bar{K}+N\to\Sigma+\eta$	5′. $\bar{\Sigma}+N\to K+\eta$	N
6. $\bar{K}+N\to\Lambda+\eta$	6′. $\bar{\Lambda}+N\to K+\eta$	N
7. $K+N\to K+N$	7′. $\bar{N}+N\to\bar{K}+K$	Λ or Σ
8. $\bar{K}+N\to\Xi+K$	8′. $\bar{\Xi}+N\to K+K$	Λ or Σ
9. $\pi+N\to\Sigma+K$	9′. $\bar{\Sigma}+N\to\pi+K$	Λ or Σ
10. $\pi+N\to\Lambda+K$	10′. $\bar{\Lambda}+N\to\pi+K$	Σ
11. $\bar{K}+N\to N+\bar{K}$	11′. $\bar{N}+N\to K+\bar{K}$	Baryon pole with strangeness S = +1

*Δ denotes the trajectory of the baryon resonance with isotopic spin T = 3/2.

Table I have been written out in a sequence corresponding to Fig. 2a, while those in the second column follow Fig. 2b. The Regge poles indicated in the third column of the table determine the asymptotic behavior under conditions when s is large and u is small, where $s = (p_1 + p_2)^2$ and $u = (p_3 - p_1)^2$.

In Table I are listed the reactions in which only mesons with spin zero participate (π, K, η), to which formulas (25)—(29) are applicable. Of course, analogous relations hold for the creation of vector mesons (ω, ρ, K*) and baryon resonances. The extent to which these relations should be violated by the instability of these particles is not clear at present.

Another type of relation is based on the factorization of the residues [8,9]. Let us consider the three reactions shown in Fig. 3. Particles A and B are fermions, while particles a and b are bosons. The contribution of one Regge pole with definite parity to the amplitudes of these three processes has the form (4). If the quantities corresponding to $f_\pm$ in (4) are denoted for these reactions by $f_\pm$, $g_\pm$, and $h_\pm$, then formulas (10) and (11) with identical $\alpha_\pm$ hold for $f_\pm$, $g_\pm$, and $h_\pm$. The residues $r^f_\pm$, $r^g_\pm$, and $r^h_\pm$ satisfy by virtue of the unitarity condition the relation

$$r^f_\pm r^h_\pm = (r^g_\pm)^2. \quad (37)$$

When u < 0 the residues r_+ and r_-, as in πN scattering, are complex conjugates, that is,

$$r^f_\pm = r^f e^{\pm i\varphi_f}, \qquad r^g_\pm = r^g e^{\pm i\varphi_g}, \qquad r^h_\pm = r^h e^{\pm i\varphi_h}. \quad (38)$$

It then follows from (37) that

$$r_f r_h = r_g^2, \qquad 2\varphi_g = \varphi_f + \varphi_h. \quad (39)$$

If we now write $f_\pm$, $g_\pm$, and $h_\pm$ in the form (17), then it follows from (39) that

$$R_f R_h = R_g^2, \qquad \lambda_f + \lambda_h = 2\lambda_g, \quad (40)$$

with λ_f, λ_g, and λ_h having identical dependence on s, in accordance with (21), since the quantities μ and ν are the same for the three reactions.

The expressions for the cross sections of each of the three reactions have the form (25)—(29), from which it follows that

$$\sigma_f \sigma_h = \sigma_g^2, \quad (41)$$

and the polarizations P are the same in all three reactions in accordance with (31).

If we consider a larger number of processes proceeding via one pair of nucleon poles (N pole), then we can obtain relations of the type

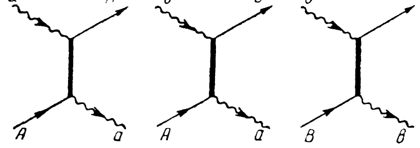
$$\sigma_1/\sigma_3 = \sigma_2/\sigma_5 = \sigma_4/\sigma_6, \quad (42)$$

where the subscripts indicate the numbers of the reactions in Table I.

The third type of relations that occur under the Regge pole hypothesis are isotopic relations due to the fact that the Regge poles have definite isotopic spins.

Let us consider, for example πN scattering. If the principal pole pair is one with T = 1/2 (nucleon trajectories), then the ratio of the scattering cross sections of $\pi^- p \to p\pi^-$ and $\pi^+ p \to p\pi^+$ should tend to zero as $s \to \infty$, $-u \lesssim m^2$. On the other hand, if the principal pole pair has T = 3/2 (Δ trajectories), then the ratio should tend to 9. These and other relations for πN scattering are listed in Table II. By investigating these relations experimentally, it is possible to establish which of the trajectories are the principal ones. A convenient method of experimentally verifying the relations between different πN cross sections is to compare data on the scattering of the same pion beam by hydrogen and deuterium targets. Thus, for example, the ratio σ_p/σ_d in the case of negative pions should tend to zero with increasing energy for an N-pole and to 9/10 for a Δ-pole. For the scattering of K mesons, the corresponding relations are listed in Table III.

Let us note that backward scattering of $\bar{K}$ mesons should proceed through a Regge pole with strangeness S = +1. No baryon states with S = +1 have been observed experimentally. This circumstance may indicate that even for negative u the "spin" of the state with S = +1 will be smaller



a b c

FIG. 3

Table II

Reaction	Pole	
	N	Δ
$\pi^+n\to n\pi^+$, $\pi^-p\to p\pi^-$	0	9
$\pi^-n\to n\pi^-$, $\pi^+p\to p\pi^+$	2	1
$\pi^+n\to p\pi^0$, $\pi^-p\to n\pi^0$	1	2

Table III

Reaction	Pole	
	Λ	Σ
$K^+p\to pK^+$, $K^0n\to nK^0$	1	1
$K^0p\to nK^+$, $K^+n\to pK^0$	1	1
$K^0p\to pK^0$, $K^+n\to nK^+$	0	4

than the "spin" of states with $S = -1$, which determine the backward scattering of the K mesons. This would mean that as $s \to \infty$ the backward scattering cross section of K^- and $\overline{K^0}$ mesons would be small compared with the backward scattering cross section of K^+ and K^0 mesons.

Another manifestation of the same fact would be an asymmetry in the angular distribution of the K mesons in nucleon annihilation. Thus, for example, in the reaction $\bar{p} + p \to K^- + K^+$ the K^- mesons should be emitted predominantly in the $\bar{p}$ direction, while the K^+ should be emitted in the p direction, since $\sigma_{11'}/\sigma_{7'} \to 0$. Analogous arguments were advanced recently by Eisenberg [10] on the basis of the polological model of single-particle exchange.

Notice should also be taken of the following relations between cross sections:

$$\sigma_{10}/\sigma_9 \to 0, \text{ if } \mu_\Sigma < \mu_\Lambda; \sigma_{10}/\sigma_9 \to \text{const, if } \mu_\Sigma > \mu_\Lambda;$$
$$\sigma_8/\sigma_9 \to \text{const always}; \quad \sigma_2/\sigma_1 \to \text{const always};$$
$$\sigma_4/\sigma_1 \to \text{const, if } \mu_N > \mu_\Delta; \quad \sigma_4/\sigma_1 \to 0 \quad \text{if } \mu_N < \mu_\Delta.$$

The same pertains to σ_3, σ_5, and σ_6.

The isotopic relations are in a certain sense quite general, since they do not depend on the details of the dynamics, and are merely expressions of the hypothesis that at large energies the amplitude is determined in the s channel by the states with definite isotopic spin in the u channel. At that it is not at all obligatory that one isolated Regge pole correspond to these states. The relations will be the same even if the structure of the singularities in the j plane is more complicated (for example, several poles or cuts). From this point of view, a check on the isotopic relations is of primary interest. An experimental confirmation of the fact that u-channel states with definite isotopic spin "are in operation" at high energies would be a serious argument in favor of stating that the amplitudes are determined at high energies by u-channel states with low energies, equal to $\sqrt{u}$, since it is known that at low energies there is no degeneracy in the isotopic spin. The same considerations pertain not only to the u channel but also to the t channel (boson states)[3)].

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3)The role of the isotopic spin in the cross channel was recently considered in [11].

1 G. Chew and S. Frautschi, Phys. Rev. Lett. **7**, 394 (1961).

2 Frautschi, Gell-Mann, and Zachariasen, Phys. Rev. **126**, 2204 (1962).

3 V. Gribov and I. Pomeranchuk, JETP **43**, 308 (1962), Soviet Phys. JETP **16**, 220 (1963).

4 V. Gribov, JETP **43**, 1529 (1962), Soviet Phys. JETP **16**, 1080 (1963).

5 T. Kinoshita, Fermion Regge Poles and the Asymptotic Behavior of Backward Meson-Nucleon Scattering, CERN, 62-33, preprint. V. Singh, Regge Poles in the πN-scattering and $\pi + \pi \to N + \bar{N}$, UCRL-10416, preprint.

6 V. Gribov and I. Pomeranchuk, JETP **43**, 1556 (1962), Soviet Phys. JETP **16**, 1098 (1963).

7 Kulakov, Lykhachev, Lubimov, Matulenko, Savin, and Stavinski, Proc. High Energy Physics Conf. (1962), p. 584. Bayukov, Leksin, Suchkov, Shebanov, and Shalamov, JETP **41**, 62 (1961). Soviet Phys. JETP **14**, 40 (1962). Jones, Lai, Perl, Ting, Cook, Cork, and Holley, Proc. High Energy Conf. (1962), p. 591. Bayukov, Birger, Leksin, Suchkov, Shalamov, and Shebanov, Proc. High Energy Phys. Conf. (1962), p. 588.

8 V. Gribov and I. Pomeranchuk, Phys. Rev. Lett. 8, 343 (1962).

9 M. Gell-Mann, Phys. Rev. Lett. 8, 263 (1962).

10 Y. Eisenberg, Phys. Rev. Lett. **10**, 60 (1963).

11 L. L. Foldy and R. F. Peierls, Phys. Rev. **130**, 1585 (1963).

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